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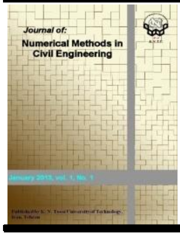

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3D finite element study of inclined soil-cement columns adjacent to shallow foundations on slopes

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Abstract:

The construction of shallow foundations adjacent to slopes is a complex geotechnical problem due to stress redistribution, reduced confinement, and development of plastic zones, which collectively reduce bearing capacity and slope stability. Ground improvement methods such as soil–cement columns installed by deep mixing or jet grouting have been widely adopted; however, most previous studies have focused on vertical columns, while the effects of inclined column installation on the coupled soil–foundation–slope system remain insufficiently investigated. This study evaluates the influence of soil–cement column inclination on the bearing capacity of a shallow foundation and the stability of an adjacent slope using three-dimensional numerical modeling in PLAXIS 3D. The nonlinear behavior of soil is simulated using the Hardening Soil model, and columns are modeled using embedded beam elements. Model validation is performed against available experimental data, showing good agreement in load–settlement response and failure mechanisms. A parametric analysis is conducted considering column inclination and spacing between columns. Results indicate that vertical columns maximize bearing capacity by improving load transfer to deeper soil layers, whereas inclined columns with an angle of approximately 45° provide the greatest improvement in slope stability by intersecting potential slip surfaces more effectively. Furthermore, as the column inclination angle relative to the vertical axis decreases, the bending moment in the foundation increases, highlighting the need for combined geotechnical and structural design considerations. An optimal configuration is proposed for balanced performance in bearing capacity and slope stability.

1. Introduction

Rapid urbanization, space limitations in developed areas, and the increasing demand for subsurface infrastructure have made construction near slopes and deep excavations a common geotechnical challenge in recent decades. In many civil engineering projects such as highways, railways, urban buildings, and underground structures, foundations are often constructed close to the crest of slopes. Under such conditions, soil behavior is strongly influenced by the presence of a free slope boundary, leading to a significant reduction in foundation bearing capacity compared to level ground conditions.

The presence of a slope alters the stress distribution pattern, reduces lateral confinement, and causes stress concentration near the slope crest. This phenomenon leads to the development of plastic zones, increased settlement, and, in critical cases, a combined failure mechanism in the soil–foundation–slope system. Moreover, foundation loading may adversely affect overall slope stability and increase the likelihood of sliding. Therefore, the coupled analysis of foundation bearing capacity and slope stability is of great importance.

To mitigate these issues, various ground improvement techniques have been developed, including piles, soil nailing, anchoring systems, retaining walls, and soil–cement columns. Among these methods, soil–cement columns have attracted considerable attention due to their ease of construction, cost-effectiveness, suitability for

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confined urban environments, and ability to simultaneously enhance soil strength and stiffness.

Soil–cement columns are typically constructed using deep mixing or jet grouting techniques, in which in-situ soil is mixed with cement slurry to form a column with higher strength and stiffness than the surrounding soil. These columns can carry part of the applied load, reduce settlement, and restrict potential failure surfaces. However, most previous studies have focused on vertical columns, while the behavior of inclined columns has received limited attention.

Inclined soil–cement columns can significantly alter load transfer mechanisms and stress distribution. While vertical columns primarily contribute to vertical load-bearing capacity, inclined columns may also act as stabilizing elements against sliding by intersecting potential failure surfaces. Therefore, investigating the effect of column inclination on the overall system performance is essential.

In recent years, three-dimensional numerical analysis has become one of the most powerful tools for investigating complex geotechnical problems. Three-dimensional finite element methods can more realistically simulate soil behavior, soil–structure interaction, and spatial stress distribution compared to two-dimensional models. Moreover, advanced constitutive models such as the Hardening Soil model allow for accurate representation of stress-dependent stiffness and stress-path dependency.

The main objective of this study is to investigate the effect of inclined soil–cement columns on the bearing capacity of shallow foundations and the stability of adjacent slopes using three-dimensional numerical analysis. The effects of column inclination and spacing on system behavior are evaluated, and an optimal column arrangement is proposed in terms of geotechnical and structural performance.

2. Literature Review

Slope stability and the bearing capacity of foundations located near slopes are among the classical yet complex topics in geotechnical engineering. Over the past decades, numerous researchers have investigated failure mechanisms and ground improvement techniques for such systems.

Early studies were primarily based on limit equilibrium methods. Although these approaches provide useful estimates of safety factors, they are unable to accurately capture stress distribution, deformation behavior, and complex soil–reinforcement interactions. With the development of finite element methods, more accurate nonlinear analysis of soil behavior has become possible.

Cheuk et al. (2013) [1] investigated the use of jet grouting for stabilizing deep excavations and showed that soil–cement columns significantly reduce shear strain concentration. Their field results demonstrated that increasing the stiffness of the improved zone effectively controls lateral deformations. Huang (2019) [2] showed that foundation boundary conditions and rotational freedom significantly influence the bearing capacity near slopes. His results indicated that failure mechanisms near slopes differ fundamentally from those in level ground, and classical bearing capacity equations may lead to significant errors.

In numerical modeling studies, Coldwell et al. (2020) [3] investigated the probabilistic behavior of soil–cement columns and showed that spatial variability of material properties can significantly affect safety factors. Their work highlighted the importance of three-dimensional and stochastic analyses. Severino et al. (2022) [4] studied the application of jet grouting in the protection of historical structures and demonstrated that soil–cement columns are effective not only in improving slope stability but also in controlling differential settlements.

Phutthananon et al. (2023) [5] provided a comprehensive comparison and argued that two-dimensional approaches cannot capture arching effects and stress concentration ratios between stiff columns and surrounding soil. In the field of dynamic behavior, Mehdizadeh et al. (2024) [6] and Ramezani et al. (2025) [7] demonstrated the effectiveness of deep mixing columns in reducing liquefaction potential and seismic displacements.

Vahedi et al. (2022) [8] conducted a laboratory study at Fasa University using 1:25 scale physical models and showed that increasing the number and depth of soil–cement columns can increase the bearing capacity of foundations near slopes by up to three times while reducing lateral displacement. These results provide a basis for validating the numerical model used in the present study.

Borazjani (2021) [9] investigated the numerical behavior of deep mixing columns under foundations located on slopes. Their results showed that increasing the distance between the foundation and the slope crest increases bearing capacity, and the presence of soil–cement columns accelerates this improvement.

2.1. Research Gap and Objectives

Despite significant advancements, a clear research gap remains regarding the static behavior of battered (inclined) soil–cement columns for foundations adjacent to slopes. Moreover, most existing static studies employ the Mohr–Coulomb model, which is incapable of capturing stress-dependent stiffness, leading to inaccuracies in predicting pre-failure deformations. In addition, systematic optimization of column inclination in a three-dimensional finite element framework has received limited attention.

To address these limitations, the present study investigates the stability of a strip foundation adjacent to a slope reinforced with inclined soil–cement columns using PLAXIS 3D and the Hardening Soil constitutive model.

The main objectives are: (1) validation of the numerical model against experimental data, (2) evaluation of the effects of column inclination angle and spacing on bearing capacity, slope stability, and foundation bending moment, and (3) development of a practical design guide for selecting the optimum column configuration.

3. Numerical Modeling and Research Methodology

3.1. Introduction

The analysis of shallow foundation behavior in the vicinity of slopes is a complex geotechnical problem, since the system response arises from the simultaneous interaction among the soil, the foundation, the slope, and the ground improvement elements. In such problems, soil behavior is highly nonlinear, and the stress distribution and deformation field are significantly influenced by the presence of a free slope boundary. In addition, the installation of soil–cement columns creates a zone with stiffness and strength properties different from those of the surrounding soil, further increasing the complexity of the system behavior.

In recent years, three-dimensional numerical analysis has become one of the primary tools for investigating such complex problems. Three-dimensional modeling enables a more realistic simulation of the spatial interaction between columns, foundations, and soil, and provides a more accurate representation of system behavior compared to two-dimensional approaches. In this study, a series of three-dimensional numerical analyses were performed using PLAXIS 3D to investigate the effect of the inclination angle and spacing of soil–cement columns on the bearing capacity of the foundation and the stability of the slope. The main focus of the modeling was on the group behavior of columns and the analysis of load transfer mechanisms and slope stabilization performance.

3.2. Numerical Model Geometry

The numerical model consisted of a soil slope with a height of 12 m and a slope angle of 35° with respect to the horizontal. These values were selected based on typical conditions encountered in civil engineering projects, where foundations are located near natural slopes or excavations, and where there is a risk of reduced bearing capacity and slope instability.

A strip shallow foundation with a width of 2 m and a length of 10 m was placed near the slope crest at a distance equal to one footing width (1B) from the edge of the crest. This distance ratio was selected based on the literature as a configuration in which the most significant foundation–slope interaction effects occur. In this configuration, the soil–cement columns are located approximately 0.8 m from the slope crest (Figure 1). Under such conditions, the stress zone induced by the foundation load overlaps with the potential slope failure zone, resulting in a significant interaction between bearing capacity and slope stability.

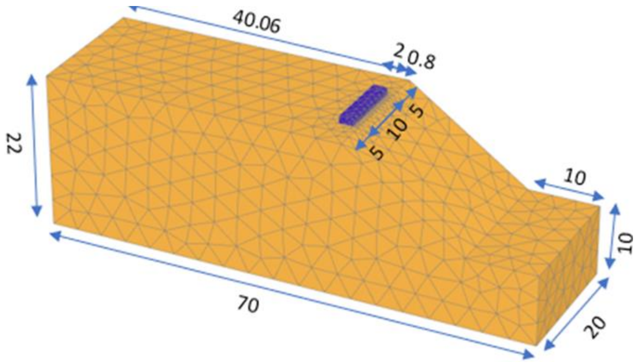

Fig. 1: Geometry of the numerical model

The overall dimensions of the model were selected to minimize boundary effects on the analysis results. To this end:

• The lateral boundaries were placed at a distance several times the footing width from the foundation;

• The model depth was chosen such that the stress influence zone induced by loading did not reach the bottom boundary.

If the model domain is too small, artificial reflection of stresses at the boundaries may lead to an apparent increase in system stiffness and errors in bearing capacity estimation. Therefore, the selection of appropriate model dimensions was critical for ensuring the accuracy of the results.

The soil–cement columns were installed as the main ground improvement element beneath and around the foundation. The purpose of the columns was to increase the stiffness of the foundation bed, transfer part of the load to deeper soil layers, and enhance slope stability. In all models, the column length was taken as 12 m. This length was selected such that the columns could extend beyond the critical failure zone and into the stable soil mass. If shorter columns were used, they would not be able to effectively contribute to resisting the potential slip surface. The column diameter was kept constant in all analyses to allow an independent assessment of the effects of column inclination angle and spacing.

To investigate the effect of column inclination, four different angles with respect to the vertical axis were considered: 0°, 30°, 45° and 60°. An angle of 0° represents vertical columns, while the other angles correspond to inclined columns.

In addition, column spacing was modeled using three values: 1 m, 1.5 m, and 2 m. These spacings were selected to evaluate the influence of column density on the group behavior of the system. Smaller spacing increases the stiffness of the improved zone but may also intensify interaction effects between adjacent columns.

The configuration naming system is defined as Tα-β-S, where α denotes the column length in meters, β represents the inclination angle measured from the vertical axis (0° = vertical), and S is the spacing between columns in meters. For example, T12-45-1.5 represents a column with a length of 12 m, an inclination angle of 45°, and a spacing of 1.5 m from adjacent columns (Figure 2).

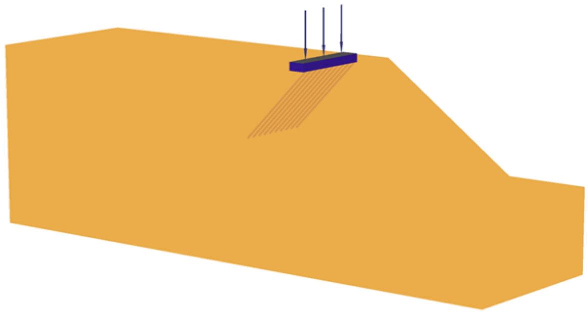

Fig. 2: Position of soil-cement columns inclined at 45° in numerical model

3.3. Soil Constitutive Model

The behavior of soil in bearing capacity and slope stability problems is highly nonlinear, and soil stiffness is a function of the effective stress level and the stress path. In many previous studies, the Mohr–Coulomb constitutive model has been widely used; however, due to its assumption of constant stiffness and linear elastic behavior up to failure, it cannot accurately capture the real mechanical response of soils. In the present study, the Hardening Soil model was employed. This model is considered one of the advanced constitutive models implemented in the PLAXIS software and is capable of realistically simulating the nonlinear stress–strain behavior of soils.

The main advantages of the Hardening Soil model include:

- Stress-dependent soil stiffness
- Simulation of nonlinear pre-failure behavior
- Distinction between loading and unloading stiffness
- Improved prediction of foundation settlement
- More realistic representation of soil deformation

In this model, soil stiffness increases with increasing stress level; therefore, the response of soil at different depths can be simulated with higher accuracy. The model is defined by three reference stiffness parameters: secant stiffness in triaxial loading (E_{50}^{ref}), unloading/reloading stiffness (E_{ur}^{ref}), and oedometer stiffness (E_{oed}^{ref}). Table 1 presents the complete soil properties of the slope. The strength and deformation parameters of the soil were derived from laboratory test results conducted as part of the thesis.

Table 1: Soil properties adopted for the slope in numerical modeling

Property	Value	Unit
Unsaturated unit weight (γ_{unsat})	18.5	kN/m³
Saturated unit weight (γ_{sat})	19.5	kN/m³
Secant stiffness (E_{50}^{ref})	32	MPa
Oedometer stiffness (E_{oed}^{ref})	32	MPa
Unloading–reloading stiffness (E_{ur}^{ref})	96	MPa
Effective cohesion (c′)	≈ 0	kN/m²
Effective friction angle (φ′)	35	degrees
Dilatancy angle (ψ)	5	degrees
Poisson's ratio (ν)	0.28	-
Drainage condition	Drained	-

3.3.1. Verification of Hardening Soil Parameters Using Triaxial Test Simulation

The constitutive parameters adopted for the Hardening Soil (HS) model [10] were derived from a comprehensive geotechnical investigation conducted for the study area. The laboratory characterization included standard index tests together with drained triaxial compression tests, from which the effective strength parameters and the stiffness parameters required for the Hardening Soil (HS) constitutive model were derived. The effective shear strength parameters (c′ and φ′) were determined directly from the laboratory tests. The reference secant stiffness (E_{50}^{ref}) was obtained from the drained triaxial test results, while the remaining stiffness parameters (E_{oed}^{ref} and E_{ur}^{ref}) were determined according to the standard relationships commonly adopted for the Hardening Soil constitutive model.

To verify the capability of the selected constitutive parameters to reproduce the actual mechanical behavior of the soil, numerical simulations of the drained triaxial compression tests were carried out using the PLAXIS SoilTest utility. The simulations were performed under effective confining pressures of 50, 100, and 200 kPa, corresponding to the laboratory testing conditions.

Figure 3 compares the experimental and numerical deviator stress–axial strain responses. The comparison indicates a close agreement between the laboratory measurements and the numerical predictions throughout the loading process. The HS model successfully reproduces the nonlinear stress–strain response, the gradual hardening behavior, and the increase in peak deviator stress with increasing confining pressure. In addition, the predicted failure stresses are consistent with the theoretical Mohr–Coulomb failure criterion for the adopted strength parameters (φ′=35°, c′≈0). This constitutive-level verification complements the validation of the overall numerical model against the physical model tests presented in Section 4 and enhances the reliability of the subsequent three-dimensional parametric analyses.

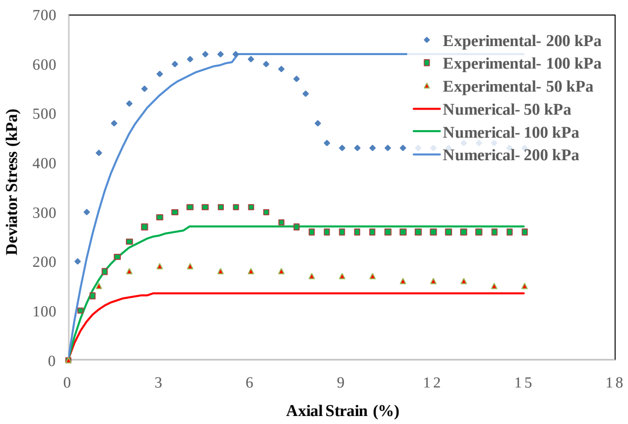

Fig. 3: Comparison between experimental and simulated drained triaxial compression responses under effective confining pressures of 50, 100, and 200 kPa

3.4. Column Modeling

The shallow reinforced concrete foundation and the soil-cement columns were both simulated using a linear elastic constitutive model. The properties used in the numerical model are summarized in Table 2. The columns were simulated as embedded beam elements with a Mohr–Coulomb interface at the soil–column contact surface. In the embedded beam approach, the column is modeled as an element embedded within the soil continuum, and force transfer between the soil and the column is governed by interaction relationships. Compared to full 3D solid modeling of columns, this approach significantly reduces computational effort while still providing reliable accuracy for simulating the group behavior of columns.

The use of these elements enables accurate simulation of the axial behavior of the columns as well as the soil–structure interaction. The elastic modulus of the columns was assumed to be 200 MPa, which is several times higher than that of the surrounding soil, in order to account for the stiffness increase induced by cement mixing in the model. This stiffness increase leads to improved stress transfer and reduced localized deformations beneath the foundation

Table 2: Material properties of shallow foundation and soil-cement columns in numerical modeling

	Property	Value	Unit
Shallow Foundation	Unit weight	24	kN/m³
	Elastic modulus (E)	28	GPa
	Poisson's ratio (ν)	0.28	-
Soil-Cement Columns	Unit weight	19	kN/m³
	Elastic modulus (E)	200	MPa

3.5. Boundary Conditions

To eliminate the influence of model boundaries on the analysis results, the dimensions of the numerical domain were chosen sufficiently large. At the lateral boundaries, horizontal displacements were restrained while vertical movements were allowed. At the bottom boundary, all translational degrees of freedom were fixed. The ground surface was considered free, allowing natural soil deformation.

3.6. Mesh Generation

A three-dimensional finite element mesh was used to discretize the model domain. To improve the accuracy of the results in critical regions, a finer mesh was applied around the foundation and the soil–cement columns, while a coarser mesh was used in regions farther from these elements. Mesh sensitivity analysis indicated that the results are independent of the selected element size.

3.7. Analysis Stages

The numerical analysis was conducted in four stages:

Stage 1: In-Situ Stress Generation

Initial in-situ stresses were generated in the soil mass. The in-situ stress state plays a crucial role in governing the subsequent mechanical response of the system; therefore, its accurate initialization is essential for achieving realistic numerical results.

Stage 2: Column Activation

Following the attainment of initial equilibrium, the soil–cement columns were activated in the numerical model. This stage represents the ground improvement operations carried out prior to foundation construction.

Stage 3: Foundation Loading

The structural loading was applied incrementally to the foundation to capture the complete response of the soil–structure system, ranging from the pre-yield (elastic) regime to ultimate failure. This step allows the development of both stiffness degradation and progressive failure mechanisms in the soil mass.

Stage 4: Slope Stability Analysis

The factor of safety was evaluated using the shear strength reduction (SSR) technique. In this approach, the shear strength parameters of the soil are progressively reduced until a global failure mechanism is triggered in the numerical model.

For slope stability assessment, the Phi–c reduction procedure implemented in PLAXIS 3D was adopted. In this method, cohesion (c) and the tangent of the internal friction angle (tan φ) are systematically reduced until the system reaches a limiting equilibrium state. The factor of safety is defined as the reduction multiplier at failure, representing the ratio between the available shear strength and the reduced strength at collapse. The ultimate bearing capacity of the foundation was derived from the intersection of the initial tangent stiffness and the post-peak asymptote of the plastic load–settlement response.

4. Verification of Numerical Model

To verify the accuracy and robustness of the developed numerical model, a validation procedure was conducted by comparing the simulation results with the physical model tests reported by Vahedi et al. (2022) [8]. In those experimental investigations, the coupled response of a sloping ground, a shallow foundation, and soil–cement columns under controlled loading conditions was examined. Key response parameters, including ultimate bearing capacity, foundation settlement patterns, and failure mechanism development, were systematically recorded.

Uniformly graded SP sand was employed as the foundation soil. The model footing consisted of a rigid steel plate with dimensions 50 × 10 × 3 cm, and the soil–cement columns were simulated using polyethylene pipes with equivalent stiffness properties representative of prototype columns.

For validation purposes, the experimental conditions—including geometry, material constitutive properties, footing dimensions, column layout, and loading scheme—were rigorously replicated within the PLAXIS 3D finite element framework. Subsequently, numerical simulations were performed under identical boundary and loading conditions, and the results were compared against the experimental observations.

Figure 4 illustrates the physical model used by Vahedi et al. (2022) [8], while Figure 5 presents the geometric configuration of the numerical validation model. Four distinct cases were considered: (1) unreinforced soil without soil–cement columns, (2) a single column adjacent to the footing, (3) three columns adjacent to the footing, and (4) five columns adjacent to the footing.

Fig. 4: Physical model of Vahedi et al. (2022) [8]

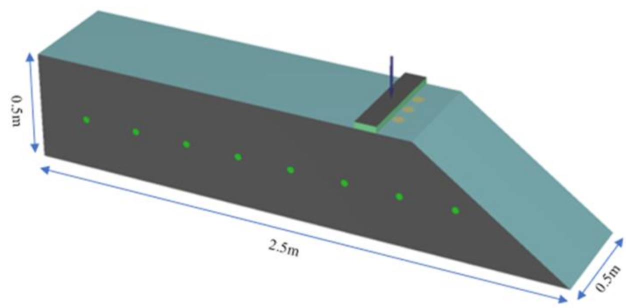

Fig. 5: Geometry of numerical models for verification against Vahedi et al. (2022)

The numerical analyses were performed using the parameters presented in Section 3.3 to generate the experimental load–settlement response. Figure 6 compares the stress–settlement responses obtained from the numerical simulations with the experimental results of Vahedi et al. (2022) [8]. Regarding the presentation of Figure 6, only a standard smoothing procedure was applied to remove minor high-frequency measurement noise from the data acquisition system and to improve graphical readability. No experimental data points or mechanical response characteristics were altered or selectively removed.

The comparison in Figure 6 demonstrates that the numerical model accurately reproduces the overall response trend, including the initial stiffness (elastic response), ultimate bearing capacity, and development of the plastic deformation zone. Moreover, the observed trends are consistent with the findings of Acharyya and Dey (2021) [11], who investigated the influence of key geotechnical parameters on the bearing capacity of strip footings on sloping ground.

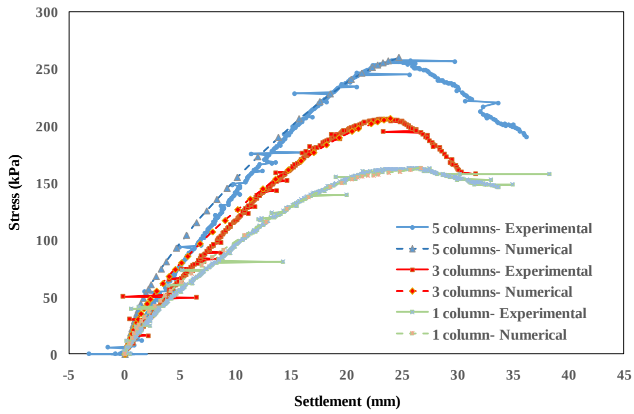

Fig. 6: Comparison of numerical and experimental stress–settlement curves for improved configurations (Vahedi et al., 2022 [8]).

Based on the performed comparisons, it can be concluded that the numerical model adopted in this study is capable of simulating the real behavior of the soil–foundation–soil–cement column system with satisfactory accuracy. Therefore, the use of this model for conducting parametric analyses and investigating the effects of the inclination angle and positioning of soil–cement columns on slope stability and foundation bearing capacity is considered reliable.

5. Results of the Parametric Study

5.1. Effect of Column Inclination Angle

The results of the parametric analyses demonstrate that the inclination angle of soil–cement columns plays a critical role in governing the behavior of the reinforced foundation–slope system. To provide a benchmark for evaluating the effectiveness of the inclusion of the soil-cement columns, the response of the unreinforced slope is included together with the reinforced cases. Figure 7 compares the stress–settlement responses of the foundation for the unreinforced slope and the reinforced models with soil–cement columns installed at different inclination angles and a spacing of 1 m. The results indicate that the unreinforced slope exhibits the lowest ultimate bearing capacity, whereas the installation of soil–cement columns significantly enhances the load-carrying performance of the foundation. This improvement is attributed to the increased stiffness of the reinforced ground and the transfer of stresses to deeper and more competent soil layers through the soil–cement columns.

As seen in Figure 7, as the column inclination relative to the vertical axis increases, the ultimate bearing capacity decreases noticeably. This reduction can be explained by the load-transfer mechanism within the columns. In the vertical configuration, the applied foundation load is aligned with the column axis, resulting in efficient axial load transfer, which represents the primary load-carrying mechanism of soil–cement columns. As the columns become inclined, the applied load is resolved into axial and transverse components. Consequently, a larger proportion of the load is carried through bending and shear, reducing the effectiveness of the reinforcement system. In addition, inclined columns introduce load eccentricity beneath the foundation, leading to increased settlements, higher internal forces, and greater structural demand.

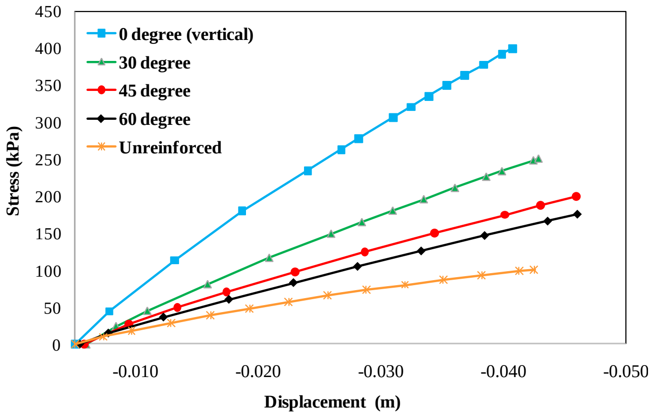

Fig. 7: Stress–settlement curves for soil-cement columns at 1.0 m spacing with various inclination angles

Figure 8 shows the ultimate bearing capacities extracted from the load–settlement curves for all analyzed cases. The ultimate bearing capacity was determined from the load–settlement curves using the tangent intersection method, which is commonly employed for interpreting foundation load–settlement behaviour when a distinct peak load is not observed [12,13]. In this method, two tangential lines are drawn to the load–settlement curve: the first tangent represents the initial response of the foundation, while the second is fitted to the plastic portion of the curve. The intersection of these two tangents marks the transition from predominantly elastic to plastic behaviour. A horizontal line projected from this intersection onto the load axis defines the corresponding ultimate bearing capacity adopted in this study. The same interpretation procedure was consistently applied to all numerical analyses to ensure an objective comparison of the different reinforcement configurations.

As shown in Figure 8, compared with the unreinforced slope, each reinforced configuration results in a noticeable increase in bearing capacity, demonstrating the beneficial contribution of the soil–cement columns. The comparison further indicates that for all spacing configurations, the vertical arrangement (0°) consistently provides the highest bearing capacity. For example, at a column spacing of 1 m, the ultimate bearing capacities for inclination angles of 30°, 45°, and 60° are 135 kPa, 90 kPa, and 83 kPa, respectively, compared with 250 kPa for the vertical configuration, corresponding to reductions of approximately 46%, 64%, and 67%.

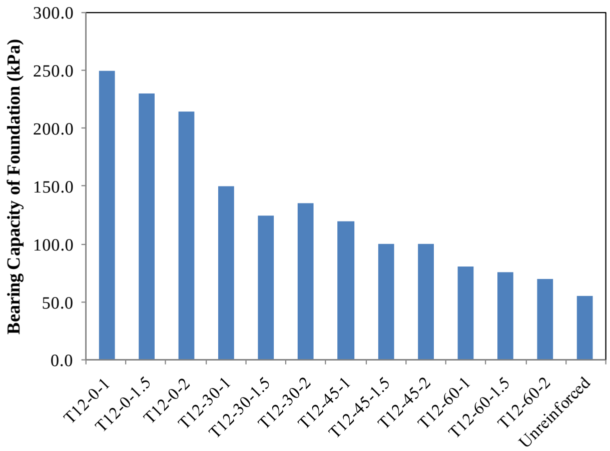

Fig. 8: Ultimate bearing capacity of the foundation for different column spacings and inclination angles

Figure 9 compares the factor of safety obtained for the unreinforced slope and all reinforced configurations with different column spacings and inclination angles. As expected, the unreinforced slope possesses the lowest safety factor, indicating the least stable condition. The introduction of soil–cement columns consistently increases the global factor of safety, demonstrating the beneficial effect of ground improvement on slope stability. Furthermore, Figure 9 shows that the highest safety factor is achieved when the columns are inclined at 45°, despite this configuration exhibiting a lower bearing capacity than the vertical arrangement. This apparent discrepancy reflects the existence of two distinct governing mechanisms. While bearing capacity is primarily controlled by the efficiency of vertical load transfer into deeper soil layers, slope stability depends mainly on the ability of the reinforcement system to intercept and restrain the critical slip surface.

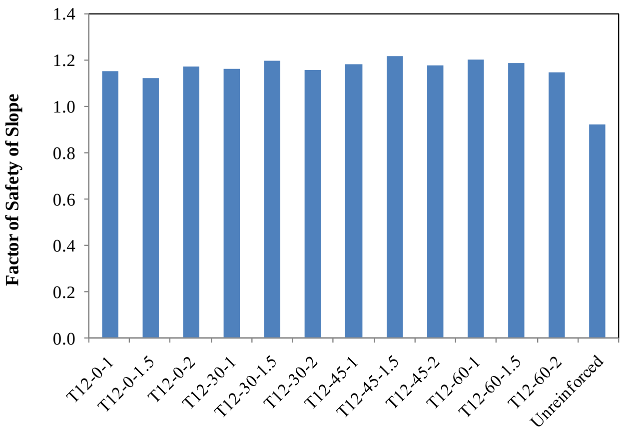

Fig. 9: Factors of safety obtained from slope stability analyses for different column spacings and inclination angles

The superior performance of the 45° configuration can be attributed to its geometric compatibility with the potential failure mechanism. Columns inclined at approximately 45° intersect the critical slip surface at a favorable angle, mobilizing larger axial resistance across the failure plane and providing more effective restraint against sliding. Furthermore, deformation is distributed more uniformly throughout the slope mass, indicating a more efficient stabilization mechanism. Based on the obtained results, inclination angles between 45° and 60° appear to represent the optimum range for maximizing slope stability.

To provide further insight into the predicted failure mechanism, the distributions of plastic points obtained from the ϕ–c reduction analyses are presented for both the unreinforced slope and the reinforced slope with vertical soil–cement columns installed at a spacing of 1 m, as shown in Figures 10 and 11, respectively. In the strength reduction method implemented in PLAXIS, failure develops progressively as the effective shear strength parameters are reduced until numerical instability is reached. Consequently, the concentration of plastic points provides a reliable indication of the location and extent of the critical failure mechanism.

Comparing Figures 10 and 11 demonstrates the beneficial effect of the soil–cement columns. In the unreinforced slope, the plastic zone develops continuously from beneath the footing toward the slope face, forming a well-defined failure mechanism. In contrast, the reinforced model exhibits a more confined plastic zone, indicating that the soil–cement columns effectively restrain the propagation of plastic deformation and modify the geometry of the critical slip surface. Consequently, the failure mechanism becomes less extensive, leading to an increase in the ultimate bearing capacity of the foundation and the global factor of safety.

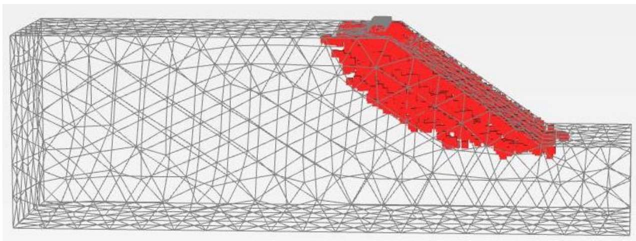

Fig. 10: Distribution of plastic points at failure for the unreinforced slope obtained from the ϕ–c reduction analysis

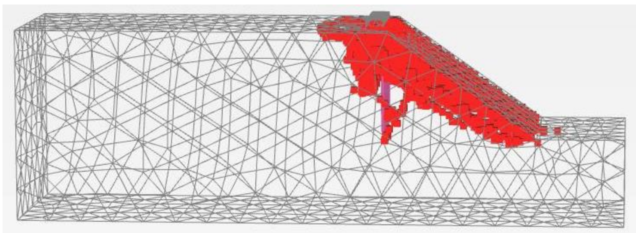

Fig. 11: Distribution of plastic points at failure for the reinforced slope with vertical soil–cement columns installed at 1 m spacing, obtained from the ϕ–c reduction analysis

Figure 12 illustrates the variation of foundation bending moment with column inclination for a spacing of 1 m.

Increasing the deviation from the vertical direction leads to a reduction in foundation bending moment. This behavior is attributed to the diminished load-transfer efficiency of inclined columns, which results in a more uniform stress distribution beneath the foundation and reduces the concentration of support reactions. Consequently, bending demands within the foundation decrease. From a structural design perspective, although vertical column arrangements provide superior load-bearing performance, they also induce larger bending moments and therefore require a higher level of flexural reinforcement in the foundation.

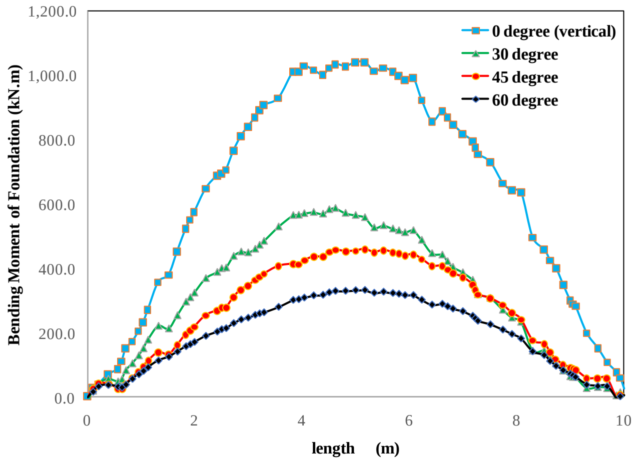

Fig. 12: Variation of foundation bending moment for different column inclinations with a spacing of 1 m.

5.2. Effect of Column Spacing

Column spacing is one of the key parameters governing the group behavior and overall effectiveness of soil–cement column reinforcement systems. Figure 13 shows the stress–settlement curves for vertical columns (0° inclination) with different spacings. The results clearly indicate that reducing the spacing between columns enhances the bearing capacity of the reinforced foundation. Among the investigated configurations, the highest bearing capacity was achieved for a column spacing of 1 m. A similar trend was consistently observed for the other inclination angles, as illustrated in Figure 8.

The observed improvement in bearing capacity with decreasing column spacing can primarily be attributed to the arching effect and the enhanced interaction between adjacent columns. At smaller spacings, the stress influence zones of adjacent columns overlap significantly, forming a relatively continuous stiffened zone beneath the foundation. Consequently, a larger proportion of the applied load is transferred to the improved ground, resulting in increased stiffness and bearing capacity. As the spacing increases, this overlap gradually diminishes, leaving larger untreated soil zones between columns and reducing the effectiveness of load transfer. It should be noted, however, that excessively small spacings may lead to stress interaction and overlapping failure mechanisms between adjacent columns, potentially limiting the efficiency of further densification.

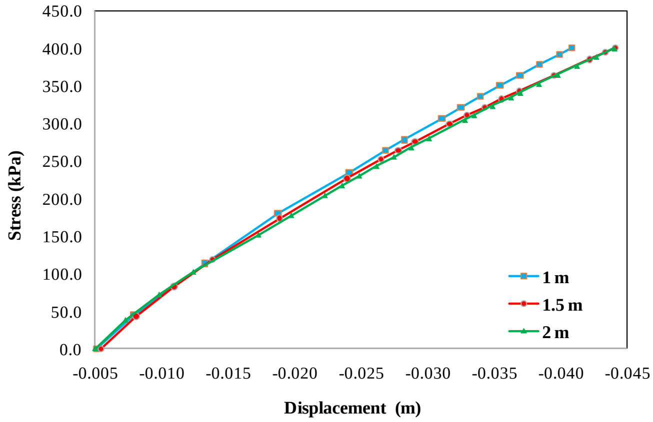

Fig. 13: Stress–settlement curves for vertical (0°) soil–cement columns at different spacings

The influence of column spacing on the structural response of the foundation is illustrated in Figure 14, which shows the bending moments in the foundation for vertical column arrangements. Interestingly, the maximum bending moment is obtained for a spacing of 1.5 m rather than for 1 m or 2 m. The maximum bending moments are approximately 1000 kN.m, 1220 kN.m, and 740 kN.m for spacings of 1 m, 1.5 m, and 2 m, respectively. Figure 15 further demonstrates that this trend persists across all investigated column inclination angles.

The observed behavior can be explained by the distribution of support reactions beneath the foundation. When the spacing is limited to 1 m, the substantial overlap of stress bulbs generated by adjacent columns promotes a relatively uniform distribution of contact stresses, thereby reducing localized reaction concentrations and associated bending moments. Conversely, at a spacing of 2 m, the smaller number of supporting columns causes the foundation to behave more like a simply supported beam with relatively distributed reactions. At the intermediate spacing of 1.5 m, however, the column locations coincide with positions that generate peak reaction forces near the critical sections of the foundation, producing the maximum bending moment.

From an engineering design perspective, this finding has important practical implications. Foundations supported by column groups with an intermediate spacing of approximately 1.5 m may require greater flexural reinforcement despite exhibiting acceptable bearing performance. Furthermore, increasing the spacing between columns increases the axial load carried by each column individually because fewer columns participate in load sharing. Therefore, both the structural capacity of the columns and the flexural design of the foundation should be evaluated simultaneously to achieve an efficient and economical reinforcement layout.

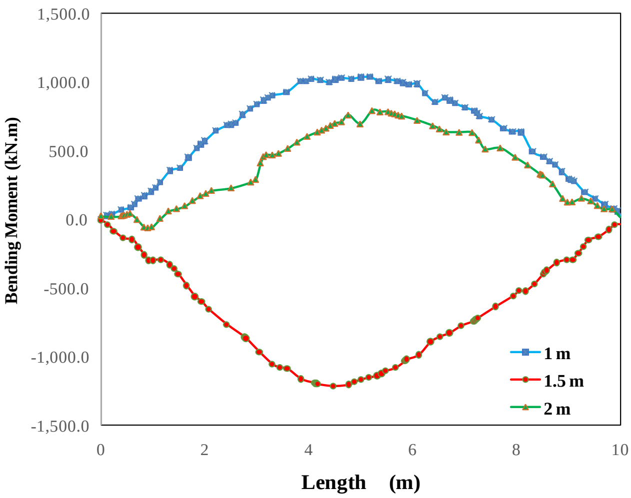

Fig. 14: Variation of foundation bending moment with column spacing for vertical (0°) soil-cement columns

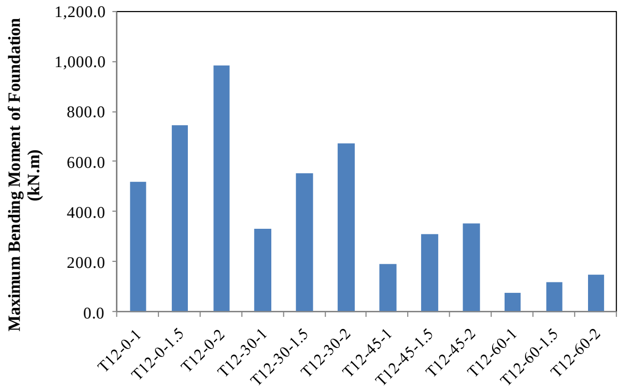

Fig. 15: Maximum foundation bending moment for different combinations of column spacing and inclination angle

6. Comparison of Embedded Beam and Solid-Element Modeling Approaches

The soil–cement columns were modeled using the embedded beam row formulation throughout this study. The embedded beam row element has been widely adopted for modelling slender embedded inclusions in geotechnical engineering because it provides an efficient representation of soil–structure interaction without requiring explicit three-dimensional discretization of each inclusion. Previous studies have shown that this approach significantly reduces the number of finite elements and computational time while maintaining satisfactory accuracy for global response predictions [14-16], making it particularly suitable for the numerous parametric analyses performed in this study involving different column layouts, spacings, and inclination angles.

Since embedded beam elements idealize the columns as one-dimensional structural members, an additional verification study was conducted to evaluate the influence of this modeling assumption on the predicted global response. A companion numerical model was therefore developed in which the soil–cement columns were explicitly represented using three-dimensional solid tetrahedral elements. Except for the column representation, all modeling conditions, including geometry, material properties, constitutive models, boundary conditions, mesh density, and loading, were kept identical to those used in the embedded beam model.

Figures 16-18 present the comparison between the embedded beam and solid-element models for a representative reference case, in which the soil–cement columns are inclined at 30° with respect to the vertical axis and installed at a spacing of 2 m. This configuration was selected solely as a representative example for comparison purposes. Figure 16 illustrates the finite element model discretized using solid tetrahedral elements. Figure 17 compares the stress–settlement responses, and Figure 18 presents the corresponding global factors of safety calculated using the strength reduction method. The close agreement observed in these comparisons further demonstrates that the embedded beam formulation is capable of reproducing the global response of the system with satisfactory accuracy while offering substantially higher computational efficiency for extensive parametric analyses.

It should also be emphasized that the embedded beam formulation was employed to represent the stiffness interaction between the soil–cement columns and the surrounding soil rather than to evaluate the structural flexural capacity of the columns.

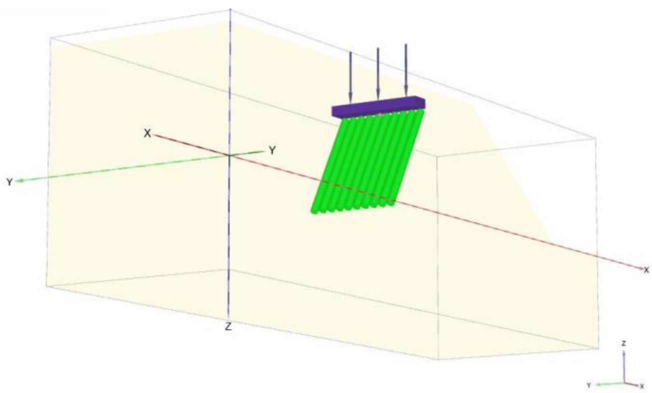

Fig. 16: Three-dimensional solid-element model used for verification of the embedded beam approach (soil-cement columns with 30° inclination and 2 m spacing)

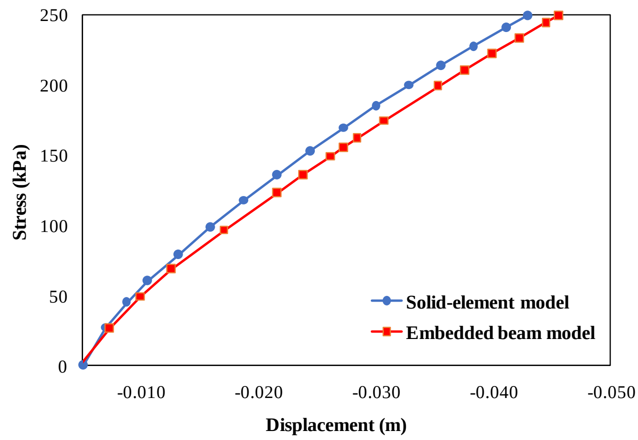

Fig. 17: Comparison of Stress-settlement curves predicted by the embedded beam and solid-element models (soil-cement columns with 30° inclination and 2 m spacing)

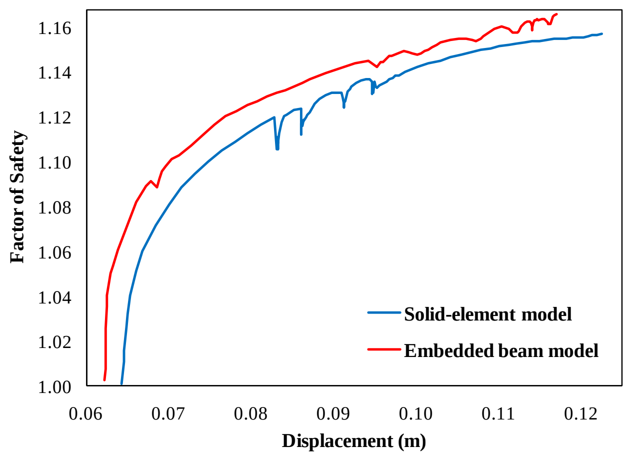

Fig. 18: Comparison of the slope safety factor obtained from the embedded beam and solid-element models (soil-cement columns with 30° inclination and 2 m spacing)

7. Mechanistic Interpretation

7.1. Fundamental Design Trade-off

The introduction of soil–cement columns fundamentally alters the stress-transfer mechanism within the soil–foundation–slope system. In the untreated condition, the majority of the foundation load is transmitted directly through the natural soil beneath the footing. For foundations located near a slope crest, this load path causes stress concentration within a region where the confining pressure is already reduced by the presence of the free surface, thereby promoting the development of plastic zones and reducing both bearing capacity and slope stability.

The inclusion of soil–cement columns modifies this mechanism by providing relatively stiff load-transfer elements that redirect a significant portion of the applied load toward deeper and more competent soil layers. Consequently, stresses acting within the critical zone adjacent to the slope crest are reduced, leading to improved system performance.

A key outcome of the present study is the identification of a fundamental design trade-off between maximizing foundation bearing capacity and maximizing slope stability. These objectives are controlled by different physical mechanisms and therefore require different reinforcement strategies.

Maximizing bearing capacity requires efficient mobilization of the axial stiffness of the soil–cement columns. This condition is best achieved when the columns are installed vertically with relatively small spacing, allowing the applied foundation load to be transferred almost entirely as axial compression. Under these conditions, bending effects are minimized, and the columns operate close to their most efficient load-carrying mode.

Conversely, improving slope stability depends primarily on the ability of the columns to intercept and restrain the potential failure mechanism. Inclined columns installed at approximately 45° intersect the critical slip surface more effectively and develop larger resisting forces normal to the sliding direction. As a result, the highest factors of safety are obtained for intermediate inclination angles rather than for vertical columns.

These findings demonstrate that no single column arrangement can simultaneously optimize all performance criteria. Instead, the optimum configuration should be selected according to the dominant design objective, whether it is settlement control, bearing capacity enhancement, or slope stabilization.

7.2. Three-Dimensional Arching Effect and Column Group Behavior

The influence of column spacing can be interpreted in terms of the three-dimensional arching effect and stress redistribution within the reinforced ground.

When adjacent columns are installed at relatively small spacings, their individual stress bulbs overlap, forming a continuous stiffened zone beneath the foundation. This interaction causes a substantial proportion of the vertical stresses carried by the intervening soil to be transferred to the stiffer soil–cement columns, thereby increasing the stress concentration ratio and enhancing the overall stiffness of the improved ground. Consequently, the bearing capacity of the reinforced foundation increases significantly.

As the spacing between columns increases, the overlap between adjacent stress fields gradually diminishes. Larger volumes of untreated soil remain between neighboring columns, reducing the efficiency of stress transfer and weakening the arching mechanism. Under these conditions, each column must resist a greater proportion of the applied load individually, explaining the observed increase in axial force with increasing spacing.

These findings are consistent with the conclusions reported by Phutthananon et al. (2023) [5], who argued that conventional two-dimensional models cannot adequately capture the spatial development of arching or the stress concentration occurring between stiff inclusions and the surrounding soil. The numerical results obtained herein clearly demonstrate that these inherently three-dimensional mechanisms play a governing role in the performance of soil–cement column groups.

8. Design Recommendations

Based on the results of the three-dimensional parametric analyses, practical design recommendations are proposed for three representative engineering scenarios. Since the governing mechanisms of bearing capacity enhancement and slope stabilization are fundamentally different, the optimum column configuration depends on the primary design objective.

Scenario 1– Bearing Capacity Optimization (Settlement-Controlled Structures)

For structures in which settlement control and maximum bearing capacity are the governing design criteria, vertical soil–cement columns with a spacing of 1 m (T12-0-1) are recommended. In this configuration, the applied foundation load is transferred almost entirely through axial compression within the columns, maximizing their load-carrying efficiency and minimizing bending demands. The close spacing also enhances the three-dimensional arching effect, resulting in a stiff and continuous reinforced zone beneath the foundation. Accordingly, this arrangement is particularly suitable for structures requiring strict settlement control, even though slope stability may not be maximized.

Scenario 2 – Slope Stability Optimization (Marginally Stable Slopes)

When slope stabilization is the primary design objective, soil–cement columns inclined at 45° with a spacing of 1.5 m (T12-45-1.5) provide the most effective solution. The inclined geometry enables the columns to intersect the critical slip surface more efficiently, thereby mobilizing greater axial resistance against potential sliding and producing the highest factor of safety. Nevertheless, this configuration is associated with a lower foundation bearing capacity compared with the vertical arrangement. Consequently, its application is most appropriate for projects in which global slope stability governs the design.

Scenario 3 – Balanced Technical and Economic Design (Recommended Configuration)

For most practical applications, vertical columns with a spacing of 2 m (T12-0-2) offer the most balanced compromise between geotechnical performance and construction economy. This configuration maintains a relatively high bearing capacity while providing an acceptable level of slope stability. Moreover, compared with the 1 m spacing arrangement, the approximately 50% reduction in the number of columns substantially lowers construction costs while preserving satisfactory overall performance. Therefore, this configuration is recommended as the preferred design option for general engineering practice.

The following practical considerations should also be incorporated into the design process:

• The column length should be approximately equal to the slope height (12 m in the present study) to ensure penetration through the critical failure zone and adequate anchorage within the stable underlying soil.

• Column spacing is the primary parameter controlling group interaction and the development of the three-dimensional arching effect and should therefore be optimized carefully.

• The assumed Young’s modulus of the soil–cement columns (200 MPa) should be verified through laboratory testing or field quality-control measurements corresponding to the adopted soil–cement mixture.

• Special attention should be given to the structural design of the foundation for intermediate column spacings (approximately 1.5 m), where localized stress concentrations may generate the maximum bending moments.

9. Limitations and Engineering Applicability

The results presented in this study are based on three-dimensional numerical analyses performed for the specific soil profile, foundation geometry, and soil–cement column configurations investigated. Therefore, the quantitative values reported herein should be interpreted within the range of the analyzed conditions. Although the observed trends regarding the influence of column inclination and spacing are expected to be applicable to similar foundation–slope systems reinforced with soil–cement columns, caution should be exercised when extending the findings to substantially different geological conditions, foundation types, or loading scenarios. In addition, the analyses were carried out using the Hardening Soil constitutive model under static loading conditions. Future research may extend the present work by considering alternative constitutive models, different soil conditions, cyclic or seismic loading, and field-scale validation to further verify and generalize the proposed design recommendations.

10. Conclusions

This study investigated the influence of soil–cement column inclination and spacing on the performance of shallow foundations constructed adjacent to slopes through three-dimensional finite element analyses using PLAXIS 3D and the Hardening Soil constitutive model. Following verification against experimental results, the numerical model was employed to evaluate different reinforcement configurations.

The analyses demonstrate that column inclination influences foundation bearing capacity and slope stability through fundamentally different mechanisms. Vertical columns maximize bearing capacity by promoting efficient axial load transfer and minimizing load eccentricity, whereas columns inclined at approximately 45° provide the greatest improvement in slope stability by intersecting the critical failure surface and mobilizing greater resistance against sliding. Consequently, the optimal inclination angle is governed by the dominant performance objective rather than by a universal design criterion.

Column spacing was also shown to play a crucial role in the overall behavior of the reinforced system. Decreasing the spacing enhances the three-dimensional arching effect and increases bearing capacity through improved stress redistribution between the columns and the surrounding soil. However, an important finding of the present study is that an intermediate spacing of 1.5 m consistently produces the largest foundation bending moments owing to localized reaction concentrations beneath the footing, emphasizing the need to consider both structural and geotechnical performance during design.

From an engineering perspective, the results indicate that vertical soil–cement columns with a spacing of 2 m provide the most appropriate balance between bearing capacity, slope stability, structural demand, and construction cost for typical practical applications.

The principal contribution of this study is the demonstration that the design of soil–cement column systems adjacent to slopes should be performance-based rather than configuration-based. Instead of adopting a single reinforcement layout for all situations, the inclination and spacing of the columns should be selected according to the governing design objective, whether it is maximizing bearing capacity, improving slope stability, or achieving an optimal balance between technical performance and economic efficiency. In addition, the three-dimensional analyses confirm that advanced constitutive modeling is essential for capturing stress-dependent stiffness and arching mechanisms that cannot be adequately represented by conventional two-dimensional or linear-elastic approaches.

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