

## Assessment of Scaled Boundary FEM-based model for solving wave interaction with $\pi$ -shape Floating Breakwaters

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### Abstract:

The principal aim of the current study is to examine a Scaled Boundary Finite Element Method (SBFEM)-based model to analyze the interaction problem between the water waves and moored floating breakwaters with sharp edges. Regarding the increasing employment of rectangular cross-section floating breakwaters with vertical side plates to their down-wave and up-wave sides ( $\pi$ -shaped floating breakwaters), it can be stated that they are used as a practical basis to examine how the model works. By comparing the present solutions to those from existing literature, without changing the mesh density compared to previous simulations used in simple configurations, the accuracy and generality of the present model in the complex configurations are evaluated. It is demonstrated that as the proposed model is a semi-analytical method, unlike conventional numerical methods, there is no need to refine the mesh around sharp corners, which can considerably save the computational time, effort, and cost in large solution domains.

## 1. Introduction

Given the importance of the continuous loads imposed by water waves on marine structures, a comprehensive understanding of their interaction with wave fields is important for their robust design. As a result, investigations into the water wave-structure interaction problems for a wide range of structures have been undertaken. Resolving wave-structure interaction for large structures such as floating breakwaters is particularly challenging and has been receiving even more greater attention.

Floating breakwaters are increasingly being used in marine environments because of their inherent benefits. In order to know the behavior of these structures in the field of incident

waves, during past decades (FYI: research is a mass noun and cannot be used in the plural form) diverse research has been carried out using a variety of numerical, analytical and experimental methods.

Among various research methods of investigating the interaction between sea waves and a floating structure, experimental studies must take the pride of place. The main advantage of experimental methods is that, if designed well, they contain and reflect most of the correct physics and there will be fewer simplifications and assumptions compared to non-experimental methods (e.g. numerical or analytical) [1–4]. However, they suffer from some drawbacks, some of which are not easy to overcome in all cases. The salient drawback associated with the experimental models is “lack of flexibility”. Usually, an experimental model is constructed for a specific purpose and will be tested under predefined conditions, but it is time consuming and expensive to make major changes to the test cases to enable optimizations. Furthermore, although all scaled laboratory models are constructed following the similarity laws, there will always be a scaling effects on the scaled physical models. Moreover, measurement inaccuracies are also

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deemed as a incorrect, though this shouldn't generally be a significant problem with available modern instruments. Although analytically derived equations to study the wave-structure interaction result in accurate solutions, they can sometimes lead to complex formulations even for a simple case [5–9]. More importantly, having been developed for a particular case, the analytical models are cumbersome and sometimes impossible to be updated for a new case. From a practical point of view, for most of the practical cases with a complicated configuration, changes in material properties, variation of loads in the field under study, and, eventually, the governing boundary conditions prevent establishing a catch-all closed form analytical solution to solve the governing partial differential equation of the wave-structure interaction. Therefore, along with non-theoretical laboratory studies, it seems necessary to develop an efficient and reliable numerical method free from the limitations of above-mentioned methods in new cases. It should be mentioned that many previous researchers have investigated a range of numerical methods to solve wave-structure interaction problems [10–19]. Although these studies have sought to develop more flexible and efficient models to provide an accurate understanding of wave-structure interaction behavior, the necessity of for more flexible and robust models to examine the behavior of floating structures with complex configurations in the wave field (and provide a platform for the development of new geometries for floating breakwaters) is still standing.

A recently developed method which has proven to be powerful for solving interaction problems in an infinite solution domain is the Scaled Boundary Finite Element Method (SBFEM), one of the most efficient and robust numerical methods that the authors of the present study have recently employed, to solve the problem of linear interaction for breakwaters with simple geometries [20–22]. This novel method was developed by merging the advantages of the finite element method and the boundary element method to create a semi-analytical method [23–25]. Hence, it leads to an exact solution in the radial direction. In the field of Fluid-Structure Interaction, Li et al., [26, 27] established a modified scaled boundary coordinate system to solve the problem of water wave radiation and diffraction in an infinite solution domain with parallel side faces. In their study, a number of rectangular floating breakwater geometries were considered, and the performance of the new coordinate system was examined. Meng and Zou, [28] solved the radiation and diffraction problem for a rectangular floating body near a vertical reflecting wall. In these three investigations regarding the situation of scaling center, defined and diverse solutions were involved; this increased the calculation efforts during the solution process. Fouladi et al., [20] improved the SBFEM models employed by previous authors by relocating the scaling center for bounded subdomains,

allowing the governing equations to be homogenized at that location. They also combined the proposed two-dimensional SBFEM-based model with an artificial neural network machine to predict the effects of wave diffraction from both ends of a structure in three-dimensional simulations. Subsequently, Fouladi et al., [21] solved the problem of full interaction (diffraction plus radiation) between water waves and a moored rectangular floating breakwater using an SBFEM model. In their study, the effects of the mooring system's nonlinear behavior were also taken into account. The efficiency and accuracy of the SBFEM-based model were examined in the case of water wave and nonrectangular floating breakwaters interaction by Qorbani Fouladi et al., [22].

According to the above discussions, the primary purpose of the current study is to scrutinize and further develop the SBFEM-based model previously outlined in Fouladi et al., [21]. As reported in Fouladi et al., [21], the model had been validated for simple rectangular floating breakwaters. However, an essential challenge arises to numerical-based models for solving the boundary value problems in the presence of sharp corner points in the solution domain. In this case, conventional numerical methods suffer from a sort of some drawbacks and, at the best-case scenario, additional computational efforts are made (such as shrinking size and increasing the number of meshes or techniques to eliminate singularity) to overcome this problem. However, sometimes it would not be technically viable or computationally feasible to solve the problem due to the effects of sharp corners in the solution domain (refer to figure 1 for sharp corners). One of the real world examples representing the solution domain with sharp corners is rectangular floating breakwaters with vertical plates to their down-wave and up-wave sides; which is referred to as  $\pi$ -shaped floating breakwaters [4]. For this reason, this type of breakwater has been selected in the present study to challenge the capability of the previously developed SBFEM model to solve sea water wave interaction in a domain with sharp edges. In addition to the fact that the  $\pi$ -shaped floating breakwaters are a kind of practical example of solution domains with sharp corners, the results of the theoretical study of these structures have already been investigated analytically by Cho [9] and can be used as the basis for validating the results of the current SBFEM-based model. Therefore, the SBFEM-based model is evaluated in solving the problem of wave interaction with a  $\pi$ -shaped floating breakwater, as a solution domain with sharp corners. It should be mentioned that the mesh density has been kept unchanged compared to the simple rectangular breakwater (i.e. no sharp edges) to demonstrate whether or not the proposed model is capable enough to deal with the sharp corners in the solution domain without increased effort.

## 2. Governing Equations and Boundary Conditions

In order to solve the problem of interaction between a moored  $\pi$ -shaped floating breakwater with side plates and incident waves, an infinite fluid domain is considered as the solution domain. As demonstrated in figure 1, the origin of the Cartesian coordinate system lies on the undisturbed water surface with the positive  $Z$  axis pointing upward and positive  $X$  axis pointing to the right direction. The breakwater is assumed to be infinite in the  $Y$  direction,

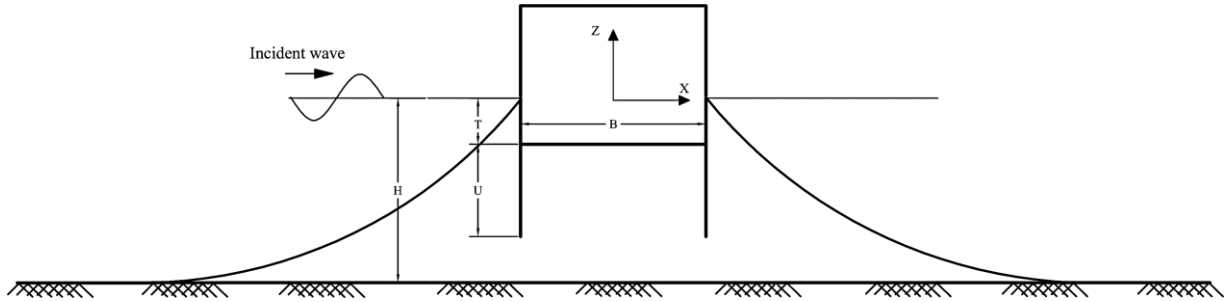


Fig. 1: 2DV definition of the wave interaction with a  $\pi$ -shaped floating breakwater in an infinite fluid domain

The incident wave is considered to be linear and the sea water is assumed to be non-viscous. The principal governing equation for the problem under study is Laplace equation as follows:

$$\nabla^2 \phi_q = 0 \tag{1}$$

Where  $q = I$  represents the incident velocity potential,  $q = S$  is the scattering velocity potential,  $q = R$  defines radiation velocity potential, and  $q = T$  denotes total velocity potential ( $\Phi_T = \Phi_I + \Phi_S + \Phi_R$ ). It should be mentioned that, for a periodic linear motion in a non-viscous fluid flow, the velocity potential ( $\Phi$ ) can be expressed as follows by segregating time and space variables:

$$\Phi(x, z, t) = \text{Re al}(\phi(x, z, t) \exp(-i\omega t)) \tag{2}$$

Also, the radiation velocity potential can be taken into account as follows:

$$\phi_R = \sum_{j=1}^3 S_j \phi_j \tag{3}$$

The term  $S_j$  denotes the amplitude of the  $j$ th mode of motion where  $j = 1, 2, 3$  represents sway motion, heave motion and roll motion, respectively. The boundary conditions considered in the current study to define the fluid domain are as follows:

$$\frac{\partial \phi}{\partial z} = \frac{\omega^2}{g} \phi \text{ at the free surface } (z=0) \tag{4}$$

which allows to simplify the problem of interaction as a two-dimensional vertical (2DV) one by ignoring the effects of wave diffraction from both ends of the structure.

In figure 1,  $B$  and  $T$  stand for the width and draft of the breakwater, respectively. The parameter  $H$ , denotes the total water depth where the breakwater is situated. Two side plates having a length of  $U$ , are considered to be protruded downward from each side of the bottom of the breakwater. As a normal practice, the thickness of the added side plates is neglected in the analyses.

$$\frac{\partial \phi}{\partial z} = 0 \text{ at the sea bottom } (z = -H) \tag{5}$$

$$\frac{\partial \phi}{\partial z} = \bar{v}_n \text{ on the solid surface} \tag{6}$$

Where  $\bar{v}_n$  is a known normal velocity in any solid surface.

## 3. Scaled Boundary Finite Element Method (SBFEM)

The mentioned governing equation synchronically with the boundary conditions is solved by employing SBFEM. Within each subdomain, the unknown velocity potentials are determined analytically along the radial lines connecting the scaling center  $(x_0, y_0)$  to any nodes on the boundaries.

However, the results are estimated numerically on the discretized boundaries  $S$  of the solution domain. By converting the governing equation from Cartesian into the scaled boundary coordinate system, which is demonstrated in figure 2, the SBFEM can be implemented. The radial coordinate  $\xi$  varies from zero at the scaling center to 1 on the boundaries. The value of circumferential coordinate,  $\beta$ , changes from -1 to 1 along the boundaries. The scaling center should be where the entire points on the boundaries are visible from that center. To achieve this objective, the whole computational domain should be divided into five subdomains.  $\Omega_2, \Omega_3$  and  $\Omega_4$  are the bounded subdomains while  $\Omega_1$  and  $\Omega_5$  are the unbounded ones.

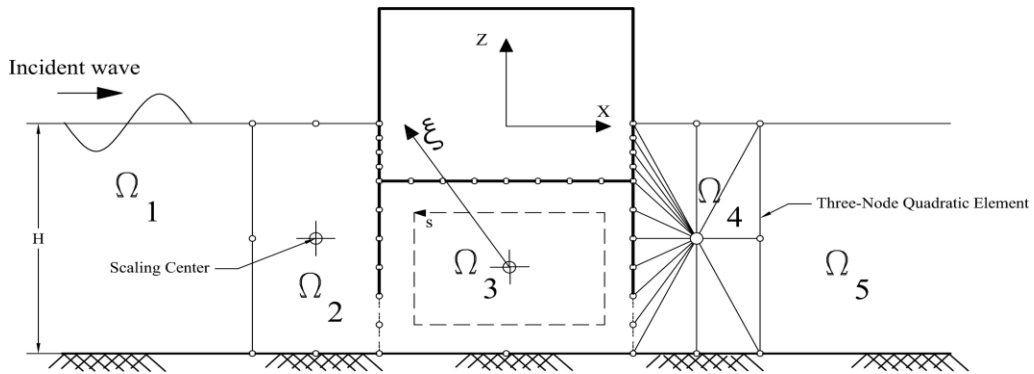


Fig. 2: Definition sketch of establishing scaled boundary coordinate system

3.1 Formulation for bounded sub-domains

As illustrated in Fig. 3, in the present study three bounded sub-domains are taken into account. The sub-domain Ω<sub>3</sub> shares the same boundary on both sides with other bounded ones. Meanwhile, subdomains Ω<sub>2</sub> and Ω<sub>4</sub> share a

boundary with Ω<sub>3</sub> and a boundary with unbounded subdomains (Γ<sub>int1</sub>, Γ<sub>int4</sub>), on the other side. Discretization patterns of the bounded subdomains' boundaries are shown in figure 3.

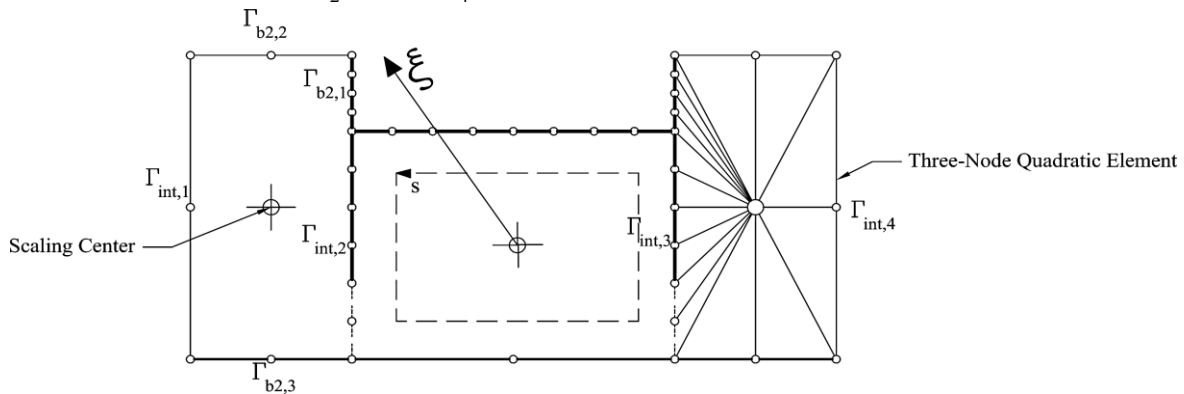


Fig. 3: Discretization of domain boundaries for bounded sub-domain

The position of any point within each sub-domain, ( $\hat{x}, \hat{y}$ ), is defined by:

$$\hat{x} = x_0 + \xi[N(s)]\{x\} \tag{7}$$

$$\hat{y} = y_0 + \xi[N(s)]\{y\} \tag{8}$$

Where  $\{x\}$  and  $\{y\}$  vectors are the Cartesian coordinates of each node that create a sector containing the mentioned point by being connected to the scaling center.  $[N(s)]$  is the shape function. It worth to mention that by applying the scaling coordinate,  $\xi$ , any arbitrary point within the domain is a linear function of a specific point on the boundary  $S$ . Similarly, the velocity potential at any arbitrary point on a sector can be defined via the potential values of the nodes as:

$$\phi(\xi, s) = [N(s)]\{a(\xi)\} \tag{9}$$

Where  $\{a(\xi)\}$  is the potential function along the lines connecting the scaling center to the nodes.

In the scaled boundary coordinate system, the governing equations in terms of the potential function  $\{a(\xi)\}$  and node flow function can be expressed as:

$$[E_0]\xi^2\{a(\xi)\}_{,\xi\xi} + ([E_0] + [E_1])^T - \tag{10}$$

$$[E_1]\xi\{a(\xi)\}_{,\xi} - [E_2]\{a(\xi)\} = 0$$

$$\{q(\xi)\} = [E_0]\xi\{a(\xi)\}_{,\xi} + [E_1]^T\{a(\xi)\} \tag{11}$$

All the parameters used in the above Equation have already been defined in Fouladi et al., [21].

3.2 Solution for the unbounded sub-domains

By employing a modified scaled boundary coordinate system with the scaling center on the interface of bounded and unbounded sub-domains, the  $s$  coordinate is along the mentioned interface boundary (defining curve) and  $\xi$  is parallel to the bottom and side faces and towards infinity.

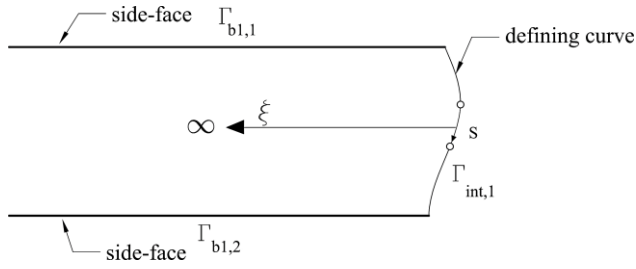
Thus,  $\xi$  is zero at the interface boundaries  $\Gamma_{int1}$  or  $\Gamma_{int4}$  and infinity at  $\Gamma_{\infty}$ .

In order to convert the Cartesian to modified scaled boundary coordinate system, the following equations are considered:

$$\hat{x} = [N(s)]\{x\} + \xi \tag{12}$$

$$\hat{y} = [N(s)]\{y\} \tag{13}$$

This coordinate system is demonstrated in figure 4.



**Fig. 4:** Applying SBFEM for semi-infinite sub-domains with parallel side-faces [20]

Following Fouladi et al., [21], the governing equations in this system are extracted as follows:

$$[E_0]\{a(\xi)\}_{,\xi\xi} + ([E_1]^T - [E_1])\{a(\xi)\}_{,\xi} + (\bar{k}^2[M_0] - [E_2])\{a(\xi)\} = 0 \quad (14)$$

$$\{q(\xi)\} = [E_0]\{a(\xi)\}_{,\xi} + [E_1]^T \{a(\xi)\} \quad (15)$$

### 3.3 Solution Process

To solve the governing equations, the problems should be turned to two eigenvalue problems. For this objective, the new variable  $\{\chi(\xi)\} = \begin{Bmatrix} a(\xi) \\ q(\xi) \end{Bmatrix}$  is introduced and combined

with Equation (11) and Equation (10) and turns into equation 16 as follows:

$$\xi\{\chi(\xi)\}_{,\xi} = [Z]\{\chi(\xi)\} \quad (16)$$

In the case of governing equations in unbounded subdomains, by utilizing Equation (15) and  $\{\chi(\xi)\}$ , Equation (14) is transformed as:

$$\{\chi(\xi)\}_{,\xi} = [Z]\{\chi(\xi)\} \quad (17)$$

Where  $[Z]$  denotes Hamilton matrix. The relations between  $\{a(\xi)\}$  and  $\{q(\xi)\}$  are extracted for the bounded and unbounded sub-domains by solving these systems of standard Eigen value problems as:

$$\{q(\xi)\} = [H^b]\{a(\xi)\} \quad (18)$$

$$\{q(\xi)\} = [H^\infty]\{a(\xi)\} \quad (19)$$

Where  $[H^b]$  and  $[H^\infty]$  are defined as stiffness matrices for bounded and unbounded sub-domains, respectively. Assembling stiffness matrices is done by considering the fact that the potentials and the flows at the interface between sub-domains are identical. Once the nodal potentials are calculated, wave pressure and subsequently the exciting forces and hydrodynamic coefficients can be determined.

## 4. Equations of motion

The system of equations of motion can be expressed as follows:

$$[-\omega^2(m_{ij} + \mu_{ij}) - i\omega\lambda_{ij} + (c_{ij} + k_{ij})]\Xi_i = F_i \quad (i, j = 1, 2, 3) \quad (20)$$

Where  $i,j=1$  represents sway,  $i,j=2$  means heave and  $i,j=3$  is related to roll movements of FBW.  $\Xi_i$  represents amplitude of motion vector,  $F_i$  depicts the vector of wave exciting force.  $m_{ij}$ ,  $c_{ij}$ ,  $\lambda_{ij}$  and  $\mu_{ij}$  are the mass, hydrostatic stiffness, damping and added mass matrices, respectively.  $k_{ij}$  is the stiffness matrix of mooring system. The wave exciting forces, added mass and damping matrices is calculated and verified in section 5.1. More details can be found in Fouladi et al., [21]. The frame-work developed for the solution process is outlined in figure 5.

## 5. Results and Discussion

The results are classified in three steps as reported in three subsequent subsections. First of all, to establish equation (20), the values of wave-induced forces are estimated by solving the diffraction problem presented in section 5.1. As the values of added mass and radiation damping can be determined in a radiation problem, the second step is defined to solve the radiation problem as elaborated in section 5.2. The mooring's stiffness matrix is derived analytically and equation (20) is solved. Consequently, the values of response amplitude operators (RAOs) are calculated and reported in the final step in section 5.3.

### 5.1 Diffraction problem

For the purpose of evaluating the exciting wave forces, the proposed SBFEM-based model is employed for wave diffraction problem. Normal velocities are generated by the incident wave at left boundary named as  $\Gamma_{int1}$ . The wave-induced loads caused by the incident wave on the stationary structure in  $k$  direction is calculated according to the following equation:

$$F_k = i\omega\rho \int_{s_0} (\Phi_I + \Phi_S)n_k ds \quad (21)$$

Where  $n_k$  is the generalized normal with  $n_1 = n_z$ ,  $n_2 = n_x$  and  $n_3 = (z - z_0)n_x - (x - x_0)n_z$ .  $(x_0, z_0)$  denotes the center of rotation and  $\rho$  is the fluid density. The parameter  $S_0$  represents the wetted surface of structure, and, eventually,  $F_k$  is the force in  $k$  direction. Except for the bottom of structure which is discretized by four elements, the considered mesh consists of two numbers of three-node quadratic elements at each boundary. Therefore, the bounded sub-domain is discretized by 30 elements. In addition, each unbounded sub-domain is assigned with two elements.

For the structure with  $B/H = 1.0$ ,  $T/H = 0.25$  and  $U/B = 0.5$ , the components of non-dimensional forces are analyzed from current SBFEM-based for 100 different frequencies, and compared with the reported results by Cho [9] as depicted in figure 6. The top graphs in figure 6

represent the exciting wave forces, while the bottom ones depict the moment applied by the wave on the stationary structure.

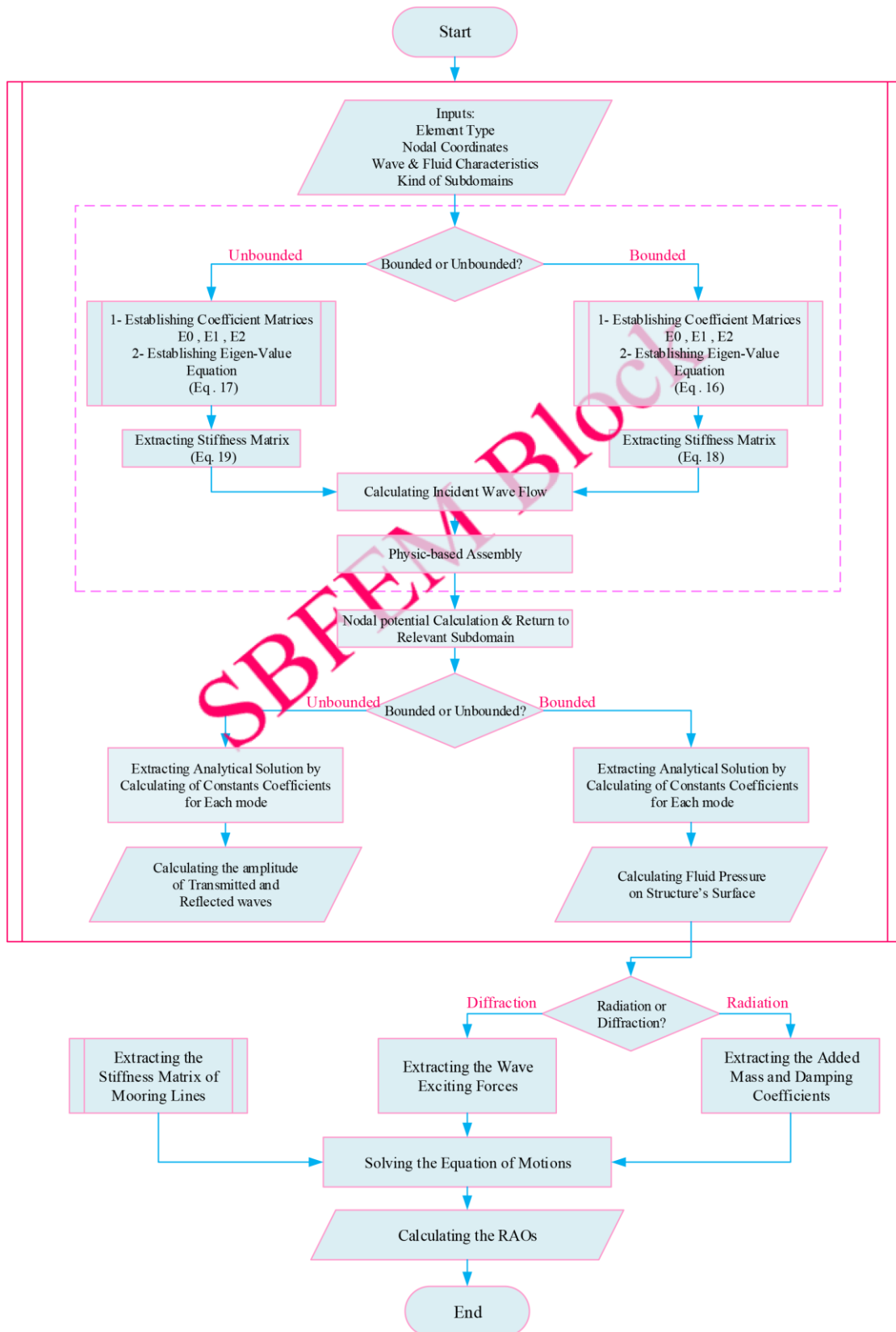
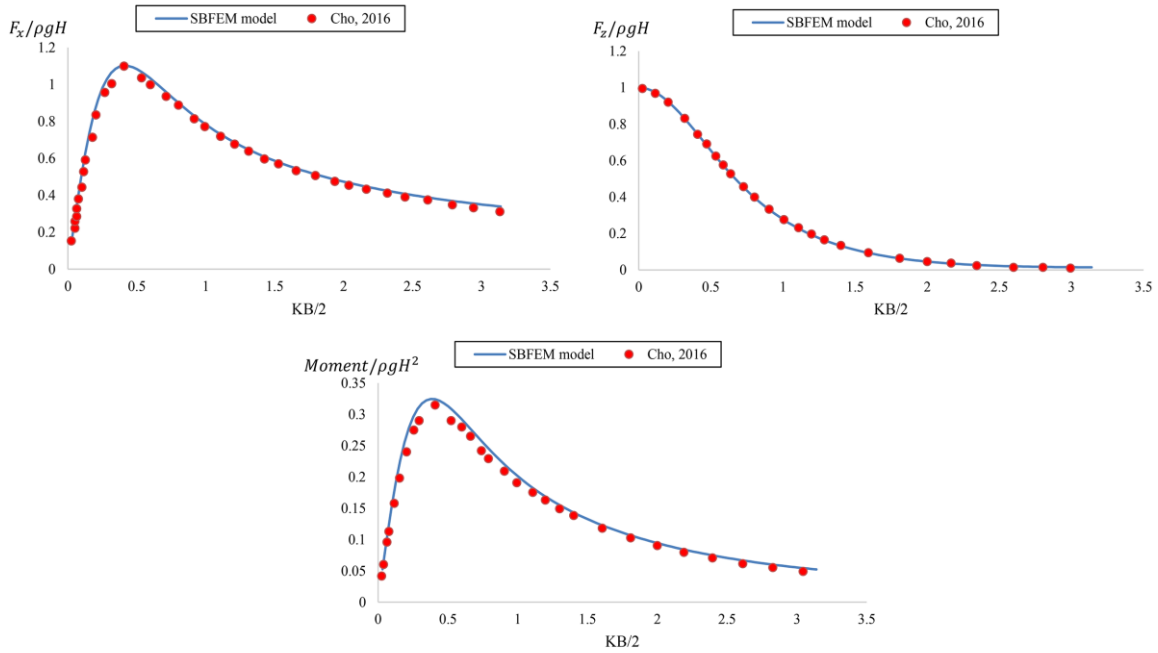


Fig. 5: The SBFEM-based solution frame-work developed in the current study



**Fig. 6:** Dimensionless wave forces for  $\pi$  - shaped MFB

As shown in figure 6, there is a good agreement between the SBFEM-based results and analytical values reported in previous studies, approving the accuracy and competence of the proposed SBFEM method.

Moreover, the presented SBFEM method offers an interesting edge over previous analytical studies in reducing the required number of mesh elements. In other words, with taking a minimum number of mesh elements to simulate the diffraction problem under consideration, even in the solution domain with different sharp corners, unlike analytical methods, the SBFEM-based solution does not suffer from any numerical deficiencies.

### 5.2 Radiation Problem

In order to estimate the values of added mass and radiation damping, the radiation problem is studied. Due to forced displacements of structure in three modes in calm water, the hydrodynamic forces applied to the MFB can be defined as:

$$F_{wj} = i\omega\rho \int_{s_0} (\Phi_w) n_j ds \quad (22)$$

Where  $F_{wj}$  is the force in  $j$  direction caused by different modes of movements ( $w = 1$  for heave,  $w = 2$  for sway, and  $w = 3$  is related to rotational movement). The added mass,  $\mu_{wj}$ , and radiation damping coefficients,  $\lambda_{wj}$ , can be calculated using the following equations:

$$\mu_{jw} = -\frac{1}{\omega^2} \text{Re}(F_{wj}) \quad (23)$$

$$\lambda_{wj} = -\frac{1}{\omega} \text{Im}(F_{wj}) \quad (24)$$

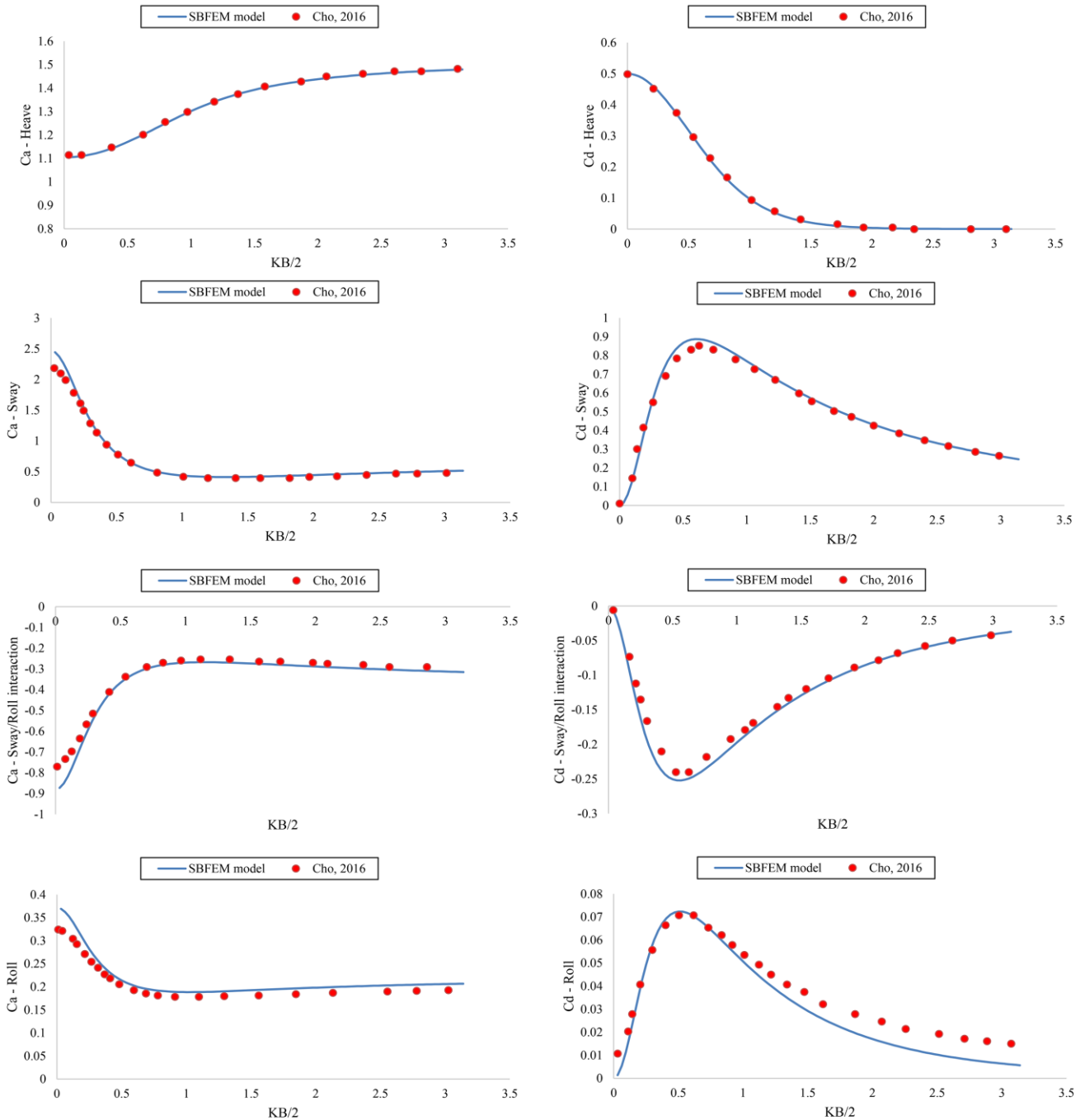
By non-dimensioning added mass,  $C_a$ , and damping,  $C_d$ , these coefficients can be expressed as follows:

$$C_{awj} = -\frac{1}{\omega^2 BTs} \text{Re}(F_{wj}) \quad (25)$$

$$C_{dwj} = -\frac{1}{\omega^2 BTs} \text{Im}(F_{wj}) \quad (26)$$

$S$  represents amplitude of the forced displacement in radiation problem.

For the MFBW under study in diffraction problem, the above-mentioned coefficients are extracted and compared with analytical ones for each mode of motion as demonstrated in figure 7. As shown in figure 7, in general, there is a good agreement between calculated results of the present model and the analytical values reported in literature. However, slight discrepancies can be seen in the graphs. The first reason for this difference is rooted in errors in digitizing the reported results. Moreover, the primary source of this difference is also noteworthy. As mentioned in the scattering and radiation hydrodynamic forces calculation, the potential velocity values should be integrated over the body's boundaries. This integration in the present model is performed numerically by the Newton-Coates method. This considerable number of errors in calculating the added mass and damping coefficients in the roll movement has increased because it is necessary to perform numerical integrals on all wet boundaries of the structure and two sides of each side plate. Hence, the amount of accumulated error, in this case, has grown.



**Fig. 7:** Comparison of dimensionless added mass coefficient derived from SBFEM with analytical value

As shown in figure 7, in general, there is a good agreement between calculated results of the present model and the analytical values reported in literature. However, slight discrepancies can be seen in the graphs. The first reason for this difference is rooted in errors in digitizing the reported results. Moreover, the primary source of this difference is also noteworthy. As mentioned in the scattering and radiation hydrodynamic forces calculation, the potential velocity values should be integrated over the body's boundaries. This integration in the present model is performed numerically by the Newton-Coates method. This considerable number of errors in calculating the added mass

and damping coefficients in the roll movement has increased because it is necessary to perform numerical integrals on all wet boundaries of the structure and two sides of each side plate. Hence, the amount of accumulated error, in this case, has grown.

### 5.3 Full interaction between wave and moored floating breakwater

Based on previous steps, the hydrodynamics forces exerted on the structure by fluid domains are calculated and verified. In this part, the interactive response of the moored structure is obtained. With regards to equation (20), by estimating the

amplitudes of the structure's movements, the Responses of Amplified Operators (RAOs) for the heave, sway and roll motions can be obtained applying the presented SBFEM-

based model. The comparison of the calculated results in the current study with previously reported theoretical values are illustrated in figure 8.

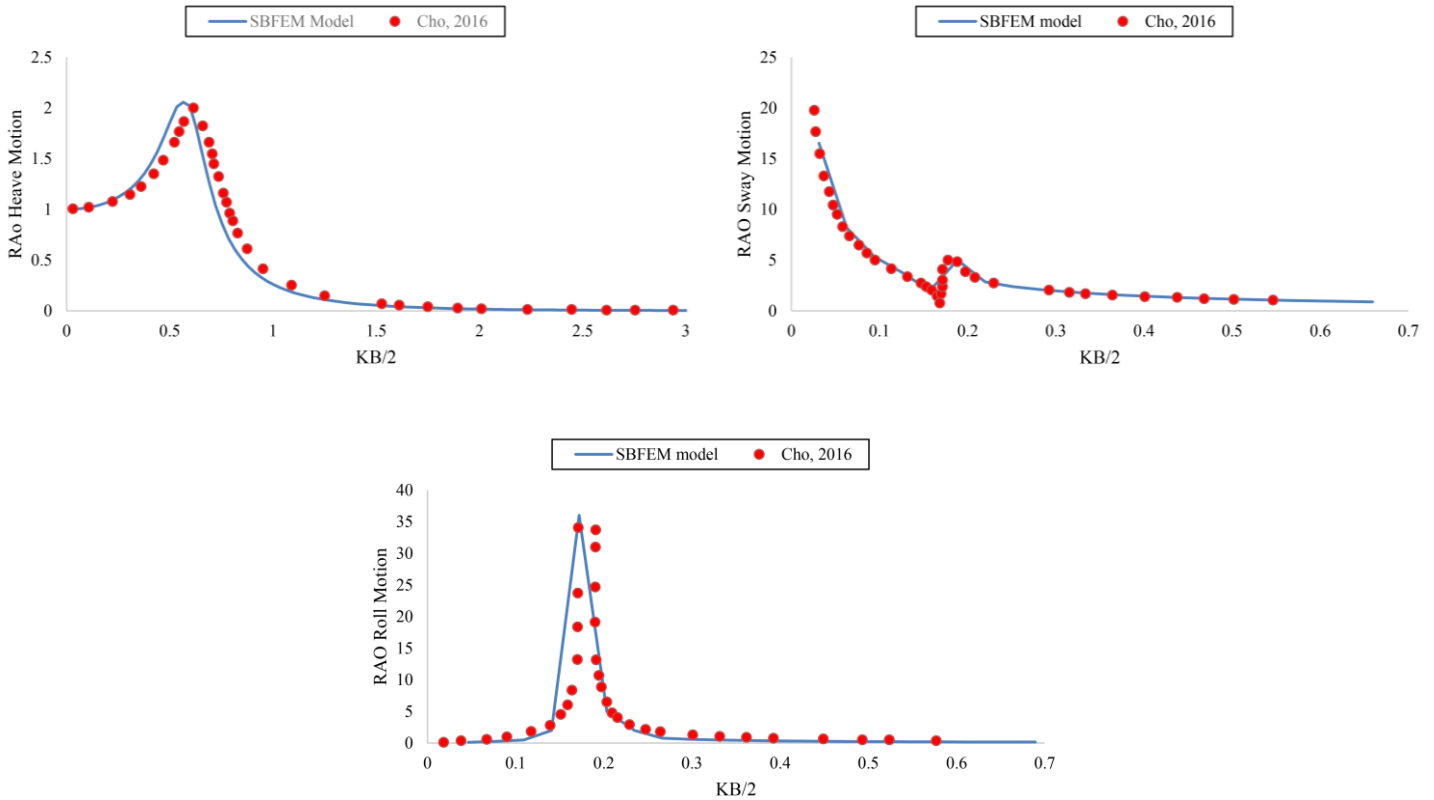


Fig. 8: Comparison of RAOs in heave, sway and roll as a function of normalized wave frequencies

Generally speaking, there is a good agreement between the calculated results of the present study and the reported ones in literatures. In the range of  $0.15 < KB / 2 < 0.25$ , a dramatic growth occurred in the RAO of the sway and roll modes, which is attributed to the robust coupling between them. The current model was well anticipated in this phenomenon. However, for roll responses, a clear difference can be observed between the resonance frequencies of two models. This divergence, first of all, is rooted in the fact that there are marginal differences between the calculated results of the current model with that of the other in diffraction and radiation problem estimations. As can be seen in equation (20), the outputs of diffraction and radiation problems are the fundamental parameters in full interaction simulation. These marginal errors resulting from numerical errors, in some way can affect the resonance frequency. It should be mentioned that in similar cases of engineering problems, this event can be traced. For instance, in the earthquake design spectrum, a range of resonance frequency is also considered in the reputable codes. This perspective of the code explains that different methods predict the resonant frequency with a slight difference. Therefore, the difference between the current model and the results of the previous study is quite normal.

Moreover, it should be mentioned that the reason why the SBFEM method is not sensitive to sharp edges and is well viable with minimum number of elements can be explained as follows. The SBFEM method is a semi-analytic method in which the calculations are performed only in the peripheral direction with the numerical method and the solutions are obtained analytically in radial direction. Therefore by using the same mesh density previously used for FBW with rectangular cross-section reported in [4], the results show good agreement with analytical value reported in previous studies. This dependency on mesh-density proves that practically the proposed SBFEM method has no sensitivity in sharp edges of the solution domain. Therefore, the proposed method works satisfactorily to solve the interaction of a floating structure in an infinite domain of fluids and not suffer from the limitations of other conventional numerical methods.

### 6. Conclusion

In current study the previously developed SBFEM-based model has been further assessed in the case of solution domains with sharp edges. It has been previously shown that the SBFEM method is a competent method to solve wave-structure full interaction in the case of simple

rectangular FBW. What has been achieved in the present study is that the proposed SBFEM method works satisfactorily not only in simple domains, but also it is viable in solution domains with sharp edges where the FE based methods face difficulties.

Accordingly,  $\pi$ -shaped floating breakwaters has been studied as a real world case of solution domains with sharp edges. In order to evaluate the dependency of SBFEM method to sharp edges, the model is implemented to solve the problem of wave interaction with a  $\pi$ -shaped floating breakwater with the same mesh density previously used for simple rectangular FBW.

Hydrodynamic wave forces and coefficients are extracted and compared with analytical values reported in literature in order to establish motion equations. The effects of mooring lines are considered in the mentioned equations as well. The results obtained in the current study are consistent with the results reported in the literature. Results shows that, the calculated results based on the SBFEM methodology proposed in the current study are satisfactory for diffraction and radiation problems as compared to the previously derived results using analytical models.

Validation of numerically-derived results with experimental values is always the most reliable way to evaluate an emerging numerical method. However, due to the limitations of laboratory studies, and in the absence of such experimental results, the consistency of the results achieved by the newly proposed method with the results obtained from other known methods can confirm the accuracy of the proposed model to some extent.

The results approve prove that the proposed SBFEM method could perform the intended tasks as a reliable model in complex configuration having sharp edges with a minimum number of elements, which implies that the proposed SBFEM method is worthwhile to develop and study more.

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