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Uncertainty Analysis of the Effect of Modulus of Elasticity on Seismic Performance of Concrete Quay Wall

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Abstract:

This paper investigated the sensitivity of the seismic performance of quay wall system to changes in the modulus of elasticity of the body concrete Monte Carlo probabilistic analysis, which is a new method for parametric study and sensitivity analysis. Monte Carlo method presents an appropriate solution to consider a specified range for various parameters effective in analyzing. The ANSYS software which is based on finite element method is applied for analysis considering fluid-structure interaction effect. In the uncertainty analysis, modulus of elasticity of the quay wall body concrete is a parameter indicating the stiffness and strength of body in design of concrete structures and has been selected as input variable parameter. Additionally, the maximum displacement of the crest and the maximum tensile principal stress in critical point of the body has been selected as output variables. The model is analyzed in time domain by applying the horizontal and vertical components of El Centro earthquake. Finally, the effect of the modulus of elasticity on the maximum responses at each stage is shown as sensitivity curves. According to the results, an optimal value is obtained for the modulus of elasticity of quay wall concrete to ensure system safety.

1. Introduction

One of the structures used to protect beaches are quay walls. A number of researchers have already studied the seismic behavior of gravity quay walls. For example, Madabhushi and Zeng (1998) numerically studied the seismic response of gravity quay walls [1].

Dynamic behavior of caisson type quay walls was studied by Kuwano et al. (1999). Parameters such as the width and the unit weight of the quay wall and the magnitude of the input excitation were examined. According to their results, with increase in the quay wall width and its unit weight, a significant reduction in wall tilting and horizontal displacement can be achieved [2]. Chen (1995) and Lee et al. (1999) conducted studies on effects of earthquakes on marine structures and quay walls. In their studies they failed to exactly incorporate the interaction effects [3-4].

Gharabaghi et al. (2007) used a finite element modelling approach to simulate the dynamic behaviour of gravity

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Gharabaghi et al. (2007) used a finite element modelling approach to simulate the dynamic behaviour of gravity quay-walls during earthquakes. They reported that the structure-fluid interaction had negligible effects on the wall seismic response [5].

One of the new techniques considered in design is based on the possibilities and a variable range for design parameters. Similarly, one important aspect of performance-based earthquake engineering is an accurate estimate of seismic response and structural capacity [6].

There are two major sources of uncertainty in seismic performance of the structure: the uncertainties associated with randomness and physical uncertainties caused by modeling assumptions, deletions or existing errors [7]. Adequate understanding of responses expected from the structure can be influential on the probabilistic analysis regarding the structure of the uncertainty [8]. Calculating different types of seismic performance, given the physical uncertainty, commonly involves the use of safety factors or standard dispersion criteria [9].

A very robust method, given both sources of uncertainty in earthquake engineering, is probabilistic analysis using Monte Carlo simulation [10]. Several studies have been

conducted on this type of analysis. Carvajal et al. (2009) proposed a probabilistic analysis for assessment of hydraulic load and shear strength acting on some hydraulic structures [11]. Altarejos-Garcia et al. (2011) conducted a study to validate this analysis on a series of hydraulic structures under hydraulic loading and estimate their probability of failure as a case study [12]. Calabrese and Lai (2016) presented sensitivity analysis of the seismic response of gravity quay walls, wherein the effects of inherent variations of ground motions and geotechnical quantities are investigated. Results showed that the uncertainties associated with the seismic input, i.e. intensity level and ground motion definition, are the most relevant ones. Then, there are the effects of the geotechnical parameters, the largest of which is given by the friction angle of the backfill and foundation soil shear modulus [13].

In the field of probabilistic method application in hydraulic structures analysis, Pasbani Khiavi (2017) investigated the influence of concrete stiffness on the seismic performance of concrete gravity dams [14]. Also, Pasbani Khiavi et al. (2020) simulated the effect of reservoir length on seismic behavior of concrete gravity dams. The Monte Carlo simulation was used in the sensitivity analysis using probabilistic model [15].

In this research, the Monte Carlo method is used to study the changes in modulus of elasticity parameter on seismic behavior of quay wall model and to optimize the dynamic performance. A key feature of the Monte Carlo simulation is that, sampling points are located in the space of random input variables. The Monte Carlo simulation produces a random sample of *N* points for each input variable and specifies the corresponding result for each of the *N* values. Then it processes and displays the results as standard deviations, sensitivity, and probability density curves.

This paper presents the Monte Carlo method as an effective tool for optimizing and designing the uncertainty space by FE-ANSYS. ANSYS is a software program based on the finite element method and is used to give more accurate results for complicated geometries. In this method, the geometry is divided into small parts and the boundary conditions are applied. Finally, the results are obtained for each node or element.

2. Governing Equation

In this section, structural and hydrodynamic considerations are described. Fluid is considered as non-viscous and uncompressible, with minor displacement, whereas, quay wall and foundation are considered as solid and elastic with linear behavior of materials.

Considering the behavior and geometry of the quay wall system, the finite element model is assumed as two-dimensional and the plane stress model is used for the quay wall body elements.

Also, the main purpose of study is to present the probabilistic model application in seismic analysis and optimization of the quay wall model. Due to the high computational effort in this field, the linear behavior is assumed for the body. The results can be generalized to nonlinear behavior.

2. 1 Fluid equation

In problems related to the acoustic interaction between structure and fluid, equation of the structure needs to be considered together with Navier-Stokes, momentum and continuity equations of the fluid. Assuming a non- viscous and compressible fluid with small displacements, continuity and momentum equation are summarized to wave equation. Furthermore, applied pressure on the structure from fluid at the interface is considered to form the interaction matrix. The hydrodynamic pressure equation will be (Helmholtz equation) [16]:

$$\frac{1}{C^2}\frac{\partial^2 P}{\partial t^2} - \nabla^2 P = 0 \tag{1}$$

In above equation, C is velocity of sound in fluid, P is hydrodynamic pressure, and t is time.

2.2 Finite element formulation of governing equations

The governing equations corresponding to the fluid-structure system can be presented in the matrix form using the finite element method.

2.2.1 Finite element equation of structure

The equation of discretized structural dynamics can be expressed with the structural components. The addition of fluid pressure load on the interface to structural equation, is currently performed for describing the fluid-structure interaction problem completely. Therefore, the structural equation is expressed subsequently as:

$$[M_e]\{\ddot{u}_e\} + [C_e]\{\dot{u}_e\} + [K_e]\{u_e\} = \{F_e\} + \{F_e^{Pr}\} \tag{2}$$

In which the fluid's compressive load vector $\{F_e^{Pr}\}$ at the contact point is obtained with vector integration (ANSYS user manual, 2007):

$$\{F_e^{Pr}\} = \int_s \{N'\} P\{n\} ds \tag{3}$$

Where $\{N\}$ and $\{N'\}$ represent the function of element shape for pressure and displacement, and {n} represents the normal vector at the contact surface. Replacing the finite element estimating function of pressure into Eq. (3):

$$\{F_e^{Pr}\} = \int_s \{N'\}\{N\}^T\{n\} ds \{P_e\} \tag{4}$$

Or

$$\{F_e^{Pr}\} = [Re]\{P_e\} \tag{5}$$

Where $[Re]^T = \int_s \{N'\}\{N\}^T\{n\}ds$. Replacing equation (5) in (4) results in the dynamic finite element equation of the structure as follows:

$$[\mathrm{M_e}]\{\ddot{\mathrm{u}}_\mathrm{e}\} + [\mathrm{C_e}]\{\dot{\mathrm{u}}_\mathrm{e}\} + [\mathrm{K_e}]\{\mathrm{u_e}\} - [\mathrm{Re}]\{\mathrm{P_e}\} = \{\mathrm{F_e}\} \quad (6)$$

2.2.2 Finite element model of water domain

The subsequent matrix operators (gradient and divergence) are presented to be used in Eq. (1) (ANSYS user manual):

$$\nabla.() = \{L\}^T = \left[\frac{\partial}{\partial x}\frac{\partial}{\partial y}\frac{\partial}{\partial z}\right] \quad (7)$$

And

$$\nabla.() = \{L\} \quad (8)$$

Eq. (1) is as subsequent:

$$\frac{1}{C^2}\frac{\partial^2 P}{\partial t^2} - \nabla.\nabla P = 0 \quad (9)$$

Using the schemes presented in Eq. (9) and (10), Eq. (11) in matrix scheme becomes:

$$\frac{1}{C^2}\frac{\partial^2 P}{\partial t^2} - \{L\}^T(\{L\}P) = 0 \quad (12)$$

The element matrices are achieved using discretization of the wave Eq. (12) by the Galerkin process. Multiplication of Eq. (12) by a virtual variation in pressure and integration over the domain volume with several manipulation results in:

$$\int_V \frac{1}{C^2}\delta P\frac{\partial^2 P}{\partial t^2}dV + \int_V(\{L\}^T\delta P)(\{L\}P)dV = \int_S\{n\}^T\delta P(\{L\}^T\delta P)ds \quad (13)$$

Where V refers to the domain volume, δP represents the virtual variation of pressure, S refers to the surface where the pressure normal derivative is acted on the surface, and {n} represents the unit normal to the interface S. In the problem of fluid-structure interaction, the treatment of surface S is performed as the interface. In order to simplify the assumptions, the equations of fluid momentum yield the subsequent relations between the slope of fluid pressure and the solid acceleration at the fluid-structure interface:

$$\{n\}.\{\nabla.P\} = -\rho_0\{n\}.\frac{\partial^2\{u\}}{\partial t^2} \quad (14)$$

In which {u} refers to the movement vector corresponding to structure at the interface.

In matrix scheme, Eq. (14) is expressed as:

$$\{n\}^T(\{L\}P) = -\rho_0\{n\}^T.\left(\frac{\partial^2}{\partial t^2}\{u\}\right) \quad (15)$$

After replacing Eq. (15) into Eq. (13), the integral is expressed as (Hanna, 1982):

$$\int_V \frac{1}{C^2}\delta P\frac{\partial^2 P}{\partial t^2}dV + \int_V(\{L\}^T\delta P)(\{L\}P)dV = \int_S -\rho_0\delta P\{n\}^T\left(\frac{\partial^2}{\partial t^2}\{u\}\right)ds \quad (16)$$

Eq. (16) comprises the pressure of fluid P and the structural movement components as the dependent parameters of the solution. The finite element estimating functions of shape for the spatial change of the pressure and movement components are expressed as follows:

$$P = \{N\}^T\{P_e\} \quad (17)$$

And

$$u = \{N'\}^T\{u_e\} \quad (18)$$

Where $\{P_e\}$ represents the vector of nodal pressure and $\{u_e\}$ refers to the vectors of nodal displacement component.

Using Eq. (17) and Eq. (18), the second time derivative of the parameters and the virtual alteration in the pressure can be expressed subsequently as:

$$\frac{\partial^2 P}{\partial t^2} = \{N\}^T\{\ddot{P}_e\} \quad (19)$$

$$\frac{\partial^2}{\partial t^2}\{u\} = \{N'\}^T\{\ddot{u}_e\} \quad (20)$$

$$\delta P = \{N\}^T\{\delta P_e\} \quad (21)$$

Allowing the matrix operator $\{L\}$ act on the functions of element shape $\{N\}$ can be indicated as:

$$[B] = \{L\}\{N\}^T \quad (22)$$

By replacing Eq. (17) over Eq. (20) into Eq. (16), the finite element statement relating to the wave equation is obtained as:

$$\int_V \frac{1}{C^2}\{\delta P\}^T\{N\}\{N\}^T dV\{\ddot{P}\} + \int_V\{\delta P_e\}^T\{B\}^T[B]dV\{P_e\} + \int_S \rho_0\{\delta P_e\}^T\{N\}\{n\}^T\{N'\}ds\{\ddot{u}_e\} = 0 \quad (23)$$

Where $\{n\}$ refers to the normal vector in the fluid boundary.

$\{\delta P\}$ represents a randomly introduced virtual alteration in nodal pressure, and can be factored out in Eq.(23). As $\{\delta P\}$ is not zero, Eq. (23) is expressed as:

$$\frac{1}{C^2}\int_V\{N\}\{N\}^T dV\{\ddot{P}\} + \int_V\{B\}^T[B]dV\{P_e\} + \rho_0\int_S\{N\}\{n\}^T\{N'\}ds\{\ddot{u}_e\} = 0 \quad (24)$$

Eq. (24) can be presented in matrix notation to obtain the equation corresponding to the discretized wave:

$$[M_e^P]\{\ddot{P}_e\} + [K_e^P]\{P_e\} + \rho_0[R_e]^T\{\ddot{u}_e\} = 0 \quad (25)$$

Where $[M_e^P] = \frac{1}{C^2}\int_V\{N\}\{N\}^T dV$ refers to the matrix of fluid mass, $[K_e^P] = \int_V\{B\}^T[B]dV$ represents the matrix of

fluid stiffness, and $\rho_0[R_e]^T\{\ddot{u}_e\} = \rho_0 \int_S \{N\}\{n\}^T\{N'\}ds$ is the matrix of mass corresponding to fluid-structure interaction.

3. Model Analysis

In this analysis, quay wall is considered as concrete type unreinforced elastic, which is made from material with homogenous, linear, and isotropic behavior; fluid behind the peripheral quay wall is considered as homogenous, compressible, non-viscous, and non-rotational with minor displacement; soil and sediment is considered as homogenous; effects of surface wave were ignored and pressure at free surface of fluid and Sommerfeld boundary condition was used for infinite truncated boundary.

Given the applicable conditions on the behavior of quay wall, this system is considered as a two-dimensional model. System specifications are summarized as follows [17-18]:

Specific weight, Poisson's ratio and modulus of elasticity of quay wall concrete is assumed to be 2400 kg/m^3, 0.2,and 30.5Gpa respectively. For water, density is 1000 kg/m^3, height is 14m ; for sediment, height is 3m and density is 1926 kg/m^3. The velocity of pressure wave in fluid was taken as 1440 m/s. For soil, modulus of elasticity is 0.1 GPa , Poisson's ratio is 0.3, density is 2000 kg/m^3. Dimensions of case study model has been shown in Figure 1

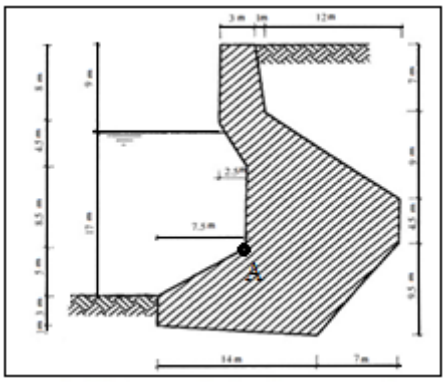

Fig. 1: Quay wall geometry

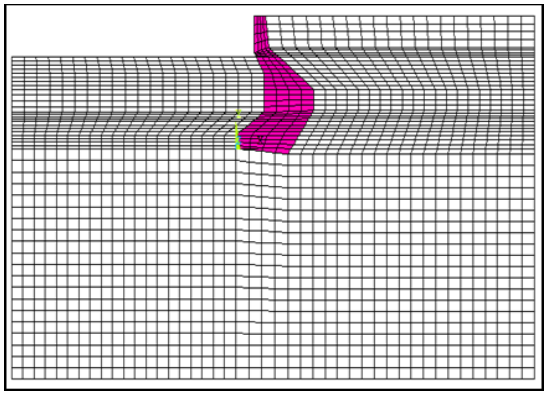

Fig. 2: Finite element model of system

The finite element model of quay wall system is shown in Figure 2. For these cases, the maximum horizontal displacement at the crest and maximum 1st principle stress at Point A were selected as the output parameters.

In this paper, the finite element method is used for seismic analysis of quay wall considering variation in modulus of elasticity.

The horizontal and vertical components of El Centro earthquake, which is one of the strongest ground motions, are applied to the entire body of the system to perform seismic performance of the studied model according to the ANSYS software capabilities. The maximum horizontal acceleration of the earthquake is 0.35 g. Newmark method was used for numerical integration, time step was selected as Δt=0.02sec. Figures 3 and 4 show El Centro earthquake records that occurred in 1940.

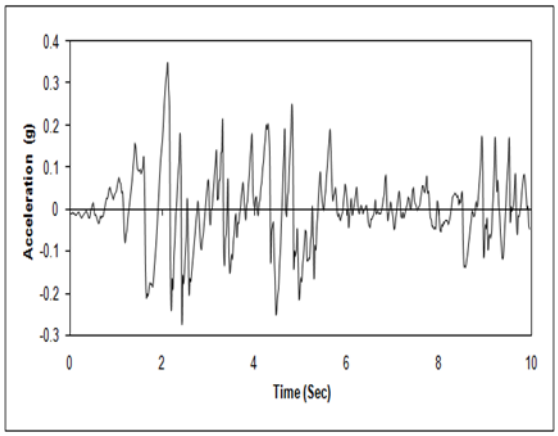

Fig. 3: Horizontal component of El Centro earthquake

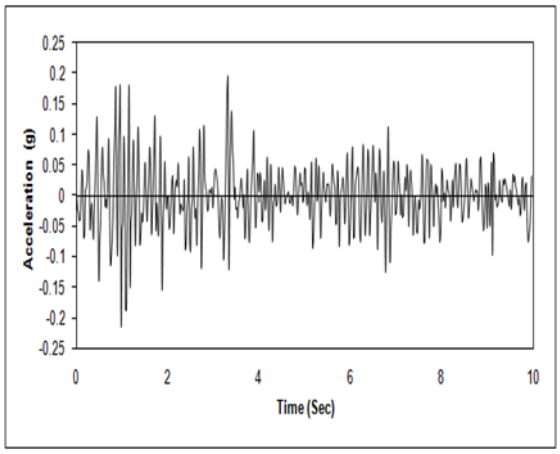

Fig. 4: Vertical component of El Centro earthquake

In this research, the modulus of elasticity of quay wall concrete was considered as input variable to evaluate the effect of concrete strength in probabilistic analysis given the logarithmic distribution as probabilistic function. The model has been analyzed using the Monte Carlo method and the following results were obtained.

To describe the scatter of the data, the lognormal distribution has been used. The lognormal distribution is

particularly suitable for phenomena that arise from the multiplication of a large number of error effects. It is also appropriate to use the lognormal distribution for a random variable that is the result of multiplying two or more random effects.

In probabilistic analysis, the number of required simulations should be such that the average value of the output variable reaches the appropriate convergence according to the number of simulations. So, in this study the ANSYS software settings are selected for Monte Carlo probabilistic method according to Table 1:

Table. 1: Monte Carlo probabilistic analysis settings in ANSYS software

Random variable	Distribution type	Number simulation loops	Number of iterations
Modulus of elasticity of concrete	Lognormal	22	2

4. Results

4.1 Finite element model validation

The finite element model is validated considering the comparison of hydrostatic pressure value obtained from analytical method and static analysis of the model under gravity acceleration. The hydrostatic pressure contour obtained from the static analysis of finite element model has been shown in Figure 5.

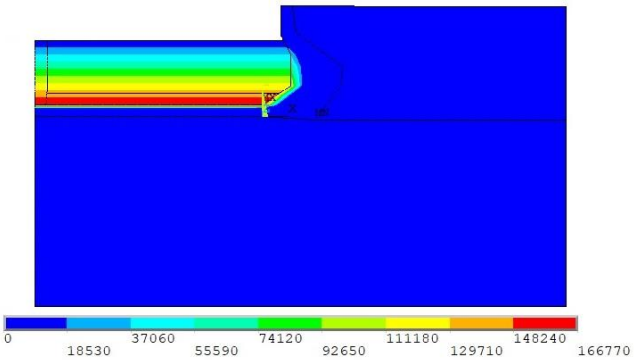

Fig. 5: Hydrostatic pressure contour obtained from the static analysis of finite element model (Pa)

The maximum value of hydrostatic pressure obtained from equation (26) compares the analytical and numerical model:

$$P = \rho g h = 1000 \times 9.8 \times 17 = 166770\ Pa \quad (26)$$

In which, P is the hydrostatic pressure (Pa), ρ is the water density (kg/m^3), and h is the height of the water in front of quay wall (meter). The comparison of the result shows that the value obtained from the static analysis of the model did not differ much from the calculation of equation (8).

4. 2 Cumulative distribution function

In this section, the cumulative distribution functions of output parameters are presented. On the other hand, CDF in the literature is referred to as distribution function, cumulative frequency function or cumulative probability function. Cumulative distribution function represents the possibility which assumes a value less than or equal to the value. In terms of continuous random variables, cumulative distribution function is obtained from probability density function by integration or by summation of discrete random variables [20]. Figures 6 and 7 show the cumulative distribution function of the selected responses of the model.

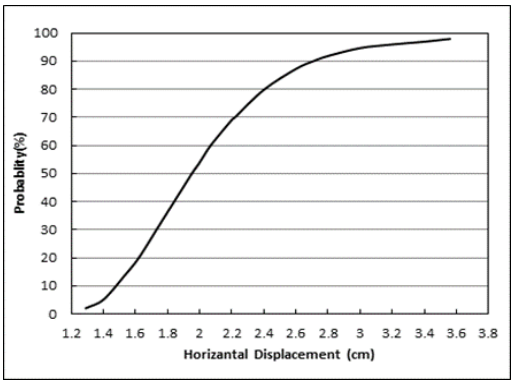

Fig. 6: The cumulative distribution function of the maximum horizontal displacement of crest

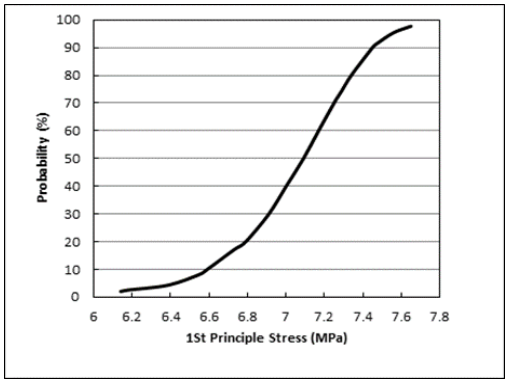

Fig. 7: The cumulative distribution function of the maximum principal tensile stress (node 46)

The cumulative distribution function is a probability which presents the values remaining lower than a certain amount. The curves show that:

- There is about 40% probability that the maximum displacement of top of quay wall remains less than 1.84 cm during an earthquake.
- There is about 70% probability that the maximum main tensile stress of quay-wall remains less than 7.25 Mpa during an earthquake.

4. 3 Sensitivity analysis of the model

Sensitivity refers generally to the changes in the output of a mathematical model due to the changes in the input quantities. Sensitivity analysis attempts to provide the ranking input assumptions of the model regarding their contribution to the variability or uncertainty [19]. The effect of modulus of elasticity of quay wall concrete as random input variable on outputs were evaluated and shown in Figures 8 and 9.

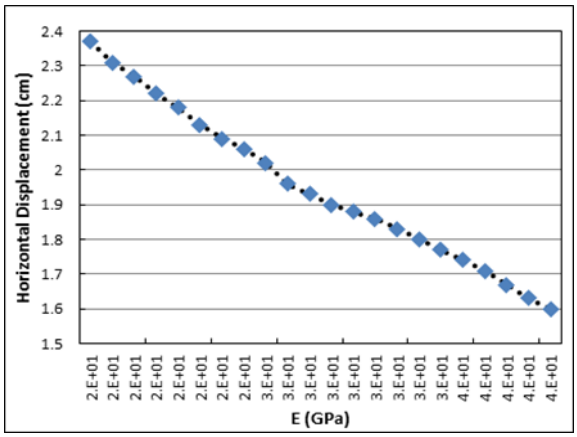

Fig. 8: Sensitivity curve of maximum horizontal displacement of crest to the modulus of elasticity of the wall

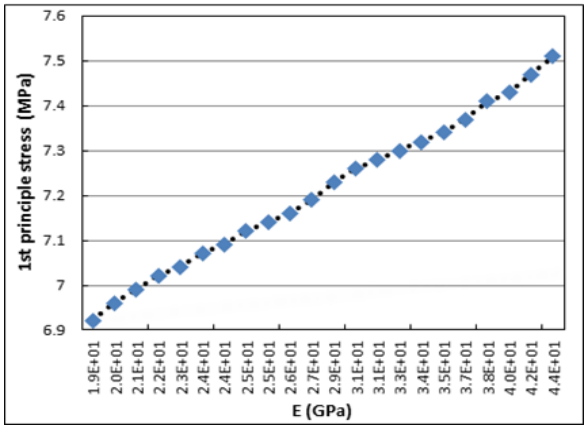

Fig. 9: Sensitivity curve of the main tensile stress to the modulus of elasticity of the wall (node 46)

Figure 8 illustrates that the graph is a linear curve whose slope is decreased from the modulus of elasticity of 26.4 Gpa onwards, and the displacement of the top of the wall is reduced due to an increase in modulus of elasticity.

As shown in Figure 9, the graph is almost a linear curve and the slight change in slope is observed in the modulus of elasticity of 26.7 Gpa onwards.

Subsequently, the maximum principal tensile stress in other parts of the structure body, which is more critical than others, was investigated according to contour of tension stress in Figure 10 and then displayed in Figure 11.

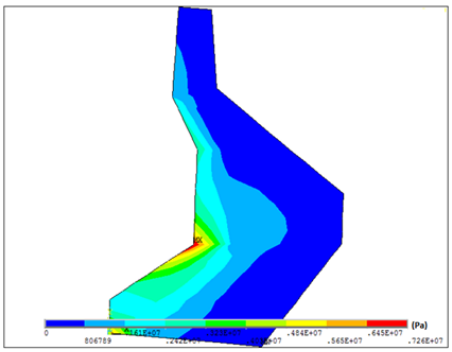

Fig. 10: The contour of the main tensile stress of the quay wall

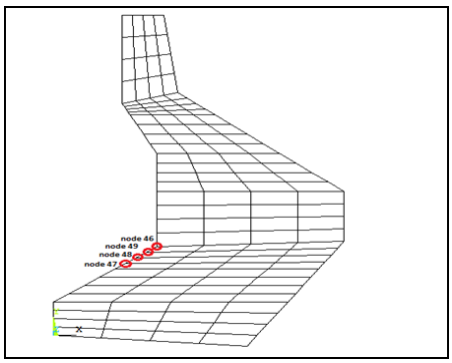

Fig. 11: Nodes with the highest main tensile stresses

The results given below allow tracing the extent of the concrete cracks compared to the principal allowed tensile stress. The modulus of elasticity of concrete approved by American Concrete Institute (ACI) is equal to:

$$E = 0.043 W_c^{1.5} \sqrt{f_c} \tag{27}$$

f_c is the compressive strength of concrete and W_c is the Specific weight of concrete.

Where the tensile strength (f_t) is equal to [20]:

$$f_t = 0.324 f_c^{2/3} \tag{28}$$

the dynamic tensile strength shall be equivalent to the direct tensile strength multiplied by a factor of 1.50 [21].

Sensitivity curves of the principal tensile stress to the modulus of elasticity of the quay wall for nodes 47 and 48 and 49 are presented in Figures 12 to 14, respectively.

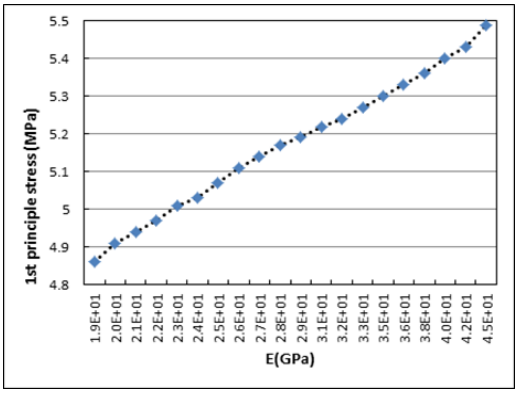

Fig. 12: The sensitivity curve of the main tensile stress to the modulus of elasticity of the wall node 47

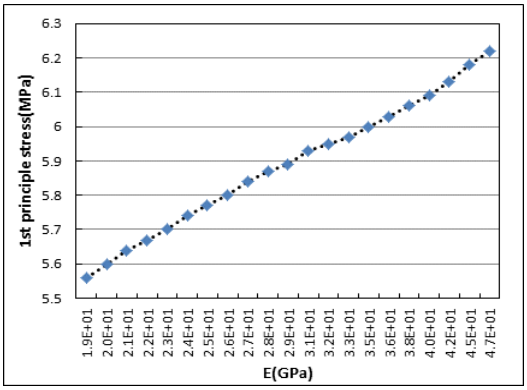

Fig. 13: The sensitivity curve of the main tensile stress to the modulus of elasticity of the wall in the node 48

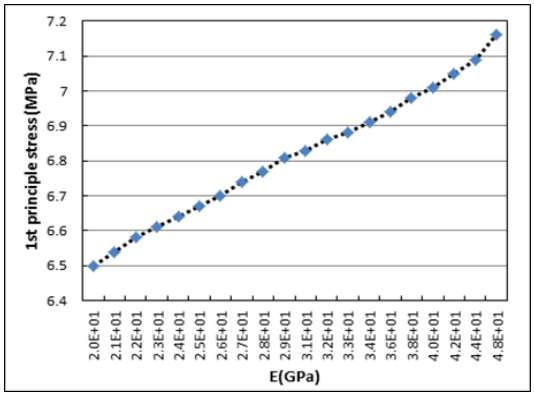

Fig. 14: The sensitivity curve of the main tensile stress to the modulus of elasticity of the wall in the node 49

Figure 15 shows the sensitivity graphs of the critical points as well as the seismic tensile stress of the concrete with respect to the modulus of elasticity.

According to Figure 15, it seen that in node 46 for the modulus of elasticity less than 39, in node 49 for the modulus of elasticity less than 37.2 Gpa, in node 48 for the modulus of elasticity less than 33 and in node 47 for the modulus of elasticity less than 30 Gpa, the concrete cracks.

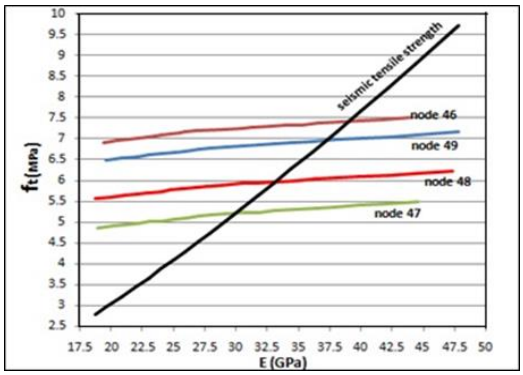

Fig. 15: Comparison of sensitivity curves with seismic tensile stress

5. Conclusion

Considering the importance of the effect of quay wall body strength on the seismic performance of model during an earthquake, this research evaluated the effect of Young Modulus of body concrete as strength parameters on seismic performance of quay wall model and examined the responses to achieve the optimal body stiffness using probabilistic analysis. Output parameters obtained from the analysis are maximum horizontal displacement of the structure and the maximum tensile stresses which are considered as the critical response. In addition, other critical points from the wall body were selected and studied to determine the location of the tensile cracks. Following results were achieved according to the model analysis and sensitivity curves:

1- Modulus of elasticity of concrete indicates the stiffness and strength of quay wall body and is an important parameter in the analysis and design of the system. Considering the sensitivity curves of responses to variation of Modulus of elasticity, it is possible to investigate the safety status of quay wall body for different values of this parameter.

2- According to the graphs it is concluded that if the maximum tensile stress is the main design criteria, the optimal value for the modulus of elasticity is about 30 Gpa. If the design criteria changes, according to the sensitivity curves, the optimal value of modulus of elasticity can be selected in a way that ensures safety.

3- According to tensile stress calculations in critical points of the quay wall and comparison with allowable seismic tensile stress, it was shown for which modulus of elasticity the cracks occur in the body.

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