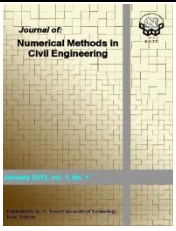
Numerical Methods in Civil Engineering, 6-1 (2021) 42-49



Numerical Methods in Civil Engineering

Journal Homepage: https://nmce.kntu.ac.ir/

Solving Wave Interaction with a Floating Breakwater in Finite Water Depth Using Scaled Boundary FEM

Meisam Qorbani Fouladi*, Hamid Heidary-Torkamani, Longbin Tao***, Behzad Ghiasi******

ARTICLE INFO

RESEARCH PAPER

Article history:
Received:
May 2021.
Revised:
June 2021.
Accepted:
July 2021.

Keywords:
Scaled Boundary Finite Element Method (SBFEM), Floating Breakwaters, Wave Interaction, Radiation, Different

Abstract:

This study aims to develop an efficient and accurate analytical-numerical model to analyze full interaction between seawater waves and cylindrical floating breakwaters in an infinite fluid domain of finite arbitrary water depth. Based on potential flow assumption, a semi-analytical Scaled Boundary Finite Element Method (SBFEM) in a two-dimensional vertical plane has been used to solve governing Laplace equations. The final equation in the scaled boundary coordinate system has been homogenized by locating the scaling center within each sub-domain. Hence, a diversity of particular solutions are omitted, leading to a unified solution process for radiation/different modes and wave diffraction problems. The accuracy, generality and robustness of the proposed SBFEM model have been evaluated by comparing the results of the proposed model with the reported results from the literature. By implementing the current SBFEM model, simulation results for radiation and diffraction problems are highly accurate compared to the result of other solutions.

1. Introduction

Floating breakwaters (FBWs) have been widely constructed all around the world for sheltering port basins, especially in deep-water areas. As these structures are fully exposed to surrounding water waves, there is always an interaction between FBWs and incident waves [1-4].

During past decades, many researchers have carried out numerical and experimental studies to investigate the wave interaction with FBWs [5-11]. Among different numerical approaches, the Scaled Boundary Finite Element Method (SBFEM) has been developed, perfected and extended in recent years to analyze the interactive systems in an infinite solution domain [12-17].

The SBFEM is a novel and emerging semi-analytical dynamics areas that combines the advantages of both the finite element and the boundary element methods [18].

In SBFEM, the spatial dimension is reduced by one, leading to the weakening of the governing differential equation in the circumferential direction, and then solving the weakened equation in the radial direction analytically. As the solution in the radial direction is analytical, the simulation is accurate and more importantly, fewer elements are needed to achieve a precise solution. The SBFEM has the inherent advantage of solving the unbounded fluid interaction problems in an accurate and efficient approach, and has been extended to solve water-structure interaction problem [17, 19].

In recent years, many researchers have carried out studies to investigate the interaction between FBWs and the wave domains using SBFEM. Critically related work can be found in [12]. Although remarkable research efforts have been performed on the wave interaction with FBWs, relatively limited attention has been given to the “full interaction” of wave domain with the FBWs. In other words, the understanding of the problem, however, is still far from being complete. One of the most evident limitations in the previous studies is that the interaction problem has not been

** School of civil engineering, college of engineering, University of Tehran, Tehran, Iran.*
*** Corresponding Author: Assistant Professor, Faculty of Civil Engineering, Sahand University of Technology, Tabriz, Iran. E-mail: hamid_heidary@sut.ac.ir*
**** Professor of Offshore Engineering, Department of Naval Architecture, Ocean & Marine Engineering, University of Strathclyde, Scotland, United Kingdom.*
***** Ph.D. of water resources, School of civil engineering, college of engineering, University of Tehran, Tehran, Iran.*

considered in a comprehensive way. The SBFEM-based studies have mainly focused on wave diffraction and radiation problems, rather than the problem of water wave full interaction with free-floated or moored FBW.

Among the rare ones, Fouladi et al. [12] studied the wave full interaction with a moored FBW with rectangular cross section. The current study aims to develop and extend the previous research by implementing the SBFEM-based model to solve the full interaction of FBW with a circular shape in a finite arbitrary depth. The results demonstrate that the proposed method by Fouladi et al. [12] can be extended efficiently to solve wave interaction problems with FBW having other types of cross sections, resulting in direct engineering applications.

As outlined in detail by Fouladi et al. [13], in the proposed model, the scaling centers of the bounded sub-domains are located within the associated solution domain, leading to the homogenization of the governing equations in the scaled boundary coordinate systems, and thus eliminating the need for organizing different forms of particular solutions.

"This study aims to develop an efficient and accurate analytical model to analyze full interaction between seawater waves and cylindrical floating breakwaters in an infinite fluid domain of finite arbitrary water depth."

2. Governing equation

The problem of interaction between a wave and a circular floating breakwater is illustrated in Fig.1 schematically. The wave is considered linear with a frequency of ω, propagating in water with a depth of H. It is assumed that the length of the breakwater is infinite, thus the effect of wave diffraction around the breakwater can be ignored. In order to define the governing equations of the surrounding wave domain, a Cartesian coordinate system $o-xyz$ is defined as shown in Fig. 1.

For a non-viscose fluid, the irrotational flow field can be expressed in terms of velocity potential. As a common practice, for a periodic motion, the time and space variables can be segregated, resulting in the following equation for velocity potential in an ideal fluid:

$$\Phi(x,z,t) = \mathrm{Real}\{\phi(x,z)e^{-i\omega t}\} \tag{1}$$

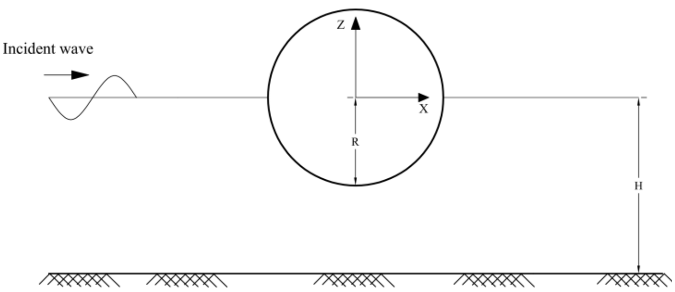


Fig. 1: Definition sketch of wave-floating breakwater interaction in infinite fluid domain

As per linear wave theory, the total velocity potential, ϕ_T, is summation of three velocity potentials as shown below:

$$\phi_T = \phi_I + \phi_s + \phi_R = \phi_I + \phi_s + \sum_{j=1}^{3} s_j \phi_j \tag{2}$$

The terms ϕ_I, ϕ_S, and ϕ_R stand for the incident wave potential, the diffraction, and the radiation velocity potentials, respectively. The term s_j denotes the amplitude of the j^{th} mode of motion where $j = 1, 2, 3$ represents heave motion, sway motion and roll motion respectively.

It is required that the potential ϕ satisfy Laplace equation within the fluid domain which can be described as follows:

$$\nabla^2\phi = 0 \tag{3}$$

The fluid domain under study can be expressed using following boundary conditions:

$$\partial\phi/\partial z = \omega^2/g\ \phi \text{ , at the free surface } (z = 0) \tag{4}$$

$$\partial\phi/\partial z = 0 \text{, at the bottom } (z = -H) \tag{5}$$

$$\partial\phi/\partial n = \overline{v}_n \text{, on the solid surfaces} \tag{6}$$

Where $\overline{v}_n$ is the normal velocity on a solid surface and g is the gravitational acceleration.

The incident wave potential, ϕ_I, can be calculated as:

$$\phi_I = -i\frac{gA'}{\omega}\frac{\cosh\ k(z+H)}{\cosh\ k(H)}e^{ikx} \tag{7}$$

Where k and A' denote wave number and wave amplitude, respectively. As the potential should result in finite values on the infinite boundary, the Sommerfeld boundary condition is applied as expressed below:

$$\lim_{x\to\pm\infty}\left(\frac{\partial\phi_i}{\partial x} \mp ik\phi_i\right) = 0 \tag{8}$$

The term ϕ_i is potential of each mode which has been defined in Equation (2).

3. Scaled boundary finite element method (SBFEM)

In the current study, SBFEM has been used to solve governing Laplace equation together with the boundary conditions discussed. In this regard, the unknown velocity potentials are solved numerically on the discretized boundaries, S, of the domain under consideration. Analytical solutions are utilized within each domain along radial lines connecting the central point called the scaling center (x_0, y_0) to any point on the boundaries. The governing equations are required to be transformed from Cartesian coordinate system into a new coordinate system (ξ, s) (shown on Fig. 2) in such a manner that the coordinate ξ is zero at the scaling center, while it is 1 on the boundaries. The circumferential coordinate, s, is defined along the boundaries. The scaling center should be placed in such a manner that all points on the boundaries are visible from the scaling center. To achieve this for the problem under study, the whole analytical domain is divided into four sub-domains. Ω_1 and Ω_4 are considered as unbounded sub-domains whereas Ω_2 and Ω_3 are bounded sub-domains. As shown in Fig. 2, the scaling center is located within each sub-domain, which leads to the homogeneity of the final equation in scaled boundary coordinate system, and can be used for non-rectangular objects (e.g. circular object).

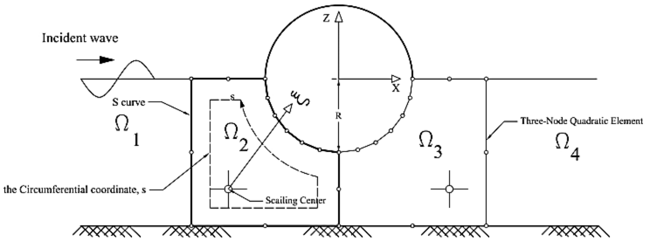


Fig. 2: Definition sketch of establishing scaled boundary coordinate system

3.1. Formulation for bounded sub-domains

For the purpose of formulation, two bounded sub-domains are considered in such a manner, that they have a common boundary at one side and each of them share a boundary with the unbounded sub-domains ($\Gamma_{\text{int}1}, \Gamma_{\text{int}3}$), on the other side (**Error! Reference source not found.**3). Considered discretization nodes on the boundaries of each bounded subdomain are shown in **Error! Reference source not found.**3.

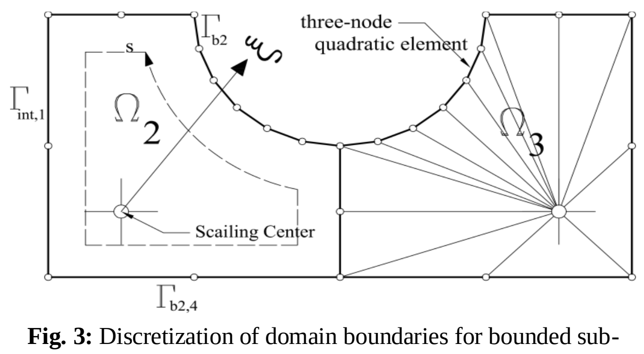


Fig. 3: Discretization of domain boundaries for bounded sub-domain

Each discretization node is defined by $\{x\}$ and $\{y\}$ vectors in Cartesian coordinate system. With the aid of shape functions, the coordinate of an arbitrary point within an element can be calculated. By applying the scaling coordinate ξ, any random point within the domain can be defined as a function of a point on the boundary S and the position of scaling center. Accordingly, the coordinate of an arbitrary point within each sub-domain $(\hat{x}, \hat{y})$, can be defined by:

$$\hat{x} = x_0 + \xi[N(s)]\{x\} \tag{9}$$

$$\hat{y} = y_0 + \xi[N(s)]\{y\} \tag{10}$$

Where $[N(s)]$ stands for the shape functions.

The velocity potential at any arbitrary point on a sector can be termed using the potential values of the nodes as follows:

$$\phi(\xi, s) = \left[N(s)\right]\left\{a(\xi)\right\} \tag{11}$$

The term $\{a(\xi)\}$ denotes the potential function along the lines connecting the discretization nodes to the scaling center.

As noted by Fouladi et al.[12], in scaled boundary coordinate system, the governing equations in terms of the potential function $\{a(\xi)\}$ and node flow function $\{q(\xi)\}$, can be expressed as:

$$\begin{aligned}&[E_0]\xi^2\{a(\xi)\}_{,\xi\xi} + ([E_0]+[E_1]^T \\ &-[E_1])\xi\{a(\xi)\}_{,\xi} - [E_2]\{a(\xi)\} = 0\end{aligned} \tag{13}$$

$$\{q(\xi)\} = [E_0]\xi\{a(\xi)\}_{,\xi} + [E_1]^T\{a(\xi)\} \tag{14}$$

3.2. Solution for the unbounded sub-domains

In order to solve far-field unbounded sub-domains using SBFEM, a modified coordinate system is defined as illustrated in **Error! Reference source not found.**4. The origin is placed on the interface of unbounded and bounded sub-domains. As shown in Fig. 4, unbounded sub-domains share parallel top and bottom side faces. It should be noted that only the boundary on the interface of unbounded and bounded sub-domains is considered to be discretized

Introducing the s coordinate along the mentioned interface boundary and ξ parallel to the bottom and to side faces and towards infinity, the modified coordinate system is formed. Thus, ξ changes from zero at the interface boundaries $\Gamma_{\text{int}1}$ or $\Gamma_{\text{int}3}$ to infinity at far end of Γ_∞.

The Cartesian and modified scaled boundary coordinate system can be related as:

$$\hat{x} = [N(s)]\{x\} + \xi \tag{15}$$

$$\hat{y} = [N(s)]\{y\} \tag{16}$$

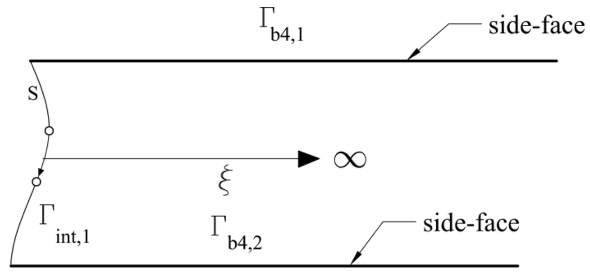


Fig. 4: Applying SBFEM for semi-infinite sub-domains with parallel side-faces

As reported by Fouladi et al. [13], the governing equations in the modified SBFEM coordinate system can be expressed as follows:

$$[E_0]\{a(\xi)\}_{,\xi\xi}+([E_1]^T-[E_1])\{a(\xi)\}_{,\xi} +(\bar{k}^2[M_0]-[E_2])\{a(\xi)\}=0 \quad (17)$$

$$\{q(\xi)\}=[E_0]\{a(\xi)\}_{,\xi}+[E_1]^T\{a(\xi)\} \quad (18)$$

3.3. Solution Process

To solve the governing equations in the scaled boundary coordinate system, a new variable $\{\chi(\xi)\}=\begin{Bmatrix} a(\xi) \\ q(\xi) \end{Bmatrix}$ has been employed in Equation (13). Accordingly, Equation (14) can be re-written as:

$$\xi\{\chi(\xi)\}_{,\xi}=[Z]\{\chi(\xi)\} \quad (19)$$

Using Equation (17) and the variable $\{\chi(\xi)\}$, Equation (18) is converted to:

$$\{\chi(\xi)\}_{,\xi}=[Z]\{\chi(\xi)\} \quad (20)$$

Where $[Z]$ stands for Hamilton matrix.

Solving this system of standard Eigen-value problems, gives the relation between $\{a(\xi)\}$ and $\{q(\xi)\}$. The algorithm developed for the process of this solution is demonstrated in Fig 5. Where $[K^b]$ and $[K^{Un-b}]$ are the stiffness matrices for bounded and unbounded sub-domains, respectively. Based on the matching conditions at the interfaces between the sub-domains n and $n+1$ (shown on eq. 21 and 22), the sub-domains are assembled.

$$\phi^n=\phi^{n+1}, \text{ at the interface between sub-domains} \quad (21)$$

$$\partial\phi^n/\partial n=-\partial\phi^{n+1}/\partial n, \text{ at the interface between sub-domains} \quad (22)$$

While the nodal potentials based on the assembled equations are solved, the wave pressure and subsequently the exciting forces and hydrodynamic coefficients can be determined.

4. Equations of motion (full wave-rigid body interaction)

The system of equations of motion describing sway, heave and roll modes, can be expressed as follows:

$$[-\omega^2(m_{ij}+\mu_{ij})-i\omega\lambda_{ij}+(c_{ij})]\Xi_i=F_i \quad (i,j=1,2,3) \quad (23)$$

Where the values associated with i and j respectively represent the components of each parameter with respect to sway, heave and roll movements of FBW. Ξ_i is amplitude of motion vector, F_i represents the resulting wave exciting force vector. m_{ij}, c_{ij}, λ_{ij} and μ_{ij} are the mass, hydrostatic stiffness, damping and added mass matrices of the floating body, respectively. Calculation of wave exciting forces, added mass and damping matrices will be discussed in section 5.1 as follows.

5. Results and Discussion

This section is divided into two parts. The first part deals with verification of the SBFEM model. For this purpose, both radiation and diffraction problems are simulated and compared against the same cases in other studies. In the second part, simulations are carried out for the problem of wave-floating breakwater interaction.

5.1. Radiation and Diffraction problems

In order to evaluate the current SBFEM-based model, the radiation problem is considered in the first step. For a structure with $H/R=2.0$, with R and H defined in Fig. (1), hydrodynamic coefficients (added mass and radiation damping) are calculated. In this modeling approach, there is a need for discretization of the structure surface boundaries. Different mesh densities are incorporated for discretization of these boundaries. Non-structural boundaries consist of two three-node quadratic elements on each boundary. The "26 Elements" and "50 Elements" mesh densities, consist of 12 and 36 three-node quadratic elements on body boundaries, respectively. Also, the unbounded sub-domains are modeled by two elements. The hydrodynamic forces applied to the structures are defined as:

$$F_{wj}=i\omega\rho\int_{s_0}(\Phi_w)n_j ds \quad (24)$$

Where n_j is the generalized normal with $n_1=n_z$, $n_2=n_x$ and $n_3=(z-z_0)n_x-(x-x_0)n_z$. (x_0,z_0) is the rotation center, ρ is the fluid density, S_0 is the wetted surface in $o-xz$ plane, and F_{wj} is the force in j direction caused by heave, sway and rotational movements ($w=1,2,3$) of the floating body, respectively. In order to calculate the integral in equation (24), the 13- points and 37-

points Newton–Cotes method were employed for the "26 Elements" and "50 Elements" mesh densities, respectively. The added mass, μ_{wj}, and damping coefficients, λ_{wj}, per unit length of the structure are calculated as follows:

$$\mu_{jw} = -\frac{1}{\omega^2}\mathrm{Re}(F_{wj}) \tag{25}$$

$$\lambda_{wj} = -\frac{1}{\omega}\mathrm{Im}(F_{wj}) \tag{26}$$

The non-dimensional added mass, C_a, and damping, C_d, coefficients can be defined as:

$$C_{awj} = -\frac{1}{0.5\rho\pi R^2}\mathrm{Re}(F_{wj}) \tag{27}$$

$$C_{dwj} = -\frac{1}{0.5\rho\pi R^2\omega}\mathrm{Im}(F_{wj}) \tag{28}$$

Where s is amplitude of forced displacement in radiation problem. As shown in **Error! Reference source not found.** 6 and Fig. 7, the results of added mass and damping coefficients in heave and sway movements are compared with the analytical solution proposed by Kwang [8] to demonstrate the accuracy of SBFEM-based model.

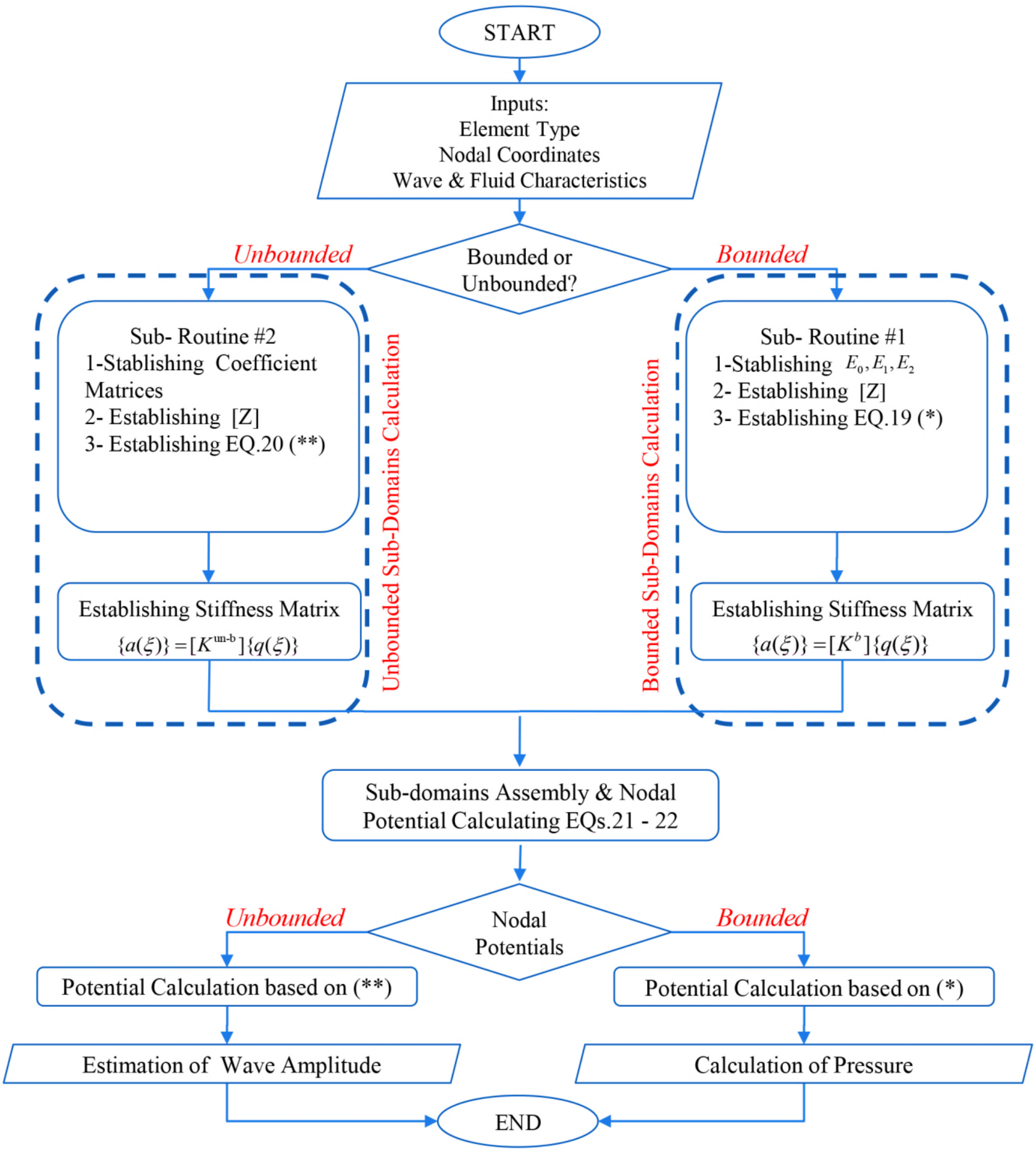


Fig. 5: The solution procedure of SBFEM-based model for 2DV analysis

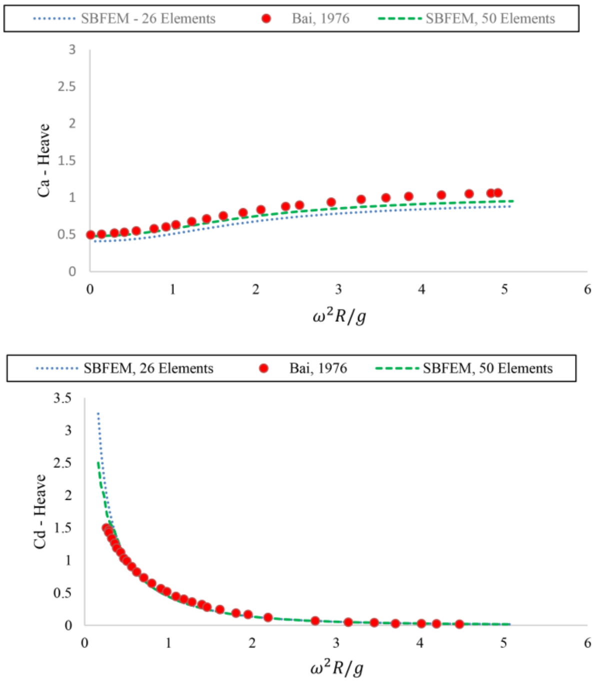


Fig. 6: Dim Dimensionless added mass and damping coefficients for circular structure heaving in calm water

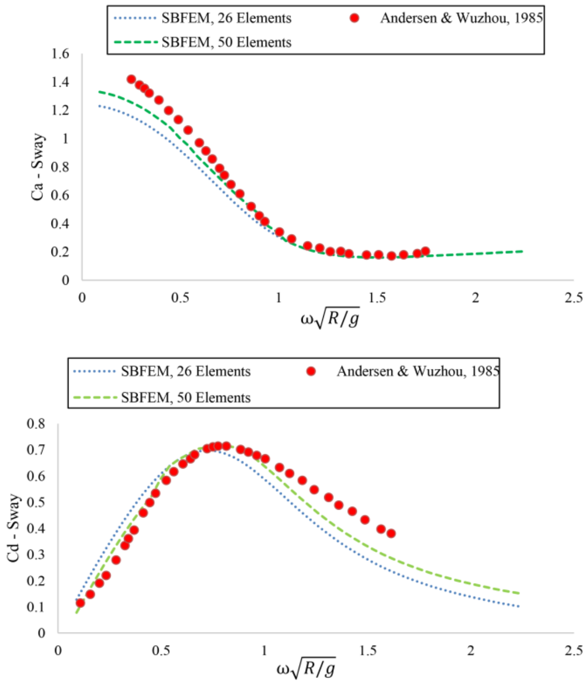


Fig. 7: Dimensionless added mass and damping coefficients for circular structure swaying in calm water

It is clear that the results are in good agreement with the bench mark [8] solution. For the purpose of evaluating the generality and robustness of the proposed model, the current SBFEM-based model is examined for wave diffraction problem with a circular cross-section. The incident wave at left boundary, Γ_{int1} generates normal velocities. Therefore, this problem could be solved using the same approach as radiation problem. The hydrodynamic loads that resulted from the incident wave on the stationary structures in k direction are calculated as:

$$F_k = i\omega\rho\int_{s_0}(\Phi_I + \Phi_S)n_k ds \tag{29}$$

The above exiting loads will be used for the simulation of full interaction between incident waves and floating breakwater. The components of forces applied on the structure are compared with the analytical results of [20] as illustrated in **Error! Reference source not found.**8. The dimensionless forces are calculated by $w_1 = \rho g A' R$. As is seen, the results of the developed model are in good agreement with the results of the analytical work.

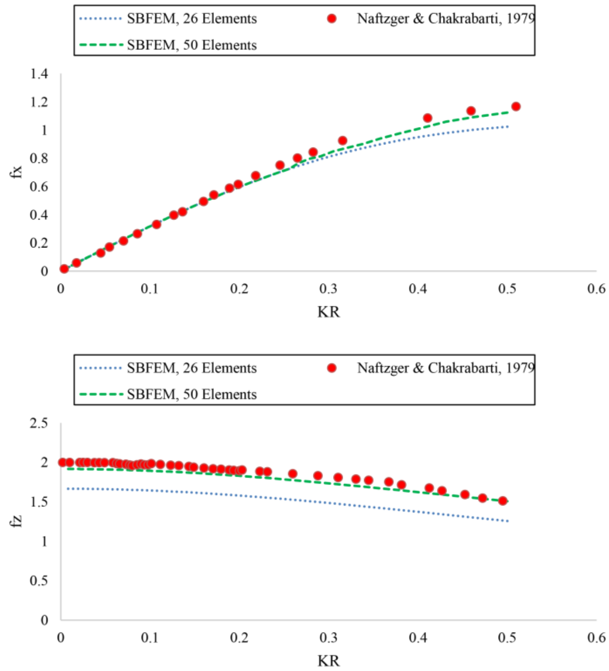


Fig. 8: Dimensionless wave forces for a circular structure

There are some differences between the calculated results and the results in the literature. Estimating the curve surface by linear elements and calculating the numerical integration in each element are the causes of these differences. These discrepancies can be eliminated by increasing the number of elements.

5.2. Full interaction between wave and floating breakwater

Considering the discussed problems in the previous sections, the hydrodynamic forces exerted on the floating object could be estimated by solving diffraction/radiation problems. In this section, the floating breakwater responses, interacting with incident waves are obtained from the proposed model and illustrated in Fig. 9.

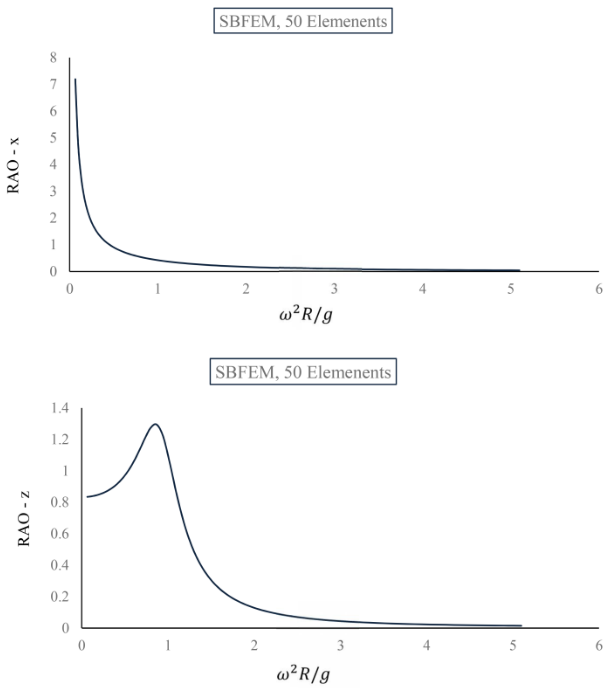


Fig. 9: RAOs in heave and sway as a function of normalized wave frequencies

6. Conclusion

The SBFEM method is used as a novel method to solve the problem of wave interaction with a circular cross section floating breakwater in an infinite fluid domain. By locating the scaling center within the bounded sub-domains, the governing equation in the scaled boundary coordinate system becomes homogeneous. This implementation eliminates the need for a variety of particular solutions and provides a unified solution process for different radiation modes and wave diffraction problems. By implementing the current SBFEM model, simulation results for radiation and diffraction problems are highly accurate when compared to the results of other solutions.

References:

[1] Abul-Azm, A.G., Gesraha, M.R., 2000. Approximation to the hydrodynamics of floating pontoons under oblique waves. Ocean Eng. 27, 365–384. https://doi.org/10.1016/S0029-8018(98)00057-2

[2] Gesraha, M.R., 2006. Analysis of Π shaped floating breakwater in oblique waves: I. Impervious rigid wave boards. Appl. Ocean Res. 28, 327–338. https://doi.org/10.1016/j.apor.2007.01.002

[3] Zheng, Y.H., Shen, Y.M., You, Y.G., Wu, B.J., Jie, D.S., 2006. Wave radiation by a floating rectangular structure in oblique seas. Ocean Eng. 33, 59–81. https://doi.org/10.1016/j.oceaneng.2005.04.005

[4] Zheng, Y.H., You, Y.G., Shen, Y.M., 2004. On the radiation and diffraction of water waves by a rectangular buoy. Ocean Eng. 31, 1063–1082. https://doi.org/10.1016/j.oceaneng.2003.10.012

[5] Cho, I.-H., 2016. Transmission coefficients of a floating rectangular breakwater with porous side plates. Int. J. Nav. Archit. Ocean Eng. 8, 53–65.

[6] Diamantoulaki, I., Angelides, D.C., 2011. Modeling of cable-moored floating breakwaters connected with hinges. Eng. Struct. 33, 1536–1552. https://doi.org/10.1016/j.engstruct.2011.01.024

[7] Jeong, K.L., Lee, Y.G., 2014. Numerical simulations of two-dimensional floating breakwaters in regular waves using fixed cartesian grid. Int. J. Nav. Archit. Ocean Eng. 6, 206–218. https://doi.org/10.2478/IJNAOE-2013-0173

[8] Kwang, J.B., 1976. The added mass and damping coefficients of and the excitation forces on four axisymmetric ocean platforms.

[9] Rahman, M.A., Womera, S.A., 2013. Experimental and numerical investigation on wave interaction with submerged breakwater. J. Water Resour. Ocean Sci. 2, 155–164.

[10] Song, H., Tao, L., 2011. Wave interaction with an infinite long horizontal elliptical cylinder. Proc. Int. Conf. Offshore Mech. Arct. Eng. - OMAE 1, 589–597. https://doi.org/10.1115/OMAE2011-49829

[11] Yamamoto, T., 1981. Moored floating breakwater response to regular and irregular waves. Appl. Ocean Res. 3, 27–36.

[12] Fouladi, M.Q., Badiei, P., Vahdani, S., 2021. A study on full interaction of water waves with moored rectangular floating breakwater by applying 2DV scaled boundary finite element method. Ocean Eng. 220, 108450.

[13] Fouladi, M.Q., Badiei, P., Vahdani, S., 2020. Extracting the Solution of Three-Dimensional Wave Diffraction Problem from Two-Dimensional Analysis by Introducing an Artificial Neural Network for Floating Objects. Lat. Am. J. Solids Struct. 17.

[14] Li, B., Cheng, L., Deeks, A.J., Teng, B., 2005a. A modified scaled boundary finite-element method for problems with parallel side-faces. Part I. Theoretical developments. Appl. Ocean Res. 27, 216–223. https://doi.org/10.1016/j.apor.2005.11.008

[15] Li, B., Cheng, L., Deeks, A.J., Teng, B., 2005b. A modified scaled boundary finite-element method for problems with parallel side-faces. Part II. Application and evaluation. Appl. Ocean Res. 27, 224–234. https://doi.org/10.1016/j.apor.2005.11.007

[16] Meng, X.N., Zou, Z.J., 2013. Radiation and diffraction of water waves by an infinite horizontal structure with a sidewall using SBFEM. Ocean Eng. 60, 193–199. https://doi.org/10.1016/j.oceaneng.2012.12.017

[17] Song, H., Tao, L. and Chakrabarti, S.K. (2010): Modelling of water wave interaction with multiplcylinders of arbitrary shape, Journal of Computational Physics, 229 (5), 1498-1513, 1 March 2010.

[18] Wolf, J.P., 2003. The scaled boundary finite element method. John Wiley & Sons.

[19] Tao, L, Song, H. and Chakrabarti, S.K. (2007): Scaled boundary FEM solution of short-crested wave diffraction by a vertical cylinder, Computer Methods in Applied Mechanics and Engineering, 197(14), 232-242, 1 December 2007.

[20] Naftzger, R.A., Chakrabarti, S.K., others, 1979. Scattering of waves by two-dimensional circular obstacles in finite water depths. J. Sh. Res. 23, 32–42.