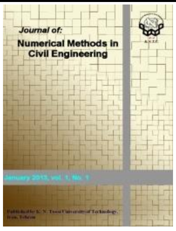

Numerical Methods in Civil Engineering

Influence of changes in the prestress force on dynamic specifications of the prestressed concrete beam

Behrouz Babakhani[*] and Hossein Rahami[**]

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Abstract:

Pre-stress techniques tend to increase cross-sectional capacity and optimize design. Such features have made these techniques rather popular. Over time, these structures tend to fail, and the reduction of pre-stress force over the lifetime of the structure is said to be one of the most significant destructive factors in these structures. This decrease in force occurs due to various reasons. In majority of cases, a cracking of pre-stressed concrete beams appears due to the reduction of pre-stress strength. Where there is permanent monitoring by the sensors buried within pre-stressed tendons, the force of the tendons can be investigated. But given the high cost of this procedure, such monitoring cannot be implemented in many structures. Nevertheless, it is possible to determine changes in pre-stress forces through investigating the dynamic behavior of structures. This paper examines the changes in the dynamic behavior of a pre-stressed beam experiencing a variety of pre-stress force levels. The idea is to monitor force changes in the pre-stressed tendons via examining certain dynamic characteristics of the beam (namely, modal damping and resonance frequency) and carry out repair prior to fail. The results show that with increasing the prestressing force, the system frequency increased and the system damping decreased.

1. Introduction

The 1950s marks the first round of research on dynamical experiments in structures. From 1956 on to 1967, small and medium-sized structures were subjected to dynamic assessments in Japan and the United States [1].Examples include the concrete structure of the 8th floor building by Mateisen and Angelkirk in 1957, and the 1962 steel framing frame by Hanison[2]. Advancements in measuring equipment made it possible in the later years to conduct research on larger structures such as dams. One of these studies typifies the vibrational study of the behavior of the Lower Crystal Spring Dam in the United States in 1979 by Wolf and his colleague [3]. In 1992, Roitman and Viero carried out a small-scale study on two fixed offshore platforms. The processes utilized in the latter study, and their results later found their way into official design codes.

**Corresponding Author: Assistant Professor, Department of Civil Engineering East Tehran branch, Islamic Azad University, Tehran, Iran. Email: babakhani2012@gmail.com*

***Associate Professor, School of Engineering Science, College of Engineering, University of Tehran, Tehran, Iran.*

In 1995, a European Research Center study investigated a concrete structure [4]. Brudy and Chery used six models of a degree of freedom and a real structure, all stacked by irregularly forced machines providing constant spectral density, and employed correlation signals to extract and analyze the dynamical specifications. Today, following the advances in computer sciences and developments of the Finite Element Method, modeling sees unprecedented, and research in this area is progressing advancing.

Many cable and suspended bridges employ pre-stressed cables externally. A number of researchers have investigated the dynamic behavior of cable and its dynamic response to pre-stress forces [5]. Irvine 1981, Shimada 1995, Russell and Lardner 1988, are among the pioneers of research in this area. There also are researchers, like Bui Khac 2006, who tend to focus on investigating and understanding ways to predict and estimate pre-stress forces and their secondary impact.

However, pioneering research on the impact of axial force on the dynamic response of beams was conducted by Saiidi et al., (1994), and Clough and Penzie (1993). Successively, various scientists have investigated the impact of axial pre-stress force on the dynamic behavior of structures, focusing on a variety of parameters[6-7].Various researchers have also applied eclectic methods in investigating changes of pre-stress force one of whom is, Tae Kim et al. (2008). Given the various factors in pre-stressing the beams and structures, the body of research in this area, especially concerning damage detection, is exponentially growing[8-10]. Some of the more recent examples of such studies are Toyota et al (2017), and Hyun Noh et al [11-12]. In the present study, the dynamic properties of a prestressed concrete beam are measured by changing the prestressing force. The results show that by increasing the prestressing force, the system frequency increased and the system damping decreased.

2. Experimental Modal Analysis

The dynamic equilibrium equation is a multi-degree of freedom system in the frequency domain according to Eq.1. This equation is then converted by the Laplace transform and turned into Eq. 2.

$$[m]\{\ddot{x}\} + [k]\{x\} = \{f\} \tag{1}$$

$$[B(\omega)]\{x(\omega)\} = \{F(\omega)\} \tag{2}$$

In Eq. 1, $[m]$ is mass matrix and $[k]$ is stiffness matrix. The equation is then solved by Pre-multiply $B[(S)]^{-1}$into Eq. 2. The reverse matrix of $B[(S)]$ is called the conversion function and is known by $[H(\omega)]$. $B[(S)]^{-1}$ can also be written as follows:

$$[B(S)]^{-1} = [H(\omega) = \frac{\mathrm{adj}[\mathrm{B}(S)]}{det[\mathrm{B}(S)]} = \frac{A(S)}{det[\mathrm{B}(S)]}] \tag{3}$$

The above equation allows us to prove, it somehow represents the mode shape. Furthermore, by expanding [H(ω)] to fractions of Polynomials, the frequency response function can be rewritten as in Eq. 4.

$$H(\omega)_{jk} = \sum_{r=1}^{n} \frac{A_{jk}}{\omega_r^2 - \omega^2} \tag{4}$$

In Eq. 1, $H(\omega)$ is Frequency response function. In the Polynomials of Eq. 4, whenever the loading frequency overlaps with the resonance frequency of the structure, the FRF sees peaks based on which dynamical characteristics of the system can be identified.

3. Examining the FDD (Frequency Domain Decomposition) in the Experimental Modal Analysis.

FDD (Frequency Domain Decomposition) is a method of extracting dynamical parameters of systems in the experimental modal analysis. In this method, unlike the classic method, there is no need to record the system input excitations. Here, parameters can be assessed by exclusively considering the recorded responses from the system. According to Brincker R. et al. (2001), the following relationship can be drawn between the spectral power density matrix of the output and the input of the system.

$$G_{yy}(\omega) = H(\omega) * G_{xx}(\omega) * H(\omega)^T \tag{5}$$

In Eq. 5, G_yy is the response power density matrix, G_xx is the system input power density matrix, and H(ω) is the frequency response function of the system. Through simplification, Equation 6 is provided. As shown in Equation 6, during resonance, only a limited number of modes take part in building the system response. When the natural frequency of the system prevails, one single mode will have the largest contribution to the system response. Thus the similarity of the response and the mode shape in this frequency. If the spectral density response power matrix for each frequency is divided into its singular vectors and values, then the frequency and damping mode shapes for each mode will be achieved [13-14]. By means of Equation 7, the input density power matrix can be reduced to singular values. The number of non-zero singular values represents resonance modes. However, to calculate the damping coefficients, the conventional half-power method is employed.

$$G_{yy}(\omega) = \sum_{k=1}^{n} \frac{d_k \emptyset_k {\emptyset_k}^T}{j\omega - \lambda_k} + \frac{\overline{d_k \emptyset_k {\emptyset_k}^T}}{j\omega - \overline{\lambda_k}} \tag{6}$$

$$G_{yy}(\omega) = U_i S U^H{}_i \tag{7}$$

$\emptyset_k$ is mode shape and ω is frequency of each mode.

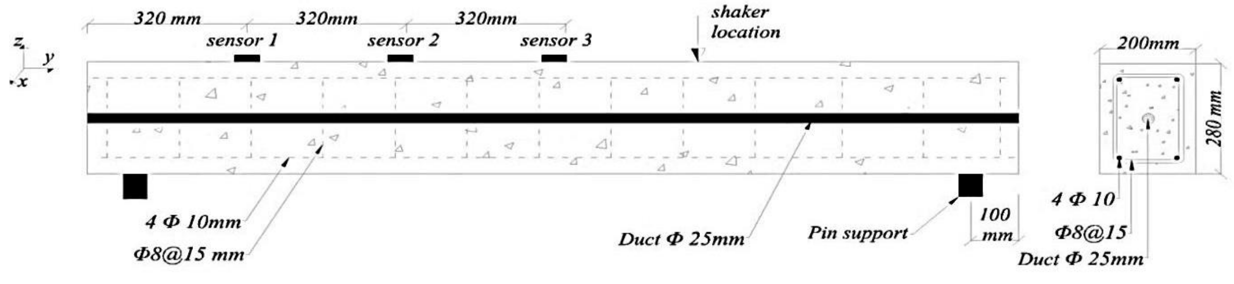

Fig. 1: Geometry and arrangement of sensors and shakers

4. Description of the tests

4.1. Geometric shape and specification of materials used in pre-stressed beam

In this research, a pre-stressed concrete beam with dimensions of 1940 x 280 x 200 mm was constructed. This beam was placed on two simple supports, placed at 10 cm from each end. Four longitudinal reinforcement bars with 10 mm diameter and stirrups of 8 mm were installed at every 15 cm. A mechanical shaker was employed to vibrate the concrete beam. This shaker was installed 30 cm to the left of the center of the beam. Also, on the other side of the beam, three accelerometers were mounted 32 cm away from each other. In Figure 1, a view of the shaker and accelerometer is shown. Given that the beam has bending and torsional vibrational frequencies in two directions, the three accelerometers on the beam allowed data to be recorded on all three X, Y, Z directions.

The concrete beam is then tested under different levels of pre-stress force via the modal experimental method. In Table 1, the names and pre-stress forces in each series of experiments is provided.

Table 1: Experiment series names

prestress force (kgf)	Test names
0	E0
5848	E5
9020	E9
12192	E12
15364	E15
18536	E18

In each series of experiments E0 to E18, tests are repeated three times to avoid possible errors. Also, each accelerometer is mapped in the three X-Y-Z directions depicted in Figure.1. In total, the series E0 to E18 provided 162 acceleration responses. In Figure 2, an overview of how the experiments and pre-stress of the beam are carried is illustrated. To apply the pre-stress force, a hydraulic jack with tensile strength of 30 tonf was employed. This jack was used in the experiments after calibration. Specifications of materials in this paper are in accordance with Table 2. In these experiments, a pre-stress (strand) of 0.6 inches with a final tensile strength of 18600 kgf/cm2 was applied to a pre-stress concrete beam. A 6-inch cable is was used to prestress the beam, but this cable disappeared through the cross section of the beam.

Table 2: Specifications of material

material	$\gamma(kg/m^3)$	$E(N/m^2)$	ν
Steel	7750	2.5E11	0.3
Concrete	2200	2.05E10	0.2

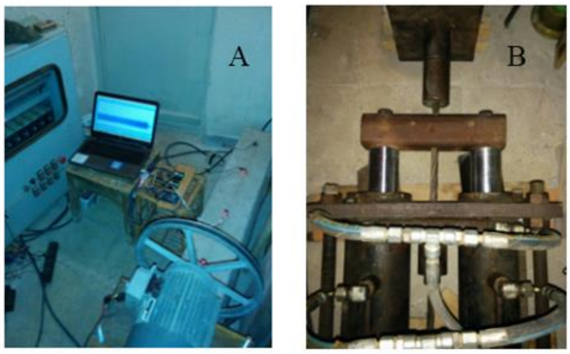

Fig. 2: A: View of the tests B: Prestress jack

4.2.Mechanical Shaker and excitation of the pre-stressed beam

In these laboratory tests, due to the flexural stiffness and the frequency range above the pre-arranged beam, this beam cannot be excited by an Impact Hammer. In order to stimulate the beam, a mechanical shaker was provided to produce a variable frequency sine wave. A single-mass mechanical shaker was made available to stimulate the concrete beam. To serve such purpose, a one horsepower three-phase engine with a speed of 1,400 rpm was utilized. At the end of the force transfer shaft, with a decentralization of 4 cm, a weight of 400 g was attached. The total weight of the shaker system was 20 kilograms. Figure 3 provides a view of the shaker from different angles.

Fig. 3: The mechanical shaker used in this

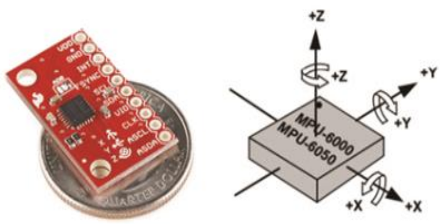

Fig. 4: MPU6050 accelerator module

A stepper motor was used to ensure that the engine speed is closely controlled and all of the tests have the same mode of propulsion.

4.3. MEMS accelerometer (MPU 6050)

In all experiments performed to record the acceleration response, the MPU6050 accelerometer module has been used. Various accelerometer modules have been designed and manufactured by various companies over the last few years. This is due to their widespread use in mobile phones, tablets, and other electronic gadgets. One of these modules is the MPU6050 -Adxl3. In some of these parts, along with the accelerometer module, a gyroscope module is implemented on the boards of handsets or tablets. As an example of the MPU6050 accelerometer module, a 6-axis IMU sensor (3 axis of acceleration - 3 axis of the gyroscope) was manufactured by Invensense. It has comprises six internal analog-to-digital (ADC) converters with a precision of 16 bits. The technology used in this sensor is based on MEMS and is powered by a 5-volt power supply where ranges can pick up acceleration. Figure 4 provides a view of the accelerometer in discussion. In this research, the Arduino control board has been used to connect the sensor to the computer[15-20]. This sensor is employed in many mobile phones, tablets and electrical appliances, due to the mass production which has made low prices and easy access to the consumers possible.

Several experiments on 15 different structures of one and a few degrees of freedom, and 3D laboratory frames were performed to verify responses received from this MPU6050 accelerometer. The aim was to ensure the accuracy of the acceleration recorded by this module. For an instance, we examined the results of the free vibration of a rod. The vibration created by the initial displacement of the head was created to vibrate in the same fashion as the first mode shape of the rod. The diameter of the rod in all experiments was 6 mm. Also, in the modeling of the rod in ABAQUS software, a wedge mesh (C3D10) of approximately 8 millimeters was arranged. The acceleration sampling rate in this experiment was 100. Table 3 shows the results of the free rotational vibration test and resonance frequency. In addition to the finite element modeling, using the analytical Equation 8, the frequency of the first-wave mode resonance was also received and the results extracted from the MPU6050 accelerometer module were tallied with these results.

$$f_n = \frac{k_n}{2\pi}\sqrt{\frac{EI}{WL^4}} \ for\ mode\ 1\ k_1 = 3.52 \tag{8}$$

In this Equation, E is the Young's modulus, I is the moment of inertia, W represents the weight of the meter, and L is the length of the beam.
As shown in Table 3, and according to many experiments on laboratory structures of different types and frequencies, the accuracy and validity of the results obtained from the MPU6050 module were testified
To check the accuracy of the results of mpu6050 accelerometer, we can refer to Determining Structural Resonance Frequency via MPU6050 Accelerometer Module (2018) Article by Babakhani - Rahmi - Karami Mohammadi.

Table 3: results from the free vibration experiment on the rod with

Height (mm)	Experiment (Hz)	Abaqus (Hz)	Eq(8) (Hz)	Error (%)
385	35.86	36.76	36.485	2.44
491	22.75	22.6	22.432	0.66
611	14.94	14.59	14.486	2.3
735	9.741	9.61	9.538	1.35

5. Results

In Figure.5, as a sample out of the 162 series of received acceleration responses from different points of the beam, results from three accelerometers where the pre-stress force is not transferred into the beam (E0 series test) were achieved.
In FDD method the SVD (Singular-value decomposition) and then the single particle number fragmentation method is employed in experimental modal analysis. Via this method, the resonance frequencies are extracted. Firstly, the power density matrix is calculated at various points where the acceleration of the points is recorded. However, due to a number of reasons, this matrix is not a symmetric one. It is not possible to calculate eigenvalues for eigenvector. Appropriate mathematical methods must be used to obtain an approximation of these values. SVD, or the singular value decomposition method is a mathematical method based on linear algebra. As samples, graphs of singular values of power density matrixes in the two directions of x and y in the E9 series of experiments are demonstrated in Figure 5.
Afterwards, according to the peak frequency method, the resonance frequency results were extracted from the above graphs. In each of the E0-E5-E9-E12-E15-E18 experiments, the trials were repeated three times. This was considered to prevent possible errors and noises. Figure 6

shows the first mode frequency of the lateral bending along X and the second mode, which shows the bending direction along Y.

To determine the modal damping of each mode via the half-power bandwidth method, and damping of each mode is calculated at the peak point of the resonant frequency. On this basis, according to Figure 7, mode damping changes in the first and second modes at different levels of pre-stress force have been investigated.

6. Conclusion

Results from the examination of the first and the second mode frequencies of the structure and the assessment of the concrete beam's behavior at various levels of the pre-stress force has shown that in both bending modes, changes in the resonance frequency do not vary much up to the 12192 kgf level .This approves the fact that the changes in the pre-stress force at lower pre-stress force levels has very little impact on the frequencies of the primary modes. However, as the pre-stress force grows larger than 12192 kgf, the resonance frequency grows accordingly. At 18536 kgf, the resonance frequency reaches 82.67 Hertz. Here, a 9.6 percent increase from the normal pre-stressless state is observed. In the second mode of the structure too, the resonance frequency reaches 96.7 Hertz at 18536 kgf, which means a 10.19 percent growth. Changes in resonance frequencies seem to be more explicit when it comes to higher modes. Furthermore, by examining the moderate damping of each mode it was observed that alterations in the pre-stress force, even at the initial loading levels, reduce the modal damping. Such decrease in damping can be observed as a drop in the damping variations diagram for both modes. As the results demonstrate, at the pre-stress level of 18536 kgf, the first and second modes respectively show 49% and 59.9% decreases. In general, changes are more pronounced as we consider higher modes. It is preferred to also consider modal damping along with resonance frequencies in damage detection. This way, diagnoses regarding the damaged parts will have higher validity because modal damping alterations show greater sensitivity to changes in the pre-stress force and prove more explicit in results.

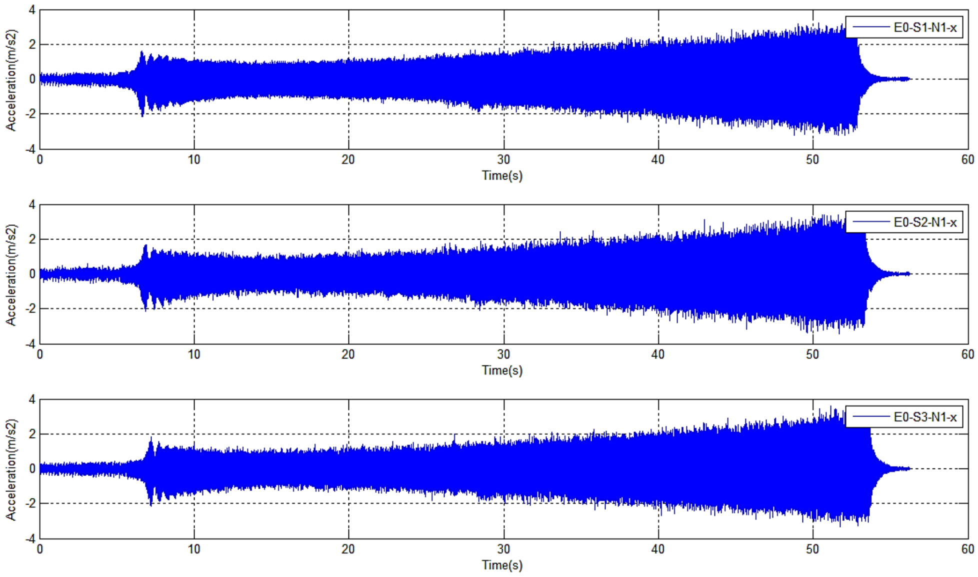

Fig. 5: Acceleration response in the E0 series experiments in direction

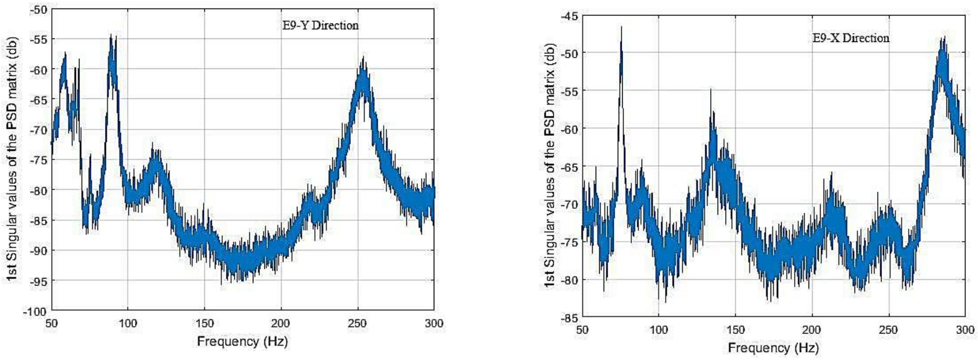

Fig. 6: singular values Spectrum of the Power Density Matrix in Y and X Directions in E9 Series

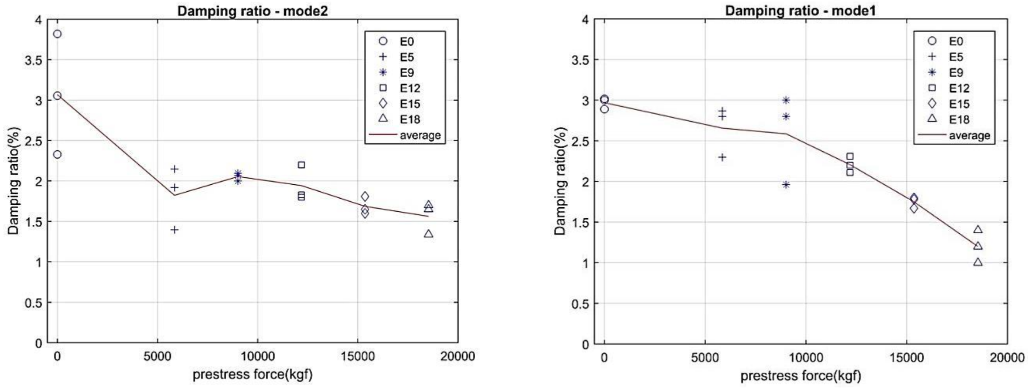

Fig. 7: Changes in the resonance frequency of the first and second modes due to the prestress force changes

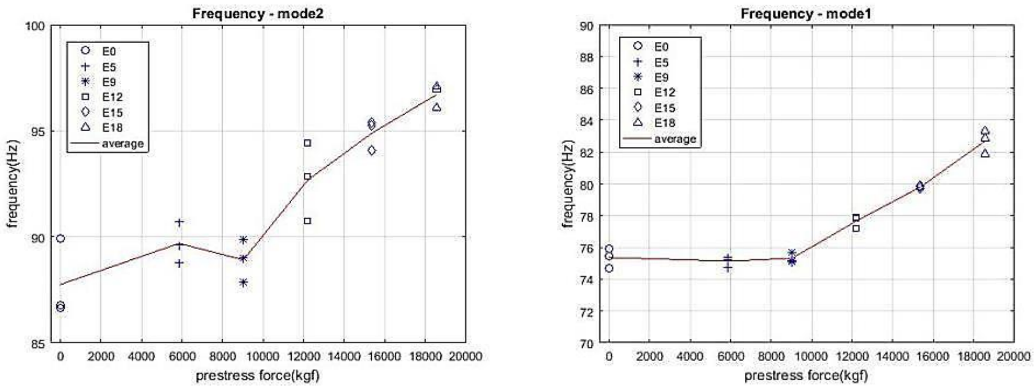

Fig. 8: mode damping at different prestress force levels

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