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Strain based panel elements for shear wall analysis

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Abstract:

The finite element method (FEM) can be applied to practically analyze the tall buildings in which the shear walls are used to resist the lateral loads. Accordingly, a variety of displacement and strain-based as well as frame macro elements have been proposed for analysis of the tall buildings. With respect to application of the lower order plane stress elements, analytical problems may arise within the numerical process of the finite element analysis. The analytical problems caused by the parasitic shear effects in finite elements and definition of an incompatible rotational coupling at the beam-column and beam-wall joints are the two major issues involved in analysis of the tall buildings. Moreover, such effects can give rise to shear locking based on definition of an incompatible rotational coupling at beam-wall joints. Subsequently, in this paper, new non-linear strain based finite elements are proposed to overcome some of the complications occurring due to above mentioned parameters. These strain-based panel type elements are comprised of eight degrees of freedom and have been formulated on the basis of the general beam elements. In conclusion, the proposed elements can be utilized to accurately analyze the shear walls on the condition that elements are of coarse size of mesh. In addition, a set of numerical analyses are conducted to evaluate the results and indicate that, changes in the power of the strain functions greatly affect the processor in structural modeling.

1. Introduction

Shear walls are commonly used in construction of tall buildings to increase resistance against the lateral loads, such as wind or seismic earthquakes acting on the buildings (Taranath 1998[20]). One of the common types of such walls are the coupled shear walls being formed because of the rows of openings that account for the architectural aspects such as windows, doors etc. Behavior of the coupled shear walls can be improved by incorporating stiffening beams at various levels. This induces additional axial forces, and thus reduces the bending moments in the walls as well as assisting to reduce the lateral deflection of the structures.

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There are a number of computational methods which have been developed in order to analyze the behavior of tall buildings (Ghalamzan Esfahani et al 2017[5], Moslehi Tabar et al 2017[14]). These analytical procedures can be categorized into the continuum methods, frame method to be classified in the solid wall and the wide column analogies, the finite element method (FEM) and the finite strip method (FSM) (Coull et al. 1991[3], Macleod et al. 1977[12], Kim et al. 2005[6]).

Application of the FEM is typically based on two main categories of displacement and strain-based finite elements. To date, all developed finite elements are not able to model and analyze buildings with great height. In this respect, the most serious issues include the absence of an exact definition for in-plane rotational freedom at each node, probable existence of parasitic shear effects in lower order elements as well as the inefficiency of using simple finite elements to model the coupling beams in such buildings (Macleod 1969[13], Thambiratnam et al. 1995[21], Öztorun 2006[15], Paknahad et al. 2007[16]).

To overcome these issues in analysis of tall buildings with low computational efforts involved, two newly developed strain-based finite elements are presented in this study. These finite elements were developed based on the governing displacement functions of a beam element and its corresponding strain functions. Both of these strain-based elements can demonstrate the internal shear-flexure interaction of shear wall panels in tall buildings. The degrees of freedom which were defined to both finite elements include horizontal and vertical translations as well as two in-plane rotations. The analytical concept employed to define the drilling degrees of freedom is the rotation of vertical fibers at connecting nodes. The novel panel elements are capable of accurately analyzing the shear walls with coarse size of mesh using a rational computational pace.

2. Formulation of the Proposed Panel Elements

As depicted in Fig. 1, the strain-based elements P1 and P2 proposed in this study, have been developed according to definition of bending behaviour of the beam elements. It should be noted that both panel elements P1 and P2 have the same analytical structure and shape. The nodal degrees of freedom pertained to the elements P1 and P2 include one horizontal and two vertical translations as well as one in-plane rotation which are defined at both chords of this element. According to Fig. 1, the degrees of freedom u_1, $\omega_1=-(\partial u_1)/\partial y$, v_1, v_2 and u_2, $\omega_2=-(\partial u_2)/\partial y$, v_3, v_4 are defined at lower and upper chords of the panel elements, respectively. According to literature, this is the only correct definition by which rotational compatibility can be ensured in modelling of the tall buildings (Desbois et al. 1989[4], Kwan et al. 1992[8], Cheung et al. 1994[1], Sabir et al. 1995[19]).

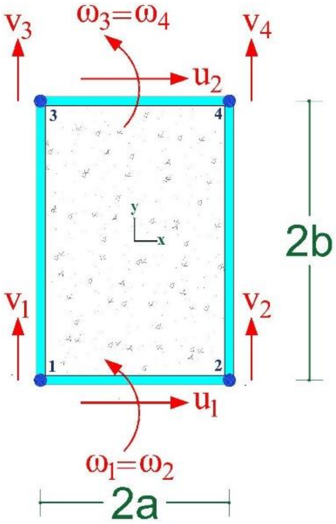

Fig. 1: The proposed strain-based panel elements P1 and P2

The strain fields of the panel elements P1 and P2 are given in the Eqs. (1a) to (1c):

$(\varepsilon_x)_{P1}=0,\ (\varepsilon_x)_{P2}=0$ (1a)

$(\varepsilon_y)_{P1}=\beta_1+\beta_2 x+\beta_3 xy+\beta_4 xy^2,$

$(\varepsilon_y)_{P2}=\beta_1+\beta_2 x+\beta_3 xy^2+\beta_4 xy^3$ (1b)

$(\gamma_{xy})_{P1}=\beta_5+\beta_4 y^3,$

$(\gamma_{xy})_{P2}=\beta_5+\beta_3 y^3$ (1c)

For both elements, horizontal strain is set to zero because of the assumption that horizontal strains are approximately negligible along the height of the panel element. This is an applicable simplification enabling to utilize the wide column elements in shear wall panels in tall structures. The axial strain along y-axis is shown by the coefficient β_1, and the strain functions ε_y are formulated by assuming a linear variation with x direction as given in Eq. (1b).
The coefficients in Eq. (1c) also denote a cubic variation for shear strain in line of y-axis. This formulation is based on the definition of a higher-order function for variation of γ_{xy} with height of the proposed panel elements which is similar to the phenomenon to be denoted for shear strain in Timoshenko beam theory (Reddy et al. 1997[18], Kwan 1993[10], Prathap 1982[17]).
The three coefficients β_7, β_8 and β_9 are required to represent the rigid body displacement functions. Both rigid body displacement functions are added to those obtained by integrating the strain functions ε_x and ε_y given in Eqs. (1a) and (1b). Hence, the displacement functions u(y) and v(x,y) are defined as follows:

$u(y)_{(P1\&P2)} = \beta_7-\beta_9 y+f_1(y)$ (2a)

$v(x,y)_{(P1\&P2)} = \beta_8+\beta_9 x+\int\varepsilon_y dy+f_2(x)$ (2b)

Two complementary functions $f_1(y)$ and $f_2(x)$ are obtained by conducting a few algebraic computations upon the analytical expression for the shear function γ_{xy} as follows:

$f_1(y)_{P1}=-\beta_9 y-\beta_2(y^2/2)-\beta_3(y^3/6)+\beta_4(y^4/6)+\beta_5 y$ (3a)

$f_1(y)_{P2}=-\beta_9 y-\beta_2(y^2/2)+\beta_3(y^4/6)-\beta_4(y^5/20)+\beta_5 y$ (3b)

$f_2(x)_{P1\&P2}=0$ (3c)

The full displacement functions u(y) and v(x,y) are presented in Eqs. (4a) and (4b):

$u(y)_{P1}=\beta_7+(\beta_5-2\beta_9)y-\beta_2(y^2/2)-\beta_3(y^3/6)+\beta_4(y^4/6),$

$u(y)_{P2}=\beta_7+(\beta_5-2\beta_9)y-\beta_2(y^2/2)+\beta_3(y^4/6)-\beta_4(y^5/20)$ (4a)

$v(x,y)_{P1}=\beta_8+\beta_9 x+\beta_1 y+\beta_2 xy+\beta_3(xy^2)/2+\beta_4(xy^3)/3,$

$v(x,y)_{P2}=\beta_8+\beta_9 x+\beta_1 y+\beta_2 xy+\beta_3(xy^3)/3+\beta_4(xy^4)/4$ (4b)

The lateral displacement function u(y) of the elements P1 and P2, is of order of four and five with respect to y direction, respectively. It can be concluded that the finite elements P1 and P2 are able to represent a bending mode in contrast to the bilinear finite elements. Therefore, the finite elements P1 and P2 are not affected by the parasitic shear effects.

The eight coefficients β_1 to β_8 in two governing displacement functions associated with both panel elements P1 and P2, are determined by equating the nodal translations and rotations to the eight degrees of freedom defined for each of the elements and solving the resulting equations. Following this approach and using Eqs. (2a) and (2b), the strain-displacement matrix [B] associated with the elements P1 and P2 is developed as indicated in the appendix.

The stiffness matrix of the elements is achieved by the standard expression as noted in Eq. (5).

$$[K]=t\int[B]^t[D_m][B]dA \quad (5)$$

$$[D_m]=diag[E,F,G] \quad (6)$$

Parameter t represents thickness of the panel element shown in Fig. 1. It should be noted that because of assuming $\varepsilon_x=0$, the material matrix $[D_m]$ would be a diagonal matrix of rank three as presented in Eq. (6). The diagonal components are E_x, E_y and G_{xy}, respectively. In the case of isotropic material, both E_x and E_y are equal to E as Young's modulus of elasticity. Hence, the parameter G_{xy} should also be equal to G, named as shear modulus of elasticity.

3. Application of the Proposed Panel Elements

As mentioned previously, presence of the parasitic shear effects in the finite elements compel them to behave in an excessively stiff manner in the case of bending mode. Importantly, this problem has not been resolved by using the Q4 finite element unless the elements are of fine size of mesh. Given the fact that tall buildings are subjected majorly to bending actions, this is indeed an evident problem. This drawback arises from the incapability of the elements with lower order to adapt themselves to the deformed shape of the structure which becomes more intense in the case of bending actions (Cook 1975[2], Desbois et al. 1989[4], Kwan 1993[9,11], Sabir et al. 1995[19]). However, the best approach to cope with the parasitic shear effects is to avoid them through applying the finite elements which are able to represent the strain state of pure bending. Consequently, it can be concluded that the panel elements P1 and P2 are not subjected to parasitic shear effects.

Accordingly, these elements are rectangular with four nodes and four degrees of freedom at the upper and lower chords. The nodal degrees of freedom at each chord consists of one horizontal translation, two vertical translations, and an in-plane rotation representing the nodal rotation of the vertical fiber. Both elements P1 and P2 acceptably fulfill the aforementioned criteria to act as proper elements for modelling process of the tall buildings.

4. Numerical Examples

Four example structures were chosen for verification of the proposed approach. The numerical results of two well-converged finite elements are used and compared with those obtained from the proposed elements as presented below:

1. SC: The four-node displacement-based quadrilateral element with 8 DOFs (Cheung and Kwan 1994[1]).
2. Kwan: The four-node strain-based quadrilateral element with 8 DOFs (Kwan 1992[8]).

In the case of stiffened coupled shear wall, the results have been compared with the continuum method (Kuang et al 1998[7]) in addition to the above elements. In all examples, the following properties have been considered for the materials:

Young's modulus: $E=2\times10^5$ (kg/cm^2)

Poisson's ratio: $\upsilon=0.25$

It is important to note that only one layer of elements was used per each story except for the second example dealing with mesh sensitivity analysis.

4.1. Four-bay shear wall structure

As shown in Fig. 2, this example deals with a coupled shear wall with four bays. The total height of the structure is equal to 64.00m, height of each storey is 4.00m, the wall width is 4.00m and the free span of connecting beams is set equal to 2.00m. The height of connecting beams is 80cm and the wall thickness is 40cm, respectively. Table1 presents the maximum horizontal displacement and stress related to this structure at the reference levels. It is acceptable to see small discrepancies among the results. The main source of the differences among the results can be attributed to the assumptions made for each element.

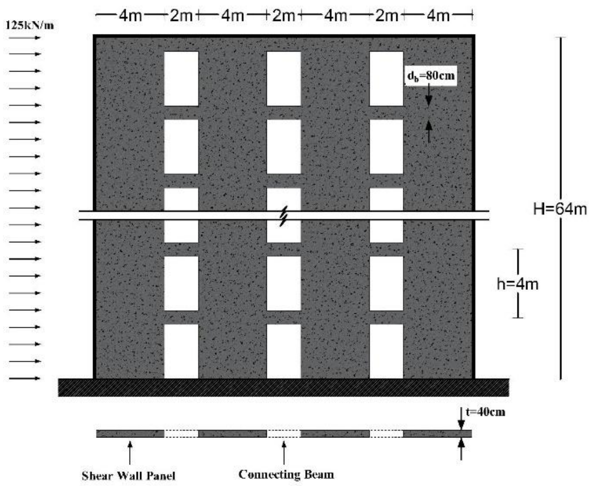

Fig. 2: Four-bay shear wall structure

As shown in Fig. 2, this example deals with a coupled shear wall with four bays. The total height of the structure is equal to 64.00m, height of each storey is 4.00m, the wall width is

4.00m and the free span of connecting beams is set equal to 2.00m. The height of connecting beams is 80cm and the wall thickness is 40cm, respectively. Table 1 presents the maximum horizontal displacement and stress related to this structure at the reference levels. It is acceptable to see small differences among the results. The main source of the variations among the results can be attributed to the assumptions made for each element.

Table 1: Analytical results for the four-bay shear wall structure

	SC	Kwan	P1	P2
$\Delta_{max}\ (cm)$	7.0183	6.9695	6.9719	6.9338
$\sigma_{max}\ (kg/cm^2)$	130.7124	160.9789	182.7383	163.8474

4.2. Single shear wall structure

The second example is a 10-storey single shear wall structure as shown in Fig. 5. The structural dimensions and applied loads are depicted in Fig. 3. To evaluate the rate of convergence for the proposed elements, mesh sensitivity analysis has been performed in this example. In this respect, five types of meshes have been chosen for each panel as shown in Fig. 3. In Table 2 and Fig. 4, the trend of convergence for displacement responses in relation to various types of meshes has been illustrated for panel elements P2 and P1, respectively.

Given the fact that mesh size plays an important role in rate of convergence and independence of the response of the proposed elements from the mesh type, it is concluded that these elements can be properly converged in the case of coarse meshes with an acceptable rate.

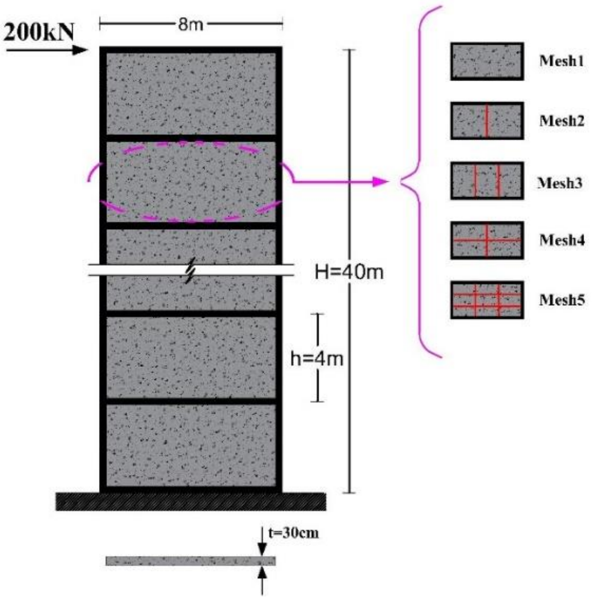

Fig. 3: Single shear wall model

Table 2: Analytical results for the single shear wall structure-Panel Element P2

Height (m)	$\Delta_{max}\ (cm)$				
	Mesh1	Mesh2	Mesh3	Mesh4	Mesh5
0	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.0266	0.0280	0.0301	0.0279	0.0288
8	0.1002	0.1028	0.1071	0.1015	0.1032
12	0.2134	0.2169	0.2233	0.2150	0.2176
16	0.3617	0.3663	0.3749	0.3636	0.3670
20	0.5400	0.5456	0.5563	0.5422	0.5465
24	0.7432	0.7499	0.7628	0.7458	0.7509
28	0.9665	0.9742	0.9893	0.9694	0.9753
32	1.2048	1.2136	1.2308	1.2079	1.2147
36	1.4530	1.4629	1.4823	1.4565	1.4642
40	1.7063	1.7172	1.7388	1.7101	1.7186

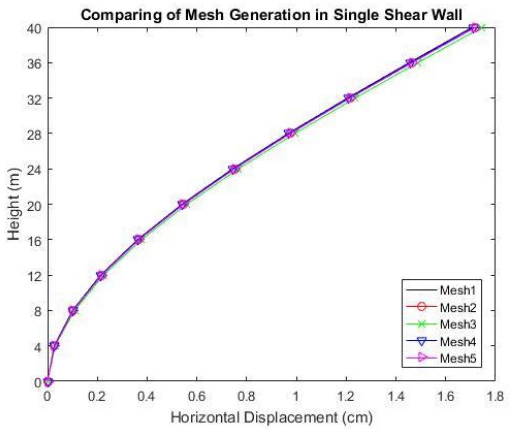

Fig. 4: Comparison of the results of the example structure-Panel Element P1

4.3. Stiffened coupled shear wall structure

In this case, a 14-storey coupled shear wall structure is considered as shown in Fig. 3. This structure is comprised of one stiffening beam at level of 16.00m. The shear wall is fixed at the base. The total height of the example structure is 56.00m, the storey height is 4.00m, the wall widths are 4.00m and the free span of connecting beams is 2.00m. The height of connecting beams is 80cm and the wall thickness is also 40cm.The height of stiffening-beam is 160cm. The problem in the study is analysed in both aspects using continuous connection and finite element methods. The numerical results are presented in Table 3 and Table 4. It can be seen from the results obtained by the panel elements and those calculated by the continuum method (Kuang et al 1998[7]) agree acceptably with each other.

The differences arise from the fact that in the laminar analogy, the wall panels are connected at the corners continuously along the height. Moreover, in the finite element procedure, the wall panels are typically connected at the floor levels, particularly when employing the panel elements. In addition, the assumption of neglecting shear deformation effects, might lead to errors in the case of tall buildings.

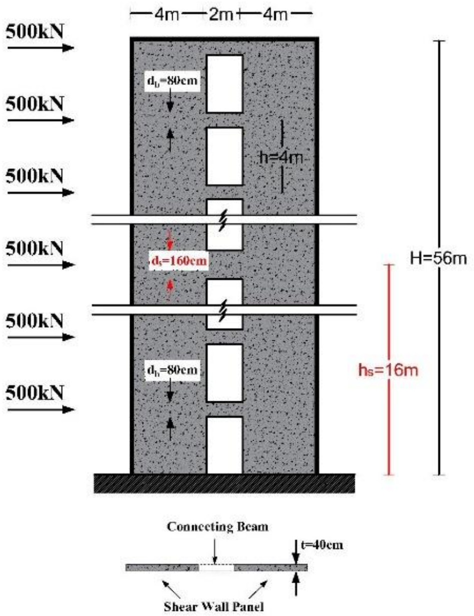

Fig. 5: Single shear wall model

uniform in height. The total height of the example structure is 95.00m, storey height is 3.80m and both wall widths are 6.00m. The free span of all connecting beams is 2.00m. The wall thickness is 30cm and the cross section of connecting beams is 30x30cm respectively. The dynamic characteristics of this coupled shear wall have already been analyzed by Kuang (Kuang et al 1998[7]). According to the analysis of this example structure which is based on using both panel elements P1 and P2, the resulted structural parameters were obtained.

Table 4 shows a number of resulted first natural frequencies of vibrations due to the example structure. It is worth mentioning that Kuang's research has been accomplished based on the discrete-continuum methodology. Therefore, it is acceptable to see differences among the results from various researches.

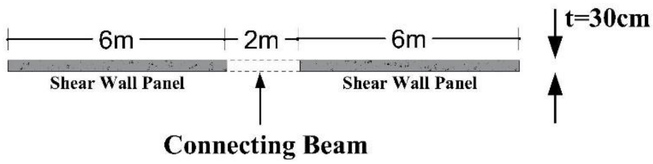

Fig. 6: Coupled shear wall model

Table 3: Analytical results for the coupled shear wall structure

	Coupled Shear Wall	
	$\Delta_{max}\ (cm)$	$\sigma_{max}\ (kg/cm^2)$
The Panel Element P1	29.1515	453.1397
The Panel Element P2	29.0990	415.2383
The Continuum Method	29.2899	419.2103
The Displacement-Based Element SC	29.1262	347.9820
The Strain-Based Element Kwan	29.1470	409.5487

Table 4: Analytical results for the stiffened coupled shear wall structure

	Stiffened at Elevation of 16m	
	$\Delta_{max}\ (cm)$	$\sigma_{max}\ (kg/cm^2)$
The Panel Element P1	28.6124	450.7852
The Panel Element P2	28.5599	412.3530
The Continuum Method	28.2388	412.0527
The Displacement-Based Element SC	28.5959	345.5464
The Strain-Based Element Kwan	28.6030	407.1902

4.4. Natural frequencies of coupled shear wall structure

The presented panel elements P1 and P2 have been used to assess the natural frequencies of a coupled shear wall structure as shown in Fig. 6. The example structure is

Table 5: Natural frequencies (Hz) of the example coupled shear wall structure

	Vibration Mode Number		
	1	2	3
P1	0.60	2.84	6.62
P2	0.63	2.90	6.74
Kuang et al 1998 (The Discrete-Continuum Method)	0.67	2.93	7.16

5. Conclusion

The lower order finite elements are likely to be influenced by the parasitic shears especially when meshes are of coarse sizes, and this problem leads the elements to behave in an unduly stiff manner under a bending mode. The proposed finite elements P1 and P2 are able to represent the state of pure bending and, they were also found to be free of parasitic shear effects. The lateral displacement function of the proposed elements does not linearly vary within height and can represent a lateral deflected shape of order of two and higher with respect to variable y. The capability of simulating the plane panels forming the shear walls and ability to generate compatible rotations with lintel beams in tall buildings, are the most remarkable reasons to apply the finite elements proposed herein. It is worthwhile that the strain-based finite elements act markedly well under both shear and bending modes. The in-plane rotational degrees of freedom are defined as the rotation of the vertical fibres at upper and lower chords of the panel elements. The application of the proposed panel elements are convenient and flexible to analyse the tall buildings. Accordingly, the results are found to be in an acceptable agreement with those obtained from similar analytical methods. The results indicate that applying this panel element requires only one layer of elements per story. It is worthwhile that the proposed element has been developed for analysis of the shear walls and if used for the other structural elements, its performance has to be evaluated.

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APPENDIX

The stiffness matrix due to strain-based panel elements P1 and P2 are given as follows:

The strain-displacement matrix [B] of the strain-based panel elements P1 and P2 in the formulations are given as follows: The displacement vector {D} includes all the eight degrees of freedom, defined in the Fig. 1. This vector is identical for both panel elements P1 and P2:

$\{D\}=\{u_1,\omega_1,v_1,v_2,u_2,\omega_2,v_3,v_4\}$^T

$$[K]_{P1} = Et\begin{pmatrix}
\frac{a^3}{b^3} & & & & & & & \\
-\frac{a^3}{b^2} & \frac{139a^3}{135b}+\frac{4ab}{7(1+v)} & & & & \text{Sym.} & & \\
0 & \frac{2a^2}{135b}+\frac{2b}{7(1+v)} & \frac{46a}{135b}+\frac{b}{7a(1+v)} & & & & & \\
0 & -\frac{2a^2}{135b}-\frac{2b}{7(1+v)} & \frac{43a}{270b}-\frac{b}{7a(1+v)} & \frac{46a}{135b}+\frac{b}{7a(1+v)} & & & & \\
-\frac{a^3}{b^3} & \frac{a^3}{b^2} & 0 & 0 & \frac{a^3}{b^3} & & & \\
-\frac{a^3}{b^2} & \frac{131a^3}{135b}+\frac{3ab}{7(1+v)} & -\frac{2a^2}{135b}+\frac{3b}{14(1+v)} & \frac{2a^2}{135b}-\frac{3b}{14(1+v)} & \frac{a^3}{b^2} & \frac{139a^3}{135b}+\frac{4ab}{7(1+v)} & & \\
0 & -\frac{2a^2}{135b}+\frac{3b}{14(1+v)} & -\frac{46a}{135b}+\frac{3b}{28a(1+v)} & -\frac{43a}{270b}-\frac{3b}{28a(1+v)} & 0 & \frac{2a^2}{135b}+\frac{2b}{7v+7} & \frac{46a}{135b}+\frac{b}{7a(1+v)} & \\
0 & \frac{2a^2}{135b}-\frac{3b}{14(1+v)} & -\frac{43a}{270b}-\frac{3b}{28a(1+v)} & -\frac{46a}{135b}+\frac{3b}{28a(1+v)} & 0 & -\frac{2a^2}{135b}-\frac{2b}{7(1+v)} & \frac{43a}{270b}-\frac{b}{7a(1+v)} & \frac{46a}{135b}+\frac{b}{7a(1+v)}
\end{pmatrix}$$

$$[K]_{P2} = Et\begin{pmatrix}
\frac{25a^3}{21b^3} & & & & & & & \\
-\frac{25a^3}{21b^2} & \frac{1153a^3}{945b}+\frac{4ab}{7(1+v)} & & & & \text{Sym.} & & \\
0 & \frac{2a^2}{135b}+\frac{2b}{7(1+v)} & \frac{46a}{135b}+\frac{b}{7a(1+v)} & & & & & \\
0 & -\frac{2a^2}{135b}-\frac{2b}{7(1+v)} & \frac{43a}{270b}-\frac{b}{7a(1+v)} & \frac{46a}{135b}+\frac{b}{7a(1+v)} & & & & \\
-\frac{25a^3}{21b^3} & \frac{25a^3}{21b^2} & 0 & 0 & \frac{25a^3}{21b^3} & & & \\
-\frac{25a^3}{21b^2} & \frac{1097a^3}{945b}+\frac{3ab}{7(1+v)} & -\frac{2a^2}{135b}+\frac{3b}{14(1+v)} & \frac{2a^2}{135b}-\frac{3b}{14(1+v)} & \frac{25a^3}{21b^2} & \frac{1153a^3}{945b}+\frac{4ab}{7(1+v)} & & \\
0 & -\frac{2a^2}{135b}+\frac{3b}{14(1+v)} & -\frac{46a}{135b}+\frac{3b}{28a(1+v)} & -\frac{43a}{270b}-\frac{3b}{28a(1+v)} & 0 & \frac{2a^2}{135b}+\frac{2b}{7(1+v)} & \frac{46a}{135b}+\frac{b}{7a(1+v)} & \\
0 & \frac{2a^2}{135b}-\frac{3b}{14(1+v)} & -\frac{43a}{270b}-\frac{3b}{28a(1+v)} & -\frac{46a}{135b}+\frac{3b}{28a(1+v)} & 0 & -\frac{2a^2}{135b}-\frac{2b}{7(1+v)} & \frac{43a}{270b}-\frac{b}{7a(1+v)} & \frac{46a}{135b}+\frac{b}{7a(1+v)}
\end{pmatrix}$$

$$[B]_{P1} = \begin{pmatrix}
0 & -\frac{3xy}{2b^3} & 0 \\
0 & \frac{xy^2}{2b^3}+\frac{3xy}{2b^2}-\frac{x}{6b} & \frac{y^3}{2b^3}-\frac{1}{2} \\
0 & \frac{xy^2}{4ab^3}+\frac{x}{6ab}-\frac{1}{4b} & \frac{y^3}{4ab^3}-\frac{1}{4a} \\
0 & -\frac{xy^2}{4ab^3}-\frac{x}{6ab}-\frac{1}{4b} & -\frac{y^3}{4ab^3}+\frac{1}{4a} \\
0 & \frac{3xy}{2b^3} & 0 \\
0 & -\frac{xy^2}{2b^3}+\frac{3xy}{2b^2}+\frac{x}{6b} & -\frac{y^3}{2b^3}-\frac{1}{2} \\
0 & -\frac{xy^2}{4ab^3}-\frac{x}{6ab}+\frac{1}{4b} & -\frac{y^3}{4ab^3}-\frac{1}{4a} \\
0 & \frac{xy^2}{4ab^3}+\frac{x}{6ab}+\frac{1}{4b} & \frac{y^3}{4ab^3}+\frac{1}{4a}
\end{pmatrix}^T$$

$$[B]_{P2} = \begin{pmatrix}
0 & -\frac{5xy^3}{2b^5} & 0 \\
0 & \frac{xy^2}{2b^3}+\frac{5xy^3}{2b^4}-\frac{x}{6b} & \frac{y^3}{2b^3}-\frac{1}{2} \\
0 & \frac{xy^2}{4ab^3}+\frac{x}{6ab}-\frac{1}{4b} & \frac{y^3}{4ab^3}-\frac{1}{4a} \\
0 & -\frac{xy^2}{4ab^3}-\frac{x}{6ab}-\frac{1}{4b} & -\frac{y^3}{4ab^3}+\frac{1}{4a} \\
0 & \frac{5xy^3}{2b^5} & 0 \\
0 & -\frac{xy^2}{2b^3}+\frac{5xy^3}{2b^4}+\frac{x}{6b} & -\frac{y^3}{2b^3}-\frac{1}{2} \\
0 & -\frac{xy^2}{4ab^3}-\frac{x}{6ab}+\frac{1}{4b} & -\frac{y^3}{4ab^3}-\frac{1}{4a} \\
0 & \frac{xy^2}{4ab^3}+\frac{x}{6ab}+\frac{1}{4b} & \frac{y^3}{4ab^3}+\frac{1}{4a}
\end{pmatrix}^T$$

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