A New Method for Estimating the Input-Energy of MDOF Structures

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Abstract:

Input-energy is the amount of energy that is imposed by an earthquake on a structure and its correlation with structural damage has been studied and demonstrated by many researchers. Since studies concerning seismic energy in multi-degree-of-freedom systems are relatively limited compared with single-degree-of-freedom, in this paper, firstly a theoretical exact method is discussed to calculate the input-energy of the multi-degree-of-freedom elastic oscillators. It is proved that unlike the general rule in mechanics, the superposition theorem is valid for input-energy in conventional modal analysis. To estimate the input-energy, an approach based on PHSA to predict the Fourier amplitude spectrum, is proposed. The results indicate that the modal mass ratio is not the only decisive parameter in input-energy. Modal input-energy decomposition also confirms the possibility of greater input-energy of higher modes in comparison with fundamental ones or the ones with the higher mass participation ratio, especially for tall buildings located in the near-field seismic zones.

1. Introduction

In recent decades, many researchers have studied the idea of using energy-based methods as more reliable scientific methods in comparison with the conventional displacement-based methods [1-3]. In fact, the idea suggesting that structures collapse when the energy demand as a design parameter for an earthquake is greater than the amount provided by the structure, was introduced from the earliest days of earthquake engineering development [4-9]. In addition, in recent years, there has been a growing focus and studies on using energy dissipating devices and new control methods in structures [10-13]. Therefore, a reliable estimation of input-energy is of great importance in energy-based seismic design methods. The energy formulation of structure can be performed using relative or absolute displacements. Although relative energy formulation has been used extensively by researchers, absolute input-energy is more significant from a physical point of view [6].

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Uang and Bertero [5, 6] found that for a given specific ductility ratio, the input-energy demand by both energy quantities demonstrates significant differences in the long and short periodic range, while they are similar in the average period range. They also studied the reliability of using a single-degree-of-freedom (SDOF) energy spectrum to predict the input-energy response of multi-degree-of-freedom (MDOF) structures by comparing analytical and experimental results which showed that the duration of earthquake excitation greatly affects the maximum input-energy. Finally, the uniqueness of the energy dissipation of the structural members was studied and it was concluded that this parameter is strongly dependent on the load and deformation path [6].

Early studies usually estimated the input-energy based on empirical approaches [14-17]. For example, in a proposed equation for energy, Housner [15] obtained the input-energy directly from the velocity spectrum, and suggested the investigation of the general behaviour of the studied system by equating this energy with the total dissipated energy. Also, many studies performed on input-energy of structures beyond the elastic limit [17-19] demonstrated that while the energy spectra tend to decrease with the increasing of

ductility levels [19], the plastic limit has a minor effect on input-energy. The energy per mass ratio is independent of the structure mass, and is mainly a function of earthquake record content [17]. The first relations for energy, often based on the peak parameters of ground motions, were independent of the structure period and consequently inadequate to express the spectral properties. So some researchers presented equations for expressing elastic input-energy in terms of spectral parameters [18, 20, and 21]. In some studies, it was correctly observed that there is also a very good connection between Fourier amplitude spectrum of the ground acceleration and the input-energy spectrum [22], where the input-energy can be calculated accurately and independently from the phase of the input record. Therefore, for the estimation of the elastic energy spectrum, employing the amplitude of Fourier spectrum of acceleration was proposed [23]. Finally, Ordaz et al. [24] proposed an explicit exact relation to calculate the input-energy of SDOF damped using the natural frequency of the SDOF system and Fourier Transform amplitude of input record. This relation confirmed that the input-energy of an SDOF oscillator is independent of the input record phase.

Since real existing systems have more than one degree of freedom, obtaining exact energy relations for MDOF systems is also of great importance. Unlike the comprehensive studies on input energy of SDOF systems, research concerning seismic energy input in MDOF systems are relatively limited [23, 25, and 26]. Akiyama [25] has shown that the relative input energy based on an SDOF system provides a reasonable estimate of the input-energy for multi-story buildings. The results of the shaking table test for a six-story concentrically braced steel structure also supported this conclusion for absolute input-energy.

In this study, firstly, the exact relation for input-energy proposed for SDOF systems is developed for MDOF systems. After verification of the suggested equation, a probabilistic method based on McGuire attenuation relation is proposed.

2. Input Energy of MDOF Structure

This section discusses relations to calculate multi-degree-of-freedom (MDOF) systems seismic energy demand.

In general, for a structure subjected to a set of loads, the internal and external forces satisfy the equilibrium equations and input-energy is the work done by external forces through the displacements of structure. In the case that several load sets act together on a structure with linear behavior, the resultant displacements and internal forces will be equal to the sum of displacements and forces of all load cases (Superposition Theorem), but it is not true for the input-energy of the structure. The input-energy is not simply the summation of input-energies of all load cases. In fact, it is

necessary to take into account the work done by forces of each load case through the displacements of other cases as:

$$W_{total} = \sum_{i=1}^{n} \sum_{j=1}^{m} F_{ij} U_{ij} + \sum_{i=1}^{n} \sum_{k=i+1}^{n} \sum_{j=1}^{m} F_{ij} U_{kj} \qquad (1)$$

The first index indicates the load case number. The second index is the location of the forces, acting on the structure at points 1, 2 …, m, for n different load cases. The first term expresses the summation of input-energy of all load cases. The second term represents the work done by the forces of each loading case through the displacements of others.

It is well-known that the motions of a linear structure or system can be described as a sum of the motions, each of which has a characteristic motion pattern known as a mode of vibration, or simply a mode. The most significant physical fact behind the modal analysis is that, the work done by the forces of each mode through the displacements of others are zero. This conclusion is the physical interpretation of the fact that the vibrational mode shape vectors are orthogonal with respect to stiffness, mass, and damping matrices in the classical damping.

The equilibrium indicates the dynamic balance of external seismic forces with the internal forces, i.e. stiffness, damping and inertial forces, leading to the equality of input-energy, which is the work of external forces through the displacement of the structure, with the summation of the dissipated energy (work of damping force), kinetic energy (work of inertia force) and elastic absorbed energy (work of stiffness force). Therefore, considering Eq. (1), the second term will be eliminated. This leads to the validity of superposition theorem for input-energy in modal analysis of an MDOF system.

The displacement vector of degrees-of-freedom (DOFs) for MDOF systems in the *ith* mode of vibration can be expressed as $\phi_i \delta U$ in which ϕ_i is ith mode shape. The works of internal forces, which are the constitutions of input-energy, are:

$$W_I = [M\phi_i \ddot{Y}_i]\phi_i^T \delta U \qquad (2)$$
$$W_S = [K\phi_i Y_i]\,\phi_i^T \delta U \qquad (3)$$
$$W_D = [C\phi_i \dot{Y}_i]\,\phi_i^T \delta U \qquad (4)$$

$M\phi_i \ddot{Y}_i$, $K\phi_i Y_i$ and $C\phi_i \dot{Y}_i$ are inertial, stiffness and damping force vectors and $\phi_i^T \delta U$ is displacement vector.

By replacing $m^* = \phi_i^T M\phi_i$, $k^* = \phi_i^T K\phi_i$ and $c^* = \phi_1^T C\phi_1$ the Eq. (2-4) can simply be interpreted as the input-energy of an SDOF. It implies that for an MDOF structure vibrating in a specific pattern i.e. mode shape, the input-energy is equal to the one of an SDOF system with particular properties that correspond to modal mass, modal damping, and modal stiffness of studied mode. So the input energy of a MDOF damped system using the relation in [14] can be expressed as:

$$E = \sum_{i=1}^{n} \frac{m_i}{\pi} \int_{-\infty}^{+\infty} |A(\omega)|^2 Re[H_v(\omega; \Omega_i; \xi_i)] d\omega \qquad (5)$$

where

$$Re[H_v(\omega; \Omega_i; \xi_i)] = \frac{2\xi_i \Omega_i \omega^2}{(\Omega_i^2 - \omega^2)^2 + (2\xi_i \omega \Omega_i)^2} \qquad (6)$$

n is the number represents the degrees of freedom. Ω_i and ξ_i are modal frequency and damping of mode i.

To verify Eq. 5, two six-story, damped and undamped structures, of Fig. (1) which represents two MDOF oscillators are presented. The earthquakes used as input records are in Table (1). In the damped structure, the damping satisfies the Rayleigh's (classical) damping condition. For each analysis, the time history response of structure using Newmark's Method with coefficients $\gamma = 0.5$ and $\alpha = 0.167$ are obtained, indicating the linear variations of acceleration in each time step. *Table 2* represents the modal properties of the studied examples.
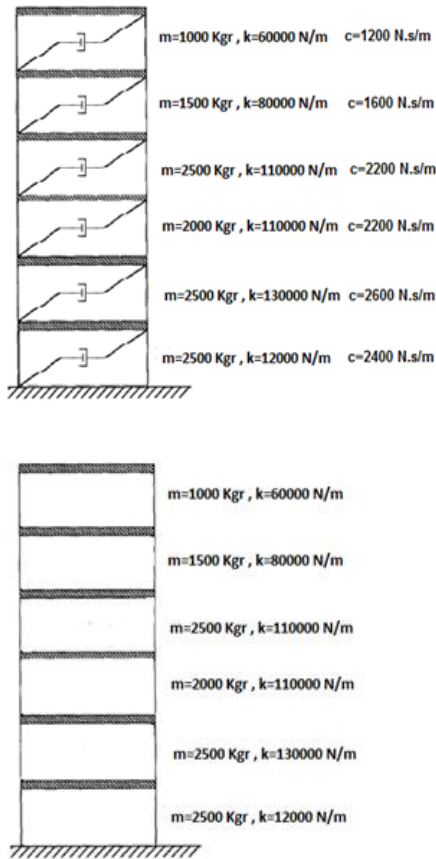

Fig. 1: Numerical examples of classically damped and undamped 6-DOF oscillators

Table. 1: Earthquake records

No	Earthquake	Date	Record
1	Northridge	January 17, 1994	Sylmar
2	Kobe	January 16, 1995	KJMA
3	Turkey	March 13, 1992	Erzincan
4	Imperial Valley	May 18, 1940	El Centro

Table. 2: Structure modal properties

Mode	ω (rad/sec)	Mass ratio	Modal mass
1	1.98	0.85	10200
2	5.1915	0.1	1200
3	7.6753	0.02987	358.44
4	10.734	0.01154	138.48
5	12.127	0.003661	43.932
6	13.53813.538	0.001719	20.628

Fig. (2) displays the input-energy of the example damped structure for three given earthquakes. *IE_time domain* is the input-energy calculated in the time domain.

The value of *IE_time domain* in t_i is:

$$W_i = \int_0^{t_i} m\ddot{a}_g ds \qquad (7)$$

Where m is mass, $\ddot{a}_g$ is ground acceleration and *ds* is displacement vector of masses. *IE_frequency domain* is the input energy that is calculated based on Eq.5. As can be seen, the results obtained by both methods are in good agreement with each other, and the difference between them is due to the precision of the numerical methods employed.

In a case $\xi = 0$, undamped oscillator, the relation of input-energy and Fourier spectra are expressed as [15]:

$$\frac{E_I(\Omega)}{m} = \frac{1}{2}|A(\Omega)|^2 \qquad (8)$$

So for an undamped MDOF system the input energy can be expressed as:

$$E = \sum_{i=1}^{n} \frac{m_i}{2}|A(\Omega_i)|^2 \qquad (9)$$

Fig. 3 demonstrates the input-energy of undamped structure for two of the given records in Table. 1. Results verify the validity of proposed method.

Fig. 4 demonstrates the input-energy decomposition, for three example records. For the Northridge earthquake, the modal mass ratio has the primary role in the modal input-energy. On the other hand, in Kobe, the input-energy of the second mode is higher than the fundamental one, while the first mode mass participation ratio is 8.5 times greater than the second mode. In Imperial Valley, the sixth mode's input-energy is 46 times greater than the second mode despite the 58 times smaller mass ratio. Therefore, it seems that even though the mass ratio plays an important role, it cannot be a sufficient parameter to judge the importance of higher modes contribution and their impact on structural response and damage.
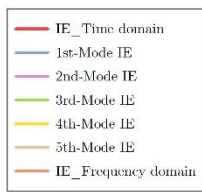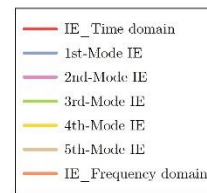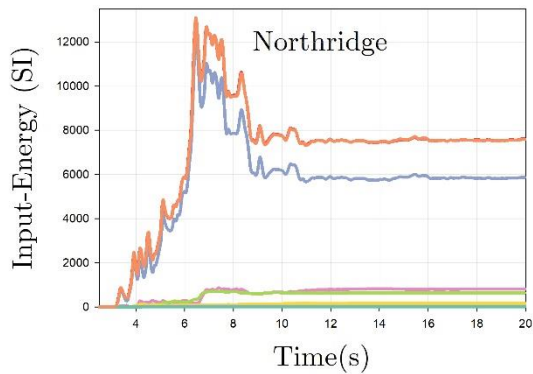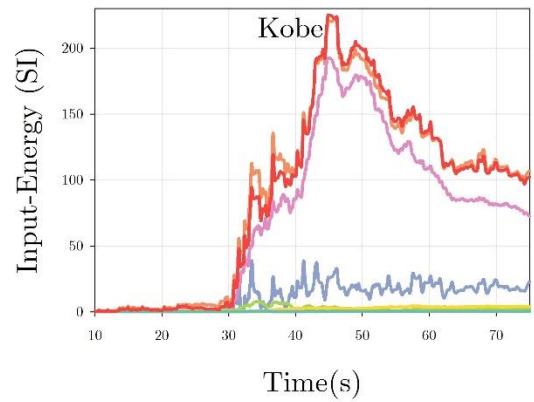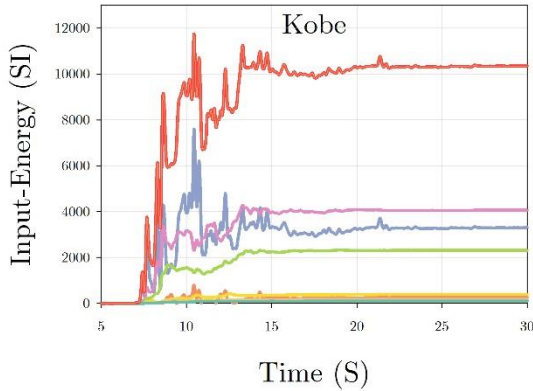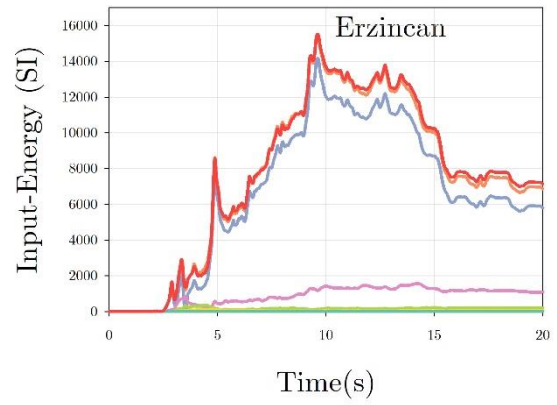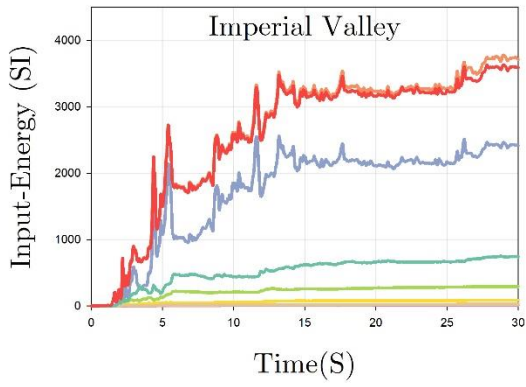
Fig. 2: verification of proposed equation for studied damped system

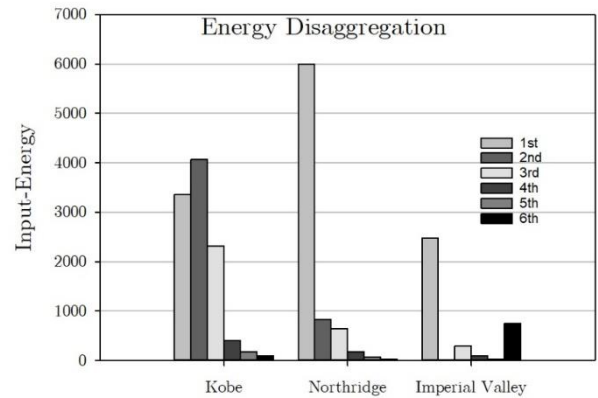**Fig. 3:** verification of proposed equation for studied undamped system

Fig. 4: modal decomposition of input-energy

3. Estimating the Input-Energy of MDOF Oscillator

Numerous investigations have been carried out on the prediction of the Fourier Amplitude of acceleration. Most of these studies are in the field of Probabilistic Seismic Hazard

Analysis (PSHA). In the following, The McGuire attenuation model [15] for predicting Fourier amplitude of acceleration is discussed and used to predict the input-energy.

Cornell equation presented in Eq. (4) calculates the probability of exceedance of the ground motion from a specified level (λ) for a seismic source [16]

$$\lambda_{IM} = \upsilon \iint G_{(IM|M.R)}(IM|M.R)\, f_{M.R}(M.R)dMdR \qquad (10)$$

M is the magnitude of the earthquake, R is the distance from the site to the source of the earthquake, υ is the average annual rate of occurrence of the earthquake, $f_{M.R}(M.R)$ is the joint probability density function of the magnitude and distance and $G_{(IM|M.R)}$ is the conditional probability function of intensity exceedance for a given magnitude and distance.

For simplicity, probability density functions of magnitude and distance assumed to be independent leads to the joint function as:

$$f_{M.R}(M.R) = f_M(M)f_R(R) \qquad (11)$$

The bounded Gutenberg-Richter equation used to calculate the probability of occurrence of earthquake magnitudes is:

$$f_M(M) = \frac{\beta\, exp\left(-\beta(M - M_{min})\right)}{1 - exp(-\beta(M_{max} - M_{min}))} \qquad (12)$$

$$M_{min} < M < M_{max}$$

The $\beta = b\ ln10$ where b is a constant in Gutenberg-Richter recurrence law represents the relative ratio of small and large magnitudes, and the typical value of b is approximately equal to 1. To perform the numerical calculation of the density function, the length of the fault divides into smaller segments, and the probability of rupture events on each of these elements is assumed uniformly. The assumed focal point location is in the middle of a rupture. An appropriate attenuation relation is essential to calculate the conditional probability of seismic intensity. In this study, the used attenuation model is McGuire model [15], a simple model for estimating Fourier amplitude spectra (FS) that is calibrated by using the horizontal components of 70 strong-motion records from California. These chosen records are so that the results are not biased by the effects of one earthquake nor by the effects of a single site. An exponential dependence of FS on magnitude M, a geometric dependence of FS on source-to-site distance R, and a soil amplification term Y are included in the model, using the form [15]:

$$FS(T) = \exp(b_1 + b_2 M + b_4 Y_s)R^{b_3} \qquad (13)$$

where the coefficients b_1, b_2, b_3, and b_4 are determined for each period T of interest.

The used example in this study is in accord with the one presented in reference [17]. Hypothetical sites located 5, 20, and 50 KM above the highest point of a Strike-slip (SS) fault with 200 km length shown in Fig .5 are assumed. For the seismicity properties of the area, the b value (Gutenberg-Richter constant) is equal to one, and the maximum and minimum magnitudes are assumed to be 7.5 and 4.5, respectively. Fourier amplitude spectrum (FS) is the studied intensity measure (IM). The hazard analysis is conducted for two return periods of 475 and 2475 years, using the Gutenberg-Richter discrete probability function. For more discussion, the input-energy for three numerical examples using Fourier amplitude spectra presented in Fig. 5 are calculated using Equation (5). These structures are in accord with the ones presented in reference. The dynamic properties of studied structures are presented in Table 3.
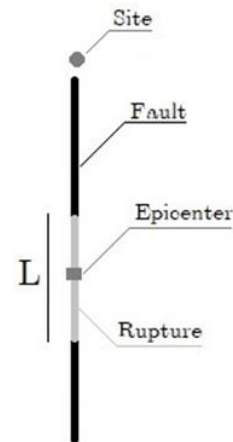

Fig. 5: Hypothetical sites of a Strike-slip (SS) fault with 200 km length

Table 3: Dynamic Properties of studied numerical examples

Mode NO	Mass Participation Ratio (%)		
	6-Story	14-Story	24-Story
1	83%	78%	71%
2	12%	11%	13%
3	3%	4%	5%
4	0.8%	2%	2%
5	0.4%	1%	2%
6	0.2%	0.7%	1%
Mode NO	Period (sec)		
	6-Story	14-Story	24-Story
1	1.78	2.43	4.65
2	0.64	0.88	1.87
3	0.37	0.53	1.13
4	0.25	0.37	0.80
5	0.19	0.28	0.61
6	0.15	0.22	0.49

Fig. 7 demonstrate the Input-Energy disaggregation of studied buildings for two different return periods.

The modal disaggregation of input-energy presented in Fig. 7 demonstrate:

1- For all structures, higher modes play a more important role in near site-to-source distance. In other words, the participation of first mode in the

input-energy reduces as the site-to-source distance decreases.

2- Site-to-source distance shows more importance in input-energy content than earthquake magnitude. In fact, the modal participation ratio in the input-energy are slightly different in the two different return periods.

3- It seems that even though the mass participation ratio plays an important role, it isn't sufficient parameter to judge the importance of the higher modes input-energy contribution, especially for tall buildings located in the near-field seismic zones.
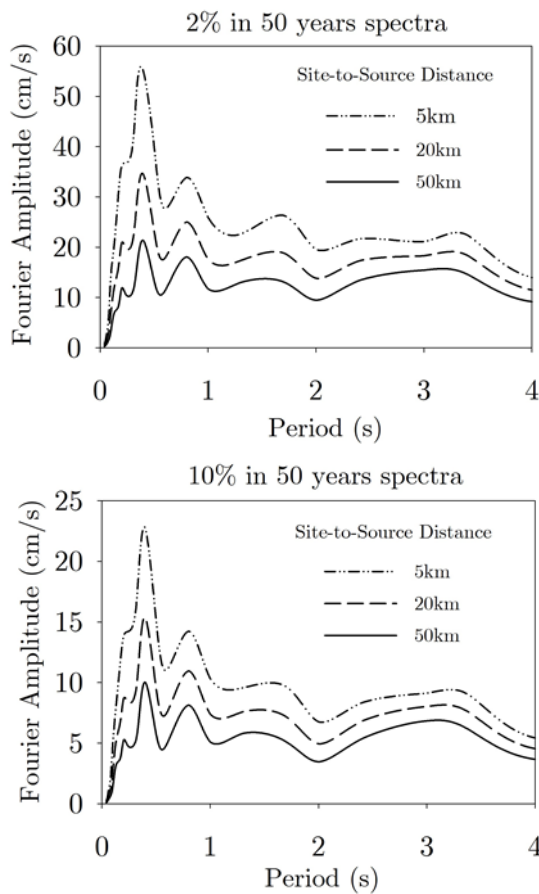

Fig. 6: Fourier Amplitude Spectra (FS) for hypothetical site

Most of the design codes (such as ASCE/SEI 7) stipulate that the analysis shall include a sufficient number of modes to obtain a combined modal mass participation of at least 90 percent of the actual mass in each of the orthogonal horizontal directions of response [18]. For the presented structures, Fig. 7 indicates that such consideration results in neglecting at least 22% and 24% of input-energy for 14- and 24-story examples for the nearest site-to-source case. For the 6-story building, the neglected amount of input-energy is 13%. Since input-energy is an intensity measure that reflects both damage measures based on peak displacement and energy-based measures. Selecting the number of modes for the seismic analysis of buildings based on solely mass

participation ratio may lead to underestimation of demand parameters, especially energy-based measures such as fatigue damage and absorbed hysteretic energy in tall buildings for the near field earthquakes. However, such underestimation cannot be considered outstanding for far field earthquakes and low-rise buildings.

4. Discussion and Conclusion

In this paper, an exact theoretical relation is investigated for computing input-energy of the multi-degree-of-freedom (MDOF) classically damped oscillators using the Fourier acceleration amplitude spectrum. It is confirmed that unlike the general rule in mechanics, the superposition theorem is valid for input-energy in conventional modal analysis. The results of two 6 degree-of-freedom damped and undamped numerical examples for several earthquake records is used as a verification for the proposed relation. It is shown that, like SDOF systems, the input-energies of MDOF structures are functions of both structural properties and record contents. It is also independent of the input record phase and is only a function of the Fourier Amplitude spectrum of input acceleration similar to the SDOF systems. In other words, the contribution of each frequency of input record to the input-energy of an MDOF system is proportional to its Fourier Amplitude, its proximity to the modal frequencies and the corresponding modal properties of the structure.

Since the input-energy is a function of earthquake content besides the modal properties of structures, the modal mass ratio it is not the only decisive parameter in the input-energy. This may result in greater input-energy of higher modes in comparison with fundamental ones or the ones with the higher mass participation ratio. Input-energy decomposition for the studied structures confirms the possibility of a higher contribution of higher-modes.

To estimate the input-energy of structures, a method based on PSHA using McGuire attenuation model to predict the Fourier amplitude spectrum is proposed. The amounts of input-energy for three structures, which are representative of buildings from low- to high-rise, and for three different site-to-source distance, which represent near-field to far-field earthquakes, are computed. The results demonstrate that site-to-source distance shows more importance in input-energy content than earthquake magnitude. In fact, the modal participation ratio in the input-energy are slightly different in the two different return periods. It is also observed that, especially for high-rise buildings located near the seismic zone, the mass participation ratio is not a sufficient parameter to judge the impact of higher modes because a great portion of work that is done by earthquake forces on the structure are neglected. This is due to neglecting the input-record impact and considering only structural properties in seismic response.
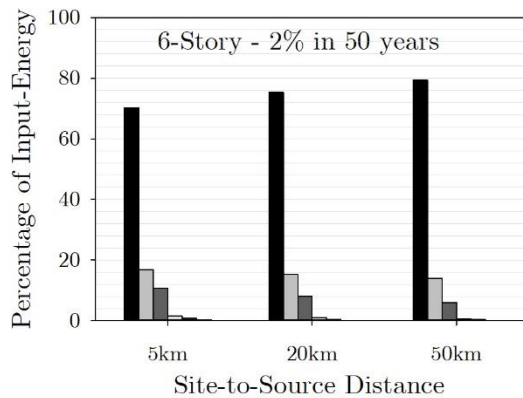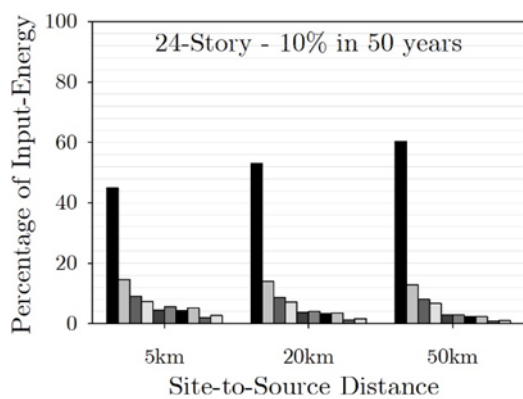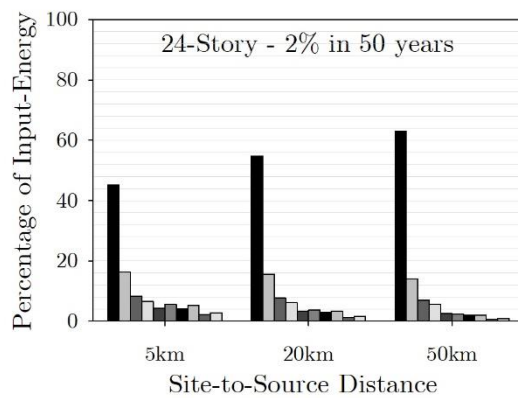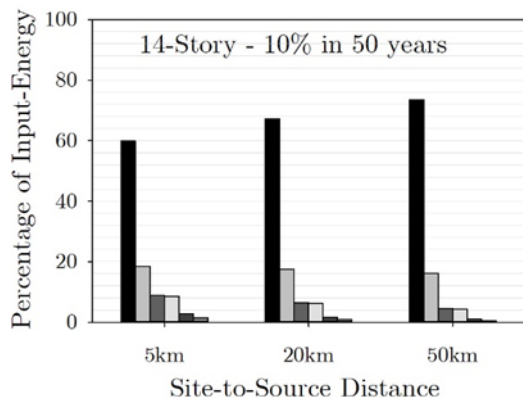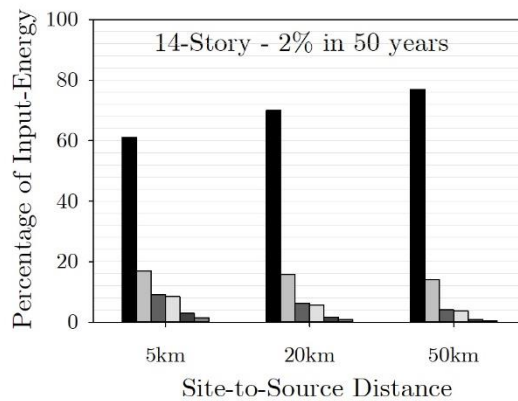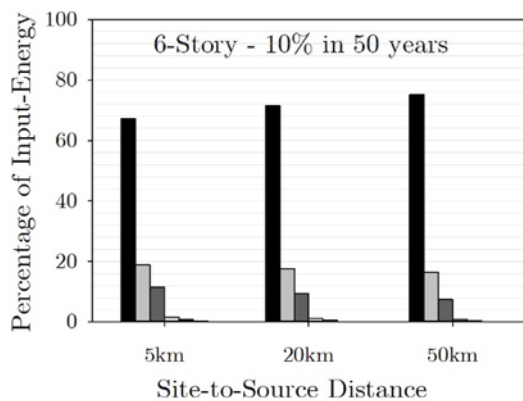
Fig. 7: Input-Energy disaggregation of the studied buildings

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