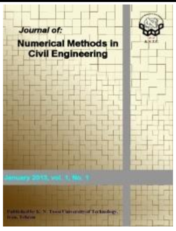

Numerical Methods in Civil Engineering

A Proposed Modeling Method in Finite Element Slope Stability Analysis

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Abstract:

Limit-equilibrium method (LEM) and finite element method (FEM) with strength reduction method (SRM) techniques are the most widely used analysis tools in slope stability assessment. Recently, researchers have reported that both factor of safety (FOS) values and failure surfaces obtained from LEM and FEM are generally in good agreement, except in some particular cases. In this paper, the FOS and the location of critical failure surfaces procured by FE-SRM with various modeling types are compared. Eventually, the outcomes of FE-SRM with high mesh density are assumed as reference. The results of this study demonstrated that in FEM, determining the shear band zone by primary analysis of slopes with coarse meshes and consequently modifying the mesh configuration by imposing the program to locate the nodes in that zone can provide accurate and low-cost results. In addition, the comparison between the results of the proposed method and Bishop's simplified approach as a limit equilibrium method (LEM) represented good agreement and it was evident that better results can be achieved with less cost and time.

1. Introduction

Slope stability analysis is often associated with considerable amount of uncertainties. In the past decades, reliability methods based on the probabilistic theory have been developed for quantifying uncertainties in slope stability analysis (Zhang et al. 2011[1]; Hong and Roh 2008[2]). This problem has drawn the attention of many investigators in the past and continues to do so (Taylor, 1948[3]; Bishop, 1955[4]; Morgenstern, 1963[5]; Griffiths and Lane, 1999[6]; Xie et al., 2003[7]). Generally, there are two major coupled tasks in the slope stability analysis: the computation of the factor of safety and the location of the critical slip surface. In the finite element slope stability analysis, there are two commonly used definitions on the factor of safety. The first is the strength reserving definition, which defines the factor of safety as the factor by which the shear strength of the soil would have to be divided to bring the slope into the state of critical equilibrium.

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The second is the overloading definition, which defines the factor of safety as the ratio of total resisting forces to total driving forces along a certain slip line (Zheng et al., 2006[8]). However, the definition based on the strength reservation seems to be widely used in engineering.

In order to obtain the factor of safety based on the strength reservation, the finite element–strength reduction technique is usually utilized. Regarding critical slip surface, since the conventional finite element method based on the compatible element technique cannot be expected to capture the shear bands (Ortiz et al., 1987[9]), some technical methods are proposed to visualize the shear bands. For instance, the adaptive mesh refinement procedure proposed by Zienkiewicz and Taylor (1991) [10], the technique of enhanced visualizing failure mechanism presented by Griffiths and Kidger (1995) [11], etc. Often, for more accuracy in FEM analysis, dense meshes which cause more cost and time inefficiency, should be applied. As a result, in many setbacks, finite element analysis with high accuracy is inadvisable. Many research articles had been published since the publication of the first method of analysis by Fellenius (1936)[12] that were either related to slope stability or involved slope stability analysis subjects. There are various methods for the slope stability analysis including the limit equilibrium methods (Nadi et al. 2014[13]; Nadi et al. 2019[14], boundary element methods (Jiang, 1990[15]), finite element methods (Matsui and San, 1992[16]), and neural network methods (Jaritngam et al., 2001[17]). Simplicity and ease of use are the main advantages of limit equilibrium methods and accordingly, these are the most commonly used methods compared to others. These methods satisfy either some or all of the equilibrium conditions. Satisfied equilibrium conditions include: (1) some or all interslice forces (Fellenius, 1936 [12]; Janbu, 1954 [18]; 1973 [19]), (2) moment and/or some forces (Taylor, 1940 [20], Bishop, 1955[4]), (3) moment and all forces (Morgenstern, 1963 [5]; Spencer, 1967 [21]; 1973 [22]; Sarma, 1973 [23]; 1979 [24]). Fellenius (1963) [12], Taylor (1940) [20], and Bishop (1955) [4] methods can be used for circular slip surfaces, while the others can be used for circular and non-circular slip surfaces. Due to the large number of possible slip surfaces, computers are used to facilitate computations. Interestingly, factors of safety obtained from stability analysis methods that satisfy all limit equilibrium conditions are within 6% difference of each other (Duncan, 1996 [25]). These methods include friction circle methods, log spiral methods, rigorous limit equilibrium methods, and finite element methods. Because Bishop's method has been validated against solutions for various particular cases and has been used extensively with satisfactory results (Verruijt A., 1995 [26]), in this investigation, it will be used as a limit equilibrium method.

Finite element method is a very powerful computational tool in engineering. It gains its power from the ability to simulate physical behaviors using computational tools without the need to simplify the problem. Indeed, complex engineering problems need finite element approaches to obtain more reliable and accurate results. Many methods for slope stability analysis using finite elements have been proposed during the last two decades. Among those methods, gravity increase method (Swan and Seo, 1999[27]) and strength reduction method (Matsui and San 1992[16]) are considered the most widely used methods. In the gravity increase method, gravity forces are increased gradually until the slope fails (g_f) then the factor of safety is defined as the ratio between the gravitational acceleration at failure (g_f) and the actual gravitational acceleration (g). In the strength reduction method, soil strength parameters are reduced until the slope becomes unstable. Therefore, the factor of safety is defined as the ratio between the initial strength parameter and the critical strength parameter. For this reason, the strength reduction method has exactly the same definition as the limit equilibrium methods (Griffiths and Lane 1999 [6]). The gravity increase method is used to study the stability of embankments during construction since it gives more reliable results while the strength reduction method is used to study the stability of existing slopes. In order to compare results of limit equilibrium methods with finite element analysis results, the strength reduction method was selected in this study, since it resembles the limit equilibrium approach more than the gravity increase method.

Also, in the finite element slope stability analysis, another definition on the factor of safety, called the overloading definition (Zheng et al., 2006[8]), is used (Farias and Naylor (1998) [28], Wang (1999) [29], Yamagami and Ueta (1998) [30], Zou and Williams (1995) [31]) that can be defined as:

$$F_O = min_{s \in S} \frac{\int_s \, [\tau] \, ds}{\int_s \, \tau \, ds} \qquad (1)$$

In which, the subscript ''o'' in F_o refers to ''overloading''. "S" is a set of some potential slip lines and "s" represents a certain slip line in set "s". In some commercial software products, "S" is usually composed of straight lines and circular arcs specified by the users. τ denotes the shear stress at a point on s and tangential to "s". $[\tau]$ is the shear strength at the point obtained by the Mohr–Coulomb's criterion. In advanced constitutive soil models, stiffness modulus is stress-dependent and changes based on the step-size computation increments. The stiffness modulus in the strength reduction method, is used as a constant stiffness modulus during computations. As a result, the advanced soil model behaves like the Mohr–Coulomb model where a constant stiffness modulus is used, as well (Brinkgreve and Vermeer, 2001[32]).

As it is stated before, in a finite element analysis, accurate results can be achieved by increasing the element numbers by defining very fine meshes. Especially, considering that in the PLAXIS software, mesh generation is automatic, defining very fine meshes is necessary in order to consider a more accurate shear band and obtain a more precise FOS. However, this approach is not cost and time effective. In this paper, a method is proposed to enhance the obtained results by modifying the mesh generation even by defining very coarse meshes. For this purpose, at first, the approximate shear band zone was estimated by primary analysis of slopes with coarse meshes. Then, the mesh configuration is modified by imposing the program to locate the nodes in that zone. It is indicated that the proposed method can provide accurate and low cost results. Eventually, the comparison between the results of the proposed method and Bishop's simplified approach as a limit equilibrium method (LEM) represented good agreement.

2. Modelling

In order to provide a proper comparison between the FEM and LEM, and also avoiding complexity in geometry and boundary conditions, a homogeneous soil slope with a slope height equal to 15 m and slope gradient equal to 1/2 is considered. In order to examine the finite element-strength reduction method (FE-SRM) accuracy, Bishop's simplified approach as a limit equilibrium method is assumed as a reference for slope stability analysis. This method has been validated against solutions for various particular cases and has been used extensively with satisfactory results (Verruijt A., 1995 [26]). The problem is investigated with 2D plane-strain models. The geometry of the slope and the soil properties are shown in Fig. 1. For this example, the Mohr–Coulomb failure criterion was used.

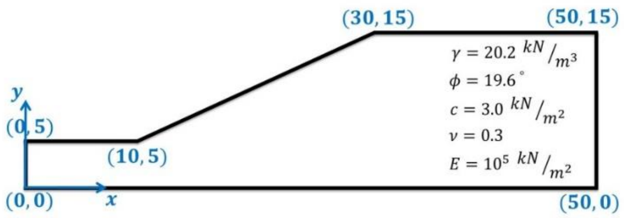


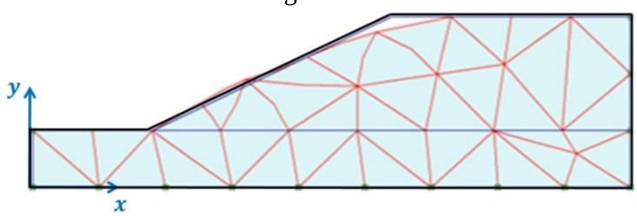
Fig. 1: Slope geometry and soil property under study

In the FEM, the factor of safety against slope failure is computed through the reduction of the strength parameters at a certain rate ($\sum Msf$), as shown in the equation below:

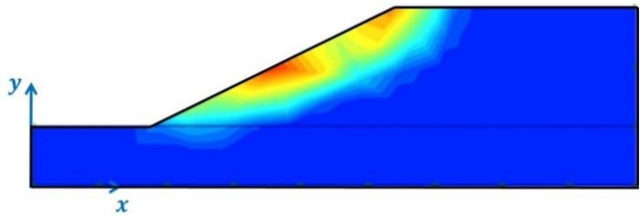
$$\sum Msf = \frac{\tan \phi_{input}}{\tan \phi_{reduced}} = \frac{c_{input}}{c_{reduced}} \qquad (2)$$

In this nonlinear deformation analysis based on the Mohr–Coulomb material model, the maximum $\sum Msf$ that provides the equilibrium is called the factor of safety (FOS). This method is also called the Phi-ci reduction method. The sliding surface does not need to be defined initially and is automatically found. Therefore, a shear surface close to the natural sliding surface is defined (Dawson et al., 1999 [33]; Griffiths and Lane, 1999 [6]).

For this analysis, plane strain fifteen-node and six-node triangular elements in five densities (i.e. very coarse, coarse, medium, fine and very fine) were used in the finite element mesh. The lowest and highest mesh density contained 41 and 810 elements respectively. The deformed mesh configuration and incremental shear strain results for very coarse and very fine meshes are shown in Fig. 2 and Fig. 3 respectively. It must be mentioned that, the illustrated results of Fig. 2 and Fig. 3 are the obtained results of the studied model that are shown in Fig. 1.

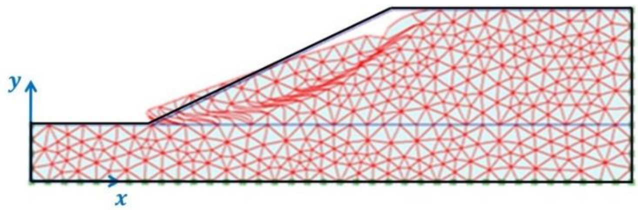


(a) Deformed mesh configuration

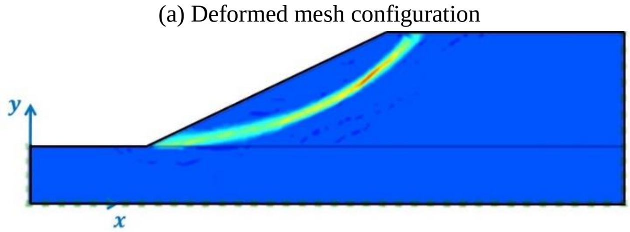


(b) Incremental shear strain

Fig. 2: Analysis results of very coarse, 6-node triangular elements



(a) Deformed mesh configuration



(b) Incremental shear strain

Fig. 3: Analysis results of very fine, 15-node triangular elements

It is well known that, based on the theory of continuum and the isoparametric interpolation technique in the finite element approach, when plastic deformation occurs in a structure, the plastic zones usually depend on the mesh configuration if the elastic–perfectly plastic model is assumed (Zheng et al., 2009 [34]). By comparison of Fig. 2-a and Fig. 3-a, it can be found that by using finer elements in slope model, the shear band is distinguished more clearly. On the other hand, factors of safety from Bishop's simplified analysis (LEM) for this problem is 0.988, and the critical slip surface of it is shown in Fig. 4. An appropriate agreement is observable with the comparison of the critical slip surface obtained from LEM and FEM with very fine meshes (Fig. 4).

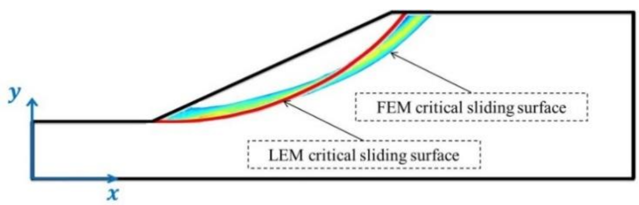


Fig. 4: Comparison of the critical sliding surface of LEM and FEM

The values of safety factors obtained from FE-SRM analysis by use of various models have deviations from the limit equilibrium method (LEM) result. These deviations are tabulated in Table 1 and demonstrated in Fig. 5.

Table 1. PLAXIS factors of safety and deviations of them from LEM results

No.	Mesh configuration	PLAXIS factor of safety	Deviation from LEM factor of safety (0.988)
1	T6- Very Coarse	1.0616	+ 7.44 %
2	T6- Coarse	1.0491	+ 6.18 %
3	T6- Medium	1.0098	+ 2.20 %
4	T6- Fine	0.9974	+ 0.95 %
5	T6- Very Fine	0.9790	- 0.91 %
6	T15- Very Coarse	0.9930	+ 0.50 %
7	T15- Coarse	0.9922	+ 0.42 %
8	T15- Medium	0.9620	- 2.63 %
9	T15- Fine	0.9992	+ 1.13 %
10	T15- Very Fine	0.9454	- 4.31 %

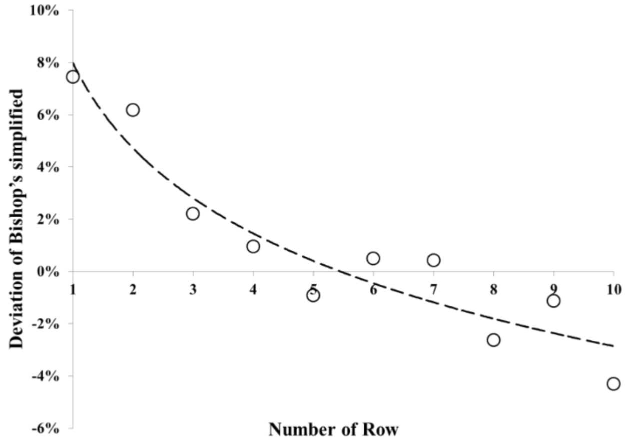


Fig. 5: Deviation of PLAXIS factor of safety from LEM results

According to Fig. 5, it can be seen that defining finer meshes would not necessarily cause reduction in the deviation from LEM results. The reason can be related to the randomness of mesh generation in PLAXIS software.

As a result of this study it can be concluded that: (a) best accuracy of safety factor in FE-SRM is obtained by use of very coarse or coarse fifteen-node triangular elements and (b) best accuracy of FE-SRM shear band is obtained by employing very fine fifteen-node triangular elements. Based on these results, the requirement of a modeling method that provides high accuracy for determining safety factor and shear band with low cost and time duration is sensed.

3. Discussion

In the previous section, a method is proposed to enhance the efficiency of FEM for predicting the shear band and FOS. The proposed method was based on the initial prediction of the slip surface with a low-cost model and locating node sets on the predicted shear band zone. For this purpose, at first, one can approximately determine the critical surface by use of analysis of the large mesh size model. Eventually, mesh regeneration is conducted and slope stability analysis is performed.

In order to clarify the application of the proposed method, an instance model is considered as shown in Fig. 6. The approximate surface is determined by use of incremental shear strain results of six-node triangular elements with very coarse mesh density. This surface is compared with LEM critical surface in Fig. 7. The difference of these surfaces is obvious in the figures below.

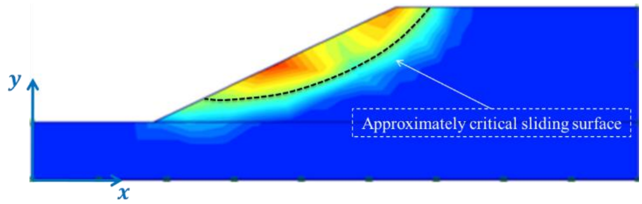


Fig. 6: Determination of approximately critical sliding surface by use of incremental shear strain results

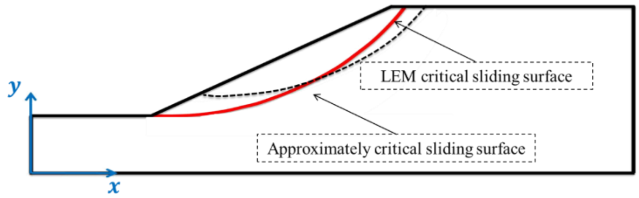


Fig. 7: Comparison of approximate critical surface with that of LEM

In the second step, before meshing the slope model, the approximate critical surface is entered into the geometry of the model by use of compressed nodes. The estimated slip surface is created by joining the nodes as shown in Fig. 8. Then, the model is meshed by use of very coarse elements.

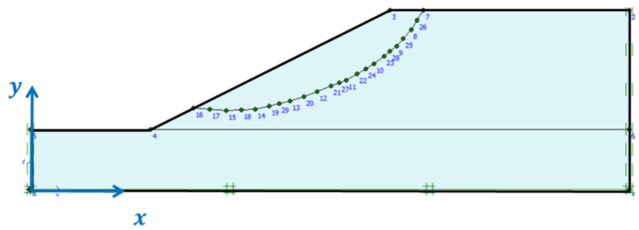
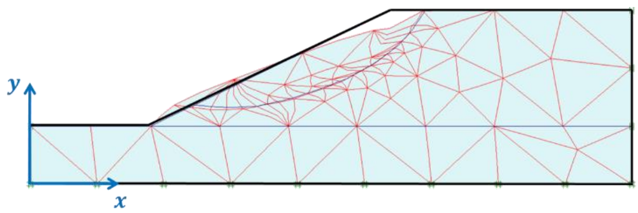
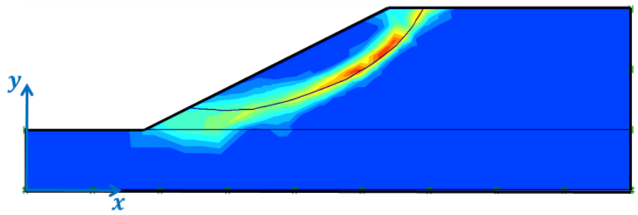


Fig. 8: Entering the approximate critical sliding surface into the slope model

The factor of safety obtained by the proposed method is 0.982, which is 0.6 % different from the LEM safety factor (0.988). As illustrated in Fig. 9, by using the proposed method, the shear band zone in Fig. 6 is converted to a slip line. This can provide a better understanding of the critical sliding surface. Therefore, by using the proposed modeling method one can obtain high accuracy slope stability results.



(a) Deformed mesh configuration



(b) Incremental shear strain

Fig. 9: Analysis results of the proposed model

Conventional finite element method (FEM) is developed on the basis of small strain assumption and is not suitable for analyzing large deformation problems. However, this can be solved by choosing the Updated Mesh option in PLAXIS, namely, upgrading the node coordinates and stiffness matrices during incremental analyses.

In order to verify the reliability of the proposed method, the results of the proposed procedure are compared with those obtained from the LEM and the results of two examples in the literature.

3.1. Example 1

Fig. 10 is a typical section of a slope of a speedway that includes two kinds of soils. This example was probed by Zheng et al. (2006, 2009) [8,34].

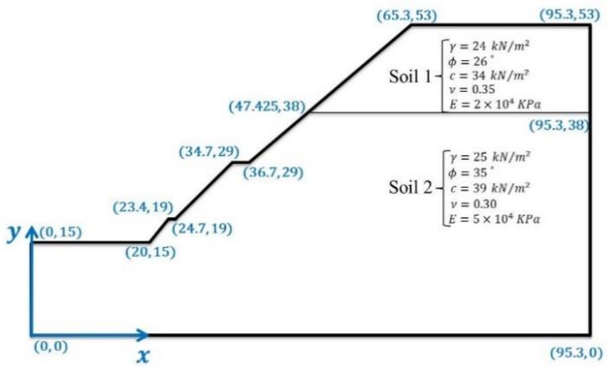


Fig. 10: Slope geometry and soil property under study

Fig. 11 illustrates the factors of safety and the critical slip lines, SL-o and SL-s, corresponding to the overloading and strength reservation definitions of the factor of safety, respectively. The critical slip line (SL-o) corresponding to Fo is shallower than SL-s corresponding to Fs. The FOSs of Spencer's method on SL-s and SL-o are 1.43 and 1.49, respectively (Zheng et al., 2006 [8]).

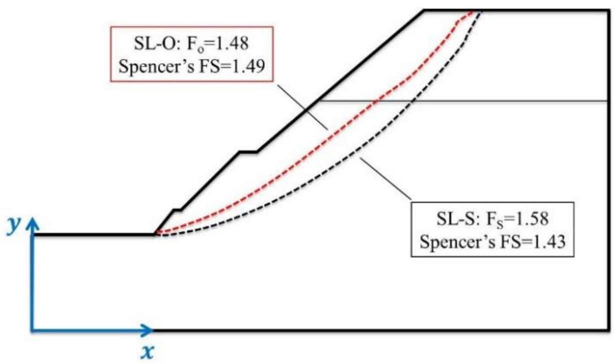


Fig. 11: Example 1: the critical slip lines of two definitions and FOSs

For applying the proposed method of modeling, the approximate critical surface is determined by use of analysis of the large mesh size model. In this problem, as shown in Fig. 12, the approximate surface is determined by use of incremental shear strain results of six-node triangular elements with very coarse mesh density.

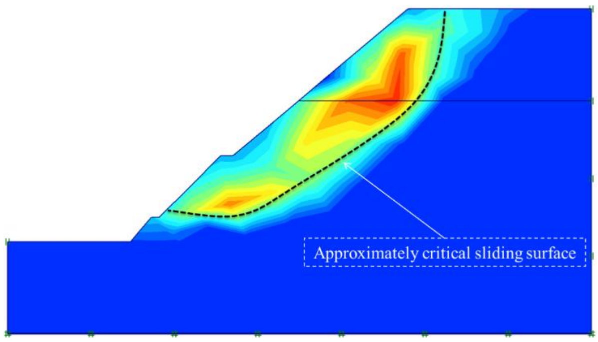


Fig. 12: Determination of approximate critical sliding surface by use of incremental shear strain results

Subsequently, before meshing the slope model, the approximate critical surface is entered into the geometry of model by use of compressed nodes and joining them to obtain a line as shown in Fig. 13. Next, the model is meshed by use of very coarse elements. By this method, 192 elements and 576 stress points are created and average element size of this method is 5.13 m. Fig. 14 shows the results of modified model analysis.

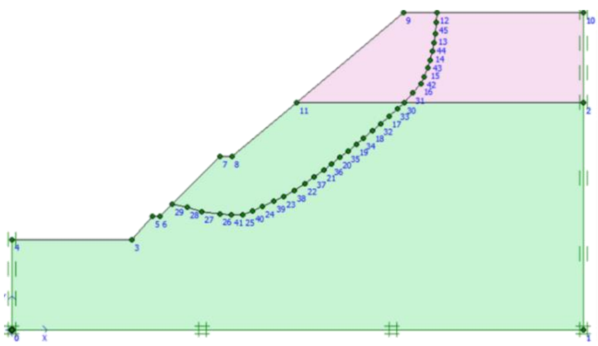


Fig. 13: Entering the approximate critical sliding surface into the slope model

The factor of safety obtained by this method (and using six-node triangular meshes) is 1.391, that is 6.64 % and 2.72 % different from LEM factors by two definitions shown in Fig. 11. Likewise, following the last points, the safety factor that is obtained from coarse fifteen-node triangular mesh (without slip surface prediction) is 1.325. Hence, the safety factor that is obtained by the proposed method is -4.74 % different from the FOS value 1.325. Also, as mentioned in the previous sections, by using very fine mesh, the shear band was clearly visible.

Fig. 15 represents the shear band in a very fine mesh model. In this model, 592 elements and 1776 stress points are used and average element size is 2.92 m. Although the shear band of the proposed method isn't as clear as the very fine mesh model, by using lower number of elements in the proposed method, the same shear band is predictable.

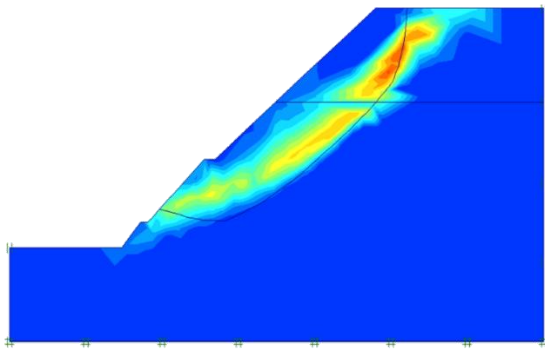


Fig. 14: Analysis results of the proposed model

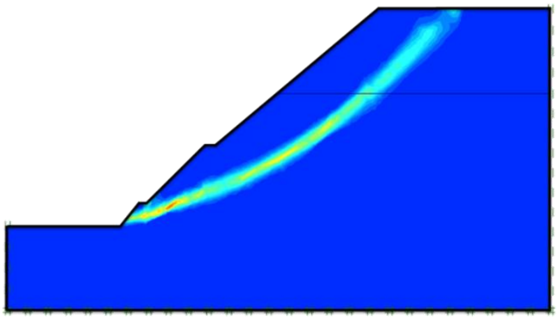


Fig. 15: The critical surface of very fine mesh

3.2. Example 2

A special slope with a soft band which has been examined by Cheng et al. (2007) [35] is considered. The geometry of the slope is shown in Fig. 16 and the soil properties are presented. It is noted that C is zero and φ is small for soil layer 2, which has a thickness of just 0.5 m. The critical failure surface is obviously controlled by this soft band, and slope failures in similar conditions have actually occurred in Hong Kong (for example, the Fei Tsui Road slope failure in Hong Kong (Cheng et al., 2007 [35])).

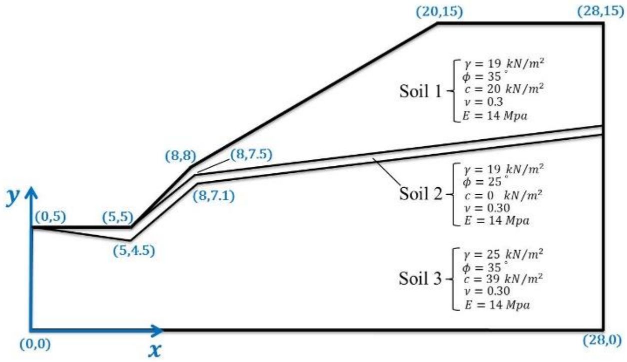


Fig. 16: Slope geometry and soil property under study

The steps of the proposed method for this problem are similar to the previous example. Fig. 17 represents the approximate critical surface by coarse mesh. Fig. 18 shows implementation of the mentioned surface in the model by dense nodes and lines. Fig. 19 shows the results of modified model analysis.

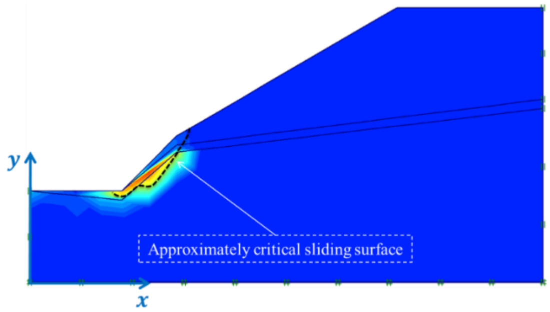


Fig. 17: Determination of approximately critical sliding surface by use of incremental shear strain results

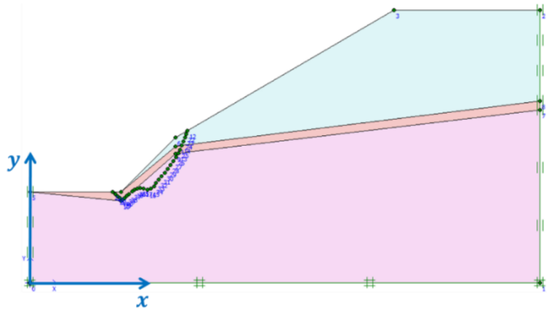


Fig. 18: Entering the approximate critical sliding surface into the slope model

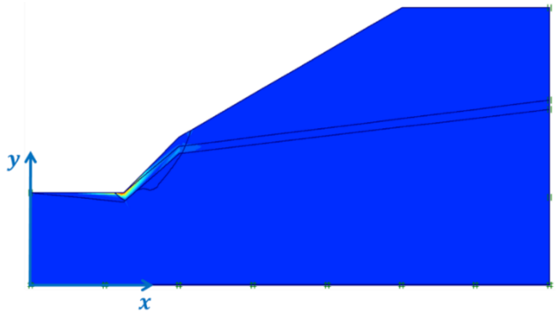


Fig. 19: Analysis results of the proposed model

The factor of safety obtained by this method is 0.506, which is 22 % different from the FOS of model with fifteen nodes-coarse mesh (0.649). Likewise, the critical sliding surface obtained by very fine mesh is represented in Fig. 20. By the proposed method, 127 elements and 381 stress nodes are created and the average element size of this method is 1.82 m. Nevertheless, by utilizing very fine mesh, 758 elements and 2274 stress points are used and the average element size of this method is 0.744 m. Accordingly, it is obvious that by the recommended method, precise results are achievable by using less and coarse elements which leads to a time and cost-effective analysis.

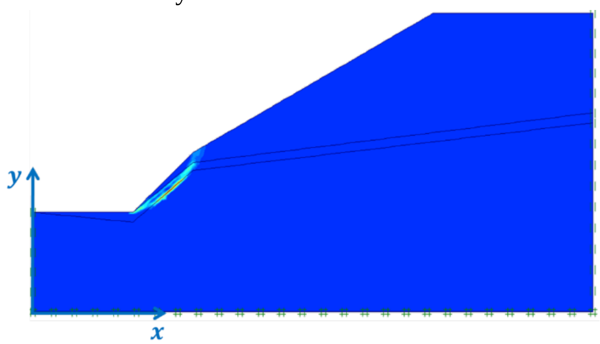


Fig. 20: The critical surface of very fine mesh

4. Conclusion

In this paper, a method is proposed to enhance the efficiency of FEM for predicting the shear band and FOS for slope stability. The proposed method was based on the initial prediction of the slip surface by a low-cost model and locating node sets on the predicted shear band zone. Consequently, an estimated slip surface is created by joining the nodes. Eventually, mesh regeneration is conducted and slope stability analysis is performed. The applicability of the proposed method is verified by several examples.

The results of this study demonstrated that best accuracy of safety factor in FE-SRM is obtained by the use of very coarse or coarse fifteen-node triangular elements. Additionally, best accuracy of FE-SRM shear band is obtained by employing very fine fifteen-node triangular elements. Verification examples revealed that by using the proposed method, the shear band zone in FEM can be converted to a slip line (similar to LEM). This can provide a better understanding of the critical sliding surface. In conclusion, it is evident that by the proposed method, precise results are achievable by using less and coarse elements which lead to a time and cost-effective analysis.

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