Mapping Historical Water-Supply Qanat Based On Fuzzy Method. An Application to the Isfahan Qanat (Isfahan, Iran)

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Abstract:

According to the geographical location of Iran, it enjoys both arid and semi-arid weather, which has been the cause of water shortage problems. Therefore, water has been one of strategic importance in Iran. When the population dramatically increased in ancient Iran, the demand for supplying water also increased simultaneously and brought about the need for the invention of a new way for supplying water from the ground by using gravity. This invention was named Qanat, and it has an important role for managing groundwater in arid and semi-arid areas like Isfahan. Qanat is a sustainable way for water supply in many regions. As qanat is a cultural heritage which is still prevalent in some regions of the country, it has been of great importance not only in the history of Iran but also recognized around the world as an unprecedented invention. The unparalleled source of this ancient invention and its popularity has led to their preservation in Iran and also made it famous around the world. In this article, due to the importance of its cultural heritage, attempts have been made to find the routes of the qanat in Isfahan using available spatial layers such as flood way, stream, ditch, watercourse, bush, etc. based on fuzzy logic method. To this regard, three different scenarios of fuzzy logic rules are proposed and consequently three paths are obtained for each group of fuzzy rules. Then, the explored routes are compared with the qanat route that was found by geophysics institute in Iran. The similarity of the most desirable map with the actual one was 73 percent.

1. Introduction

Majority of Iran is covered with arid and semi-arid land [1]. Therefore, the ancestors of the Iranians had had to deal with water shortage problems most of the time. This created conflict in ideas for preservation and collection of water. Hence, various methods for extraction, storage and use of water were considered by people [2]. Today, water management has changed fundamentally due to developments in technology and vast improvements in industrial and agricultural production.

During the development and increase of the ancient Iranian population, uncommon ways were unsuitable for supplying water to the increasing population.

Therefore, the ancestors invented a special method; they carried groundwater by using gravitational force. This invention is unique and is called Kariz or Qanat [3, 4]. However, thousands of years have passed since its invention, but this method is still being used as an important part of rural, urban and agricultural water supply. By using this method, Iranians have succeeded in sustainably exploiting groundwater, and have endured drought conditions in Iran [4].

A qanat is a sloping underground channel that is used to transfer water from spring water or underground water to the surface for irrigation and drinking [5]. This is an old water supply system from a deep well with a set of vertical shafts, effectively delivering a large amount of groundwater to the surface without the need for pumping. The aqueduct still provides reliable water for urban settlements and irrigation in arid and semi-arid regions [6, 7].

The qanat system has the benefits of resisting natural disasters, such as earthquakes and floods, and destruction in the wars. In addition, the qanat system does not relate to the

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amount of rainfall and only flows with the gradual changes from wet to dry. From the standpoint of sustainability, qanats are designed by gravity; therefore, maintenance costs are low. The qanat transmits fresh water from the mountain plateau to the lower plains with saline soils. This helps control the salinity of the soil and prevent desertification [8]. Qanat or aqueduct is one of the most amazing teamwork in human history, which has arisen with extensive planning and managing in order to solve the crucial problem of water supply. This wonderful phenomenon of water supply from the past, and the Iron Age is known as a source of supply for drinking water and agriculture issues, especially in areas facing drought risks on the Iranian plateau [9] which in turn has played a key and effective role in the economic and social life of the country. It has also boosted the economy of agriculture, and created many urban and rural activities [10]. So far, a lot of research has been done on this issues to find the history of qanat, and in some cases, authors try to find new ways to rebuild or protect them. In Europe [11], for the first time in Germany and Belgium, subsurface canals and underground channels were used to extract extra minerals, and for this purpose about 50,000 km of underground channel was drilled. Undoubtedly, many years ago and before Europeans, this method was used in western Rome, and the Europeans discovered this technique in the Renaissance and the Middle Ages. The technology then expanded to other countries in the Middle East, China, India, Japan, North Africa, Spain, and then to Latin America [8, 12].

These deep mineral water wells from fossil aquifers at a much higher rate than the replacement levels; most of which are excavated in the basin areas where the water level is near the surface. As aquifers are evacuated, the aqueducts of the waterfalls that have a common aquifer are dry and the water level is reduced and settlements disappear. Social Patterns of Social Adjustment of living together in the world of Iranian villagers has disappeared since centuries. Indeed, evidence suggests that deep wells have made many small farmers dependent on good owners, have significantly reduced agricultural production and established many long-standing settlements for long-term survival [13].

The feasibility of constructing a qanat system that can adequately supply water over a long period of time depends on several climatic, geographic and social features. Historically, the aqueducts have been built in areas with low water catchment, but they are built near high mountains and hills for underground water resources. In addition, the aqueducts provide a steady stream of water at low rates, that are suitable for dispersed rural communities with low population densities and low volatility of water demand throughout the year [8, 14].

There are also some studies related in this area around the world. For instance in 2019, Baghban and his coworkers studied about the effect of Qanats on ground settlement in Tehran due to the interference of large number of qanats with the excavation of metro lines. This problem indicates that abnormal movements were caused by pre-existent qanat chains, minor branches, and conduits in this area [15]. Moreover, in 2018, a fuzzy method was represented for water resources management with the consideration of ecological water demand. This method not only can contribute to decision makers for in-depth analysis, but are also sustainable for development of ecosystem [16]. By combination of variable fuzzy sets theory (VFST) in water quality evaluation and traditional method of calculating relative membership degree (RMD), Fang and his coworker introduced a new method for water quality assessment. The proposed method exhibited stable reliability and efficient performance in their case study [17]. Munir et al. [18] in their study remarked that Fuzzy Logic's approach helps to make smart decisions regarding irrigation requirements. They also stated that Block-chain provides the required security in their system by allowing only trusted devices to access and manage the proposed SWS. The result of the research was introduced as an efficient and safe application to efficiently manage the watering process of plants. In another study, [19] combined Discrete Wavelet Transform-Fuzzy (DWT-Fuzzy) and combined Continuous Wavelet Transform-Fuzzy (CWT-Fuzzy) models have been developed to predict the daily water level in the northern and southern borders of the Bosphorus Strait. The daily data of the observed water level are analyzed using wavelet conversion as a pre-processing tool to spectral bands (subsets) so that one can get a more accurate prediction of the daily water level with a long lead load of up to 7 days. The time series of daily water surface data is broken down into spectral bands, which are used as input to the fuzzy model. The daily water level is predicted to be the sum of the predicted components (spectral bands). A prediction model has been developed using DWT-Fuzzy hybrid and CWT-Fuzzy hybrid models to predict water level fluctuations. It has been found that the CWT-Fuzzy model performs better than the DWT-fuzzy and independent fuzzy models to predict lead time of up to 7 days at the northern and southern borders of the Bosphorus based on RMSE and CE evaluation criteria. It is concluded that CWT is a better pre-processing method because it provides a more accurate daily water level forecast with lead-times forecasting than DWT. The research [20] on water transfer planning showed overcoming water scarcity and increase water demand, achieving economic and social development, and regional balance, especially in countries with climatic variations such as Iran. The transfer of water between basins is considered as one of the methods of reducing crisis in the area. In this study, the COPRAS method, which is one of the new MCDM methods, is used for the first time to evaluate water

transfer projects between basins in three configurations; integer, fuzzy and gray (intervals). The results of this study can help decision-makers to assess water transfer projects among basins under uncertain conditions. Nayak [21] explains that, given the increasing strength of river pollution in India, common water quality indicators in Western countries, such as the National Water and Sewerage Foundation's (NSFWQI) water quality index and VWQI, cannot really show the water quality status of Indian rivers. To overcome this limitation, fuzzy modeling has been used to predict the water quality of Indian rivers. The research remarks that it has been observed, that the fuzzy water quality index values relative to NSFWQI and VWQI represent the actual water quality status of Indian rivers. This is because the approach of fuzzy logic adopted is equally sensitive to all parameters and can actually demonstrate even a slightest change in the value of each parameter, especially in cases where river extensions are more polluted.

According to the Ministry of Energy, the numbers of qanats in Iran are approximately 36300. Considering these statistics as a relative approximation for all qanats, there is 376068KM of corridor and wells of qanat available in Iran. In this research, a complete scientific model with fuzzy method is provided in order to find and determine the most reliable routes for the qanats.

One of the most challenging issues in qanat industry is to find and determine the qanat route which it has not been ignored until now. Although the qanat channels were previously identified using geophysical surveying methods, this research tried to distinguish the qanat route by the layers such as watercourse, floodway, river, stream, tree, etc., and then the obtained route is compared with the identified route explored by the geophysical institute. In this paper, the qanat routes were derived from the available data of the region using fuzzy tool in the GIS environment. The remainder of this paper is organized as follows: in the next section, the method based on fuzzy logic is introduced in this paper. Then, the experimental result is compared with the real result map. Finally, the conclusions are drawn in the last section.

2. Method

2.1. Study area

Isfahan is one of the megacities in Iran that has an arid climate [22]. It is located at the edge of the desert which is situated in the east and north of Isfahan. The Zagros Mountains are located to the west and south of Isfahan. Zayandeh Roud River is the main source and element of Isfahan's development and beauty [23]. The geographical map of Isfahan is shown in figure 1. Isfahan is the geographical region around the central desert of Iran which is also one of the most traditional agricultural areas.

It has been using the qanat technology to provide water for agriculture and human consumption for thousands of years.

Fig. 1: geography of Isfahan

2.2 Preparing data

Considering previous related studies, library and field research information, the criteria that affect the qanat route are identified in some factors. At first, raster map is prepared for each factor according to the data segmentation in GIS. These factors are divided into three proportions: the first segment related to the water features which consists of water pump, water reservoir, well, watercourse, ditch, stream, and flood way. The second segment related to the vegetation features which consists of trees, grass, orchard, and bush. The third segment is related to the geology features which includes spot height, mine, sand and pit. These features increase the possibility of qanat existence in the ground; therefore, a distance map is provided from each layer. Figures 2, 3 and 4 show the raster maps of these layers respectively.

2.3 Fuzzy logic

Fuzzy logic, the expansion of Boolean logic invented by Lotfy Zadeh in 1965, is based on the mathematical theory of fuzzy sets, which is a generalization of the classical theory of the class [24]. The term fuzzy refers to things which are vague or not clear [25]. In the real world, many times we encounter a situation when we cannot determine whether the state is true or false, so fuzzy logic provides a very valuable flexibility for reasoning in this situation. In this way, we can consider the inaccuracies and uncertainties of any situation. In Boolean system, truth value (1) represents absolute truth value and 0 represents absolute false value. But in the fuzzy system, there is no logic for absolute truth and absolute false value. In fuzzy logic, there is also an intermediate value which is partially true and partially false.
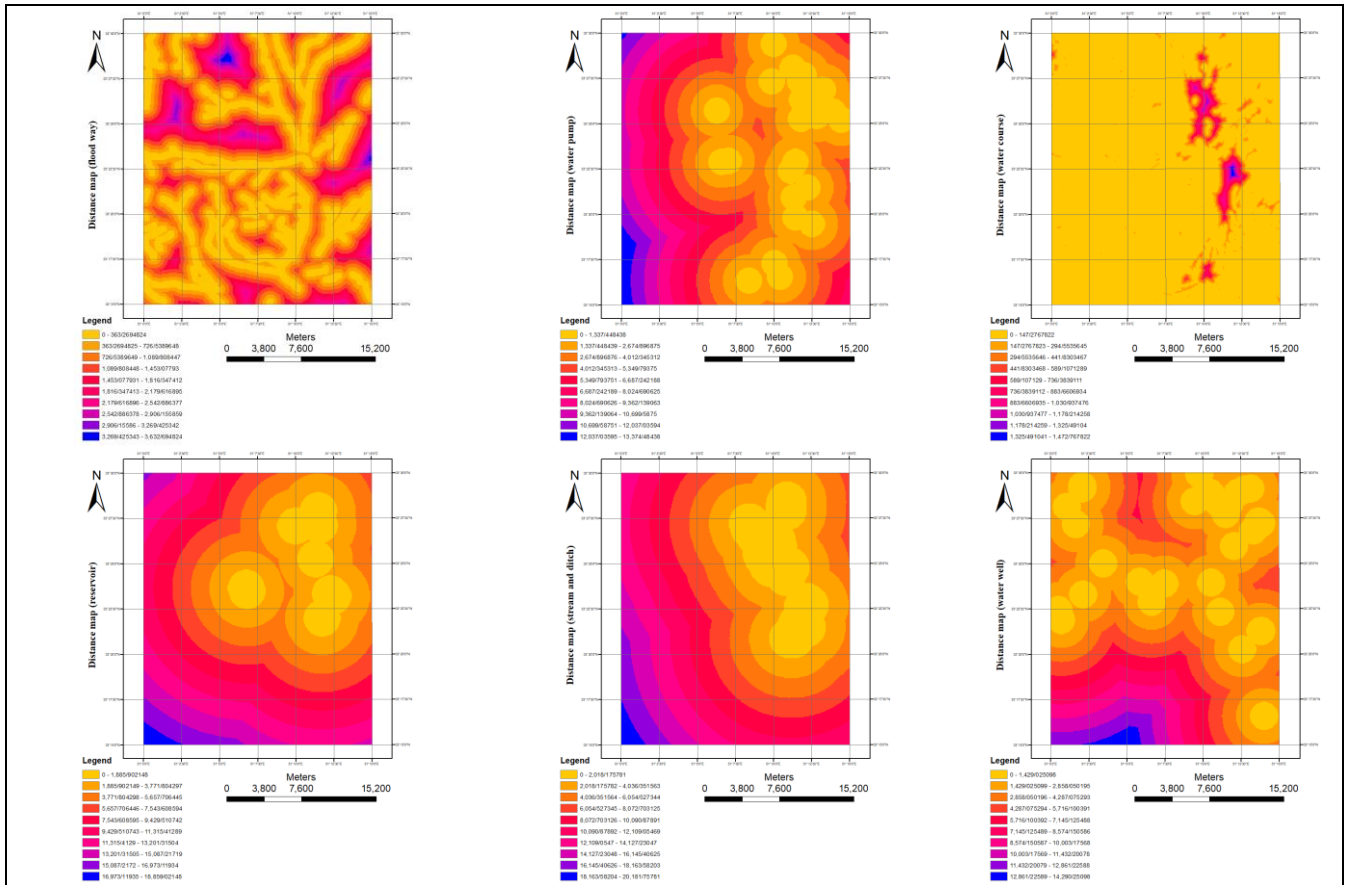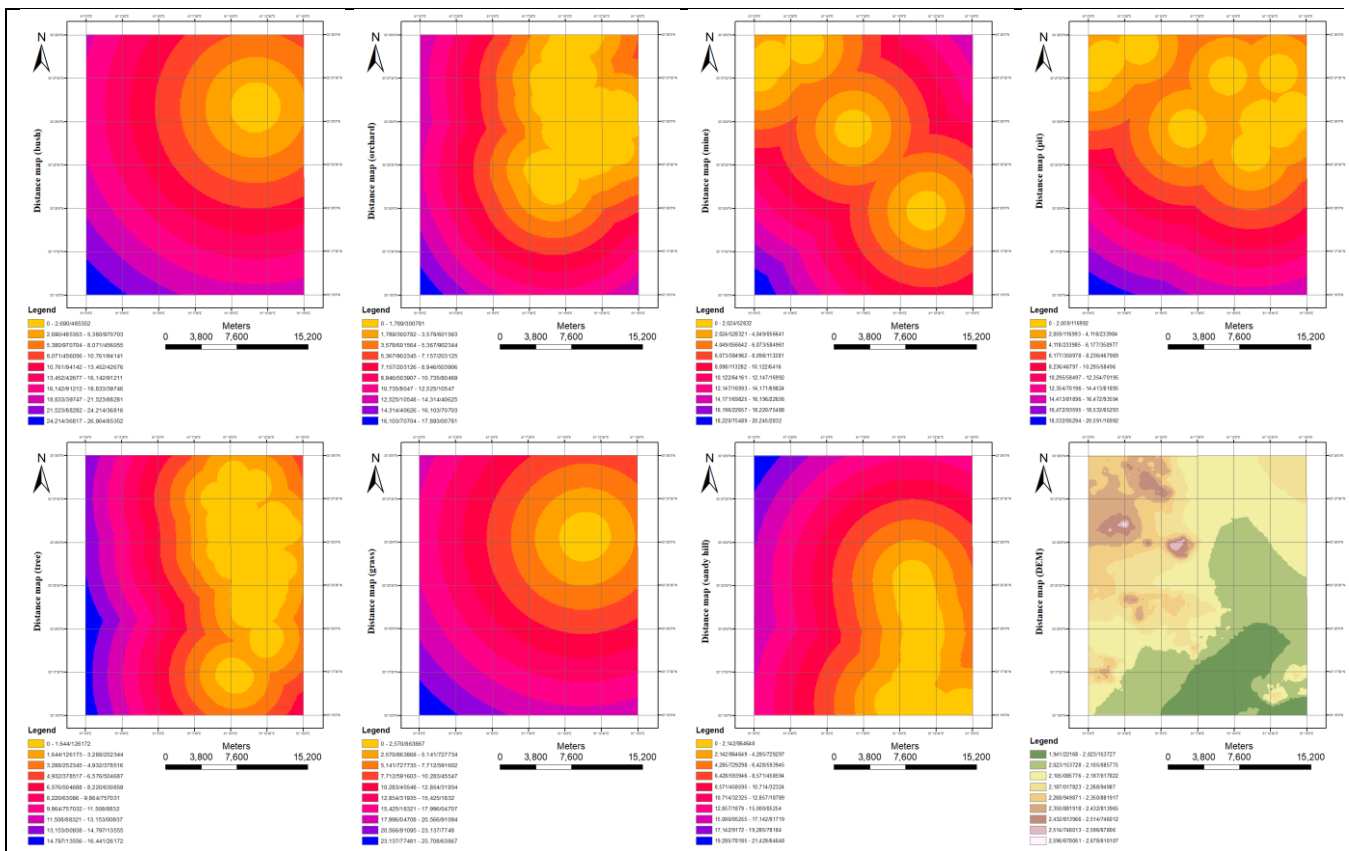
Fig. 2: water features maps

Fig. 3: vegetation features maps

Fig. 4: geology features maps

27

The architecture of fuzzy system contains four parts: 1) rule base, 2) fuzzification, 3) inference engine, and 4) defuzzification. Rule base step contains a set of rules, and the IF-THEN conditions provided by the experts to govern the decision making system, on the basis of linguistic information. Recent developments in fuzzy theory offer several effective methods for the design and tuning of fuzzy controllers. Most of these developments reduce the number of fuzzy rules. Fuzzification is used to convert inputs i.e. crisp numbers into fuzzy sets. Crisp inputs are basically the exact inputs measured by sensors and passed into the control system for processing, such as temperature, pressure, rpm's, etc. Inference engine determines the matching degree of the current fuzzy input with respect to each rule, and decides which rules are to be fired according to the input field. The fired rules are then combined to form the control actions. Defuzzification is used to convert the fuzzy sets obtained by inference engine into a crisp value. There are several available defuzzification methods, and the best suited one is used with a specific expert system to reduce the error.

By introducing the concept of degree in the verification of conditions and providing conditions in an unrealistic or false sense, fuzzy logic provides a very valuable flexibility for reasoning that allows for error and uncertainty [26]. An advantage of fuzzy logic to the recognition of human arguments is that the rules are in the natural language [26]. The fuzzy logic used in this study can be summarized in a simple conditional sentence. Eq.1 is an example of these conditional sentences

$IF\ (Distance\ to\ Sand\ is\ FAR)$
$AND\ (Distance\ to\ grass\ is\ MEDUIM)$ (1)
$THEN\ (Qanat\ is\ bad)$

In the data preparation section, after preparing the layers and making raster layers from them, three groups of distance maps are introduced as inputs in the fuzzy environment. In order to define the laws among the layers, it is necessary to make a series of rules among the sets of factors by using fuzzy logic. According to experts and the importance of features, the criteria are categorized into three groups: The first category consists of water features which are divided into four classes: very near, near, average and far. The second category consists of vegetation features which are divided into three classes: near, average and far. The last category consists of geology features which are divided into two classes: near and far. A series of criteria are also considered for output, which is divided into five classes: very low, low, medium, high, and very high. The Gaussian function is used as membership of classes because of its gentle slope. This function often provides the best results in a fuzzy environment. The Gaussian membership function used for each of the categories are shown in figs 5, 6 and 7.
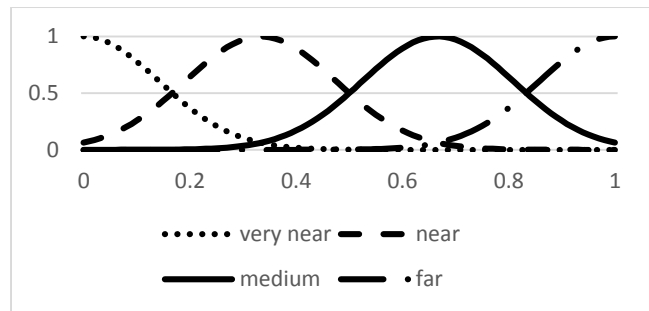

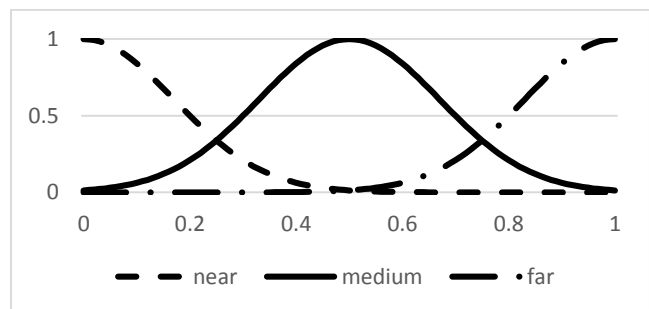**Fig. 5:** Gaussian function membership for factors of water features

Fig. 6: Gaussian function membership for factors of vegetation features
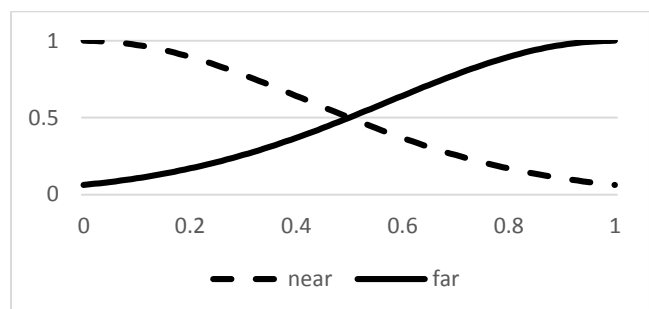
Fig. 7: tow class Gaussian function for factors of geology features

2.4 Fuzzy rules

Fuzzy rule is an important approach to infer with systems that contain real bases. Even if the observed data do not overlap with the values of any of the existing rules, fuzzy rules may still be useful [27]. The inevitable combination theory is an effective mechanism for fuzzy inference with dense dominant bases [28]. In this paper, three scenarios are considered in which three sets of fuzzy rules are developed, in each of which 14 fuzzy rules are considered for finding the qanat route. Three tables (tables 1, 2 and 3) are prepared to illustrate the rules of each scenario. Each row represents one rule in the scenario, and the columns represent indigenous criteria. The value of each cell represents the dependency of each criteria to the rule.

Table 1. First scenario of fuzzy rules

| geology features | | | | vegetation features | | | | water features | | | | | | output |
sand	mine	Pit	Spot-height	orchard	grass	bush	tree	Flood way	watercourse	Stream and ditch	Water well	Water reservoir	Water pump	Appropriate
far					medium			far						Very low
	far						near			medium				Medium
		near					near		near					High
	near			medium							medium			High
	far						near	near						High
	far			medium				near						Medium
	far				medium						medium			Very high
	far			medium					near					Very high
	far			medium						medium				Medium
far							near				medium			Very high
		far		medium								far		Very low
		far			near					medium				High
		near				far						Very near		Very high
			far				medium						medium	High

Table 2. Second scenario of fuzzy rules

| geology features | | | | vegetation features | | | | water features | | | | | | output |
sand	mine	Pit	Spot-height	orchard	grass	bush	tree	Flood way	watercourse	Stream and ditch	Water well	Water reservoir	Water pump	Appropriate
	near			far					very near					Very high
		far			medium						far			Very low
			far			near						medium		Medium
far				near								medium		Medium
		near				far			very near					Very high
	far						medium						far	Very low
near					near			near						High
		near				near							near	High
	near			near							near			High
		near					near					near		High
near						far			very near					Very high
		far			near			medium						Medium
	near						near			near				High
		far					near			medium				Medium

Table 3. Third scenario of fuzzy rules

| geology features | | | | vegetation features | | | | water features | | | | | | output |
sand	mine	Pit	Spot-height	orchard	grass	bush	tree	Flood way	watercourse	Stream and ditch	Water well	Water reservoir	Water pump	Appropriate
			far	near							medium			Medium
	near				far			very near						Very high
near						far							very near	Very high
far				medium						far				Very low
		far				medium						far		Very low
					near	near		near						High
		far			near					medium				Medium
far						medium							far	Very low
	near				far			very near						Very high
	near			near							near			High
far				medium				far						Very low
	near	far								very near				Very high
	far			near									medium	Medium
	near			far								very near		Very high

3. Result

This paper aims to determine the route of the Qanat in Isfahan. Its discharge rate was 150 m/s, which was discontinued due to negligence of authorities and it had irreparable loss on farmers and landowners. That is why the Institute of Geophysics according to gravity points has found a route by surveying which takes a lot of time. This article attempts to find this route by fuzzy logic method based on an entirely scientific approach. According to spatial nature of this issue and fuzzy nature for effective parameters in locating and confronting multiple factors for decision making, a combination of spatial information system and fuzzy logic was used to make a better decision. In each scenario, the qanat routes are identified by enforcing fuzzy rules on input layers. By applying each set of fuzzy rules on the spatial layers, a path for the aqueduct is determined. As previously stated, these routes are compared with the path that the geophysical institute has developed. The obtained and observed routes (The route specified by the geophysical institute) are shown in figure 8.
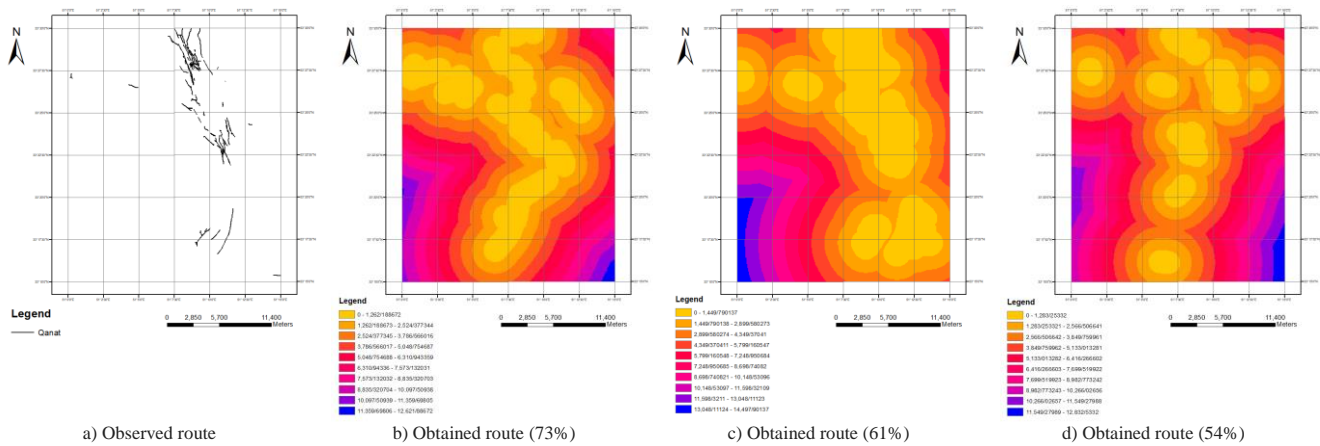

a) Observed route   b) Obtained route (73%)   c) Obtained route (61%)   d) Obtained route (54%)

Fig. 8: a) the observed qanat route (geophysics qanat route) b, c and d) the obtained qanat routes

4. Discussion

The multiplicity and importance of the qanat-related benefits emphasize the fact that this ancient technology, especially for arid irrigation in arid areas like Isfahan, needs to be reconsidered, especially in villages that deal with water shortage problems. In this study, the aqueduct route is determined in the study area based on three different scenarios. In order to evaluate the obtained routes, these results were compared with the actual aqueduct route. The aqueduct routes are exhibited in figure 8. To compare these results with reality, first the distance map of the real aqueduct is prepared, then the correlation between the distance map and the obtained results is examined. The results of the comparison are shown in Table 4.

Table 4. The comparison of three scenarios

	Scenario No.1	Scenario No.2	Scenario No.3
Correlation with reality	73%	61%	54%

The results of the comparison show that the accuracy rate in scenario 1, 2, and 3 is 73%, 61%, and 54% respectively. The best accuracy of the routes discovered by this way is approximately 73 percent similar to the route which is illustrated by the geophysical institute (figure 8). The rules described in each scenario are defined by three groups of experts. Since the research area of experts is close to each other, the obtained results are close to each other in the three scenarios.

A closer look at the results reveals that the role of mine in the aqueduct route is somewhat important. In scenario 1, the mine component is involved in almost half of the rules, while the number is reduced in the other two scenarios. Therefore, it can be concluded that mine is somewhat effective in the existence of the aqueduct route. Another important result that can be mentioned is the role of vegetation features in determining the path of the aqueduct. The role of vegetation features is the same in all three scenarios. Therefore, it can be concluded that vegetation features have played a weak role in determining the direction of the aqueduct. In other words, vegetation features can feed their own water from other sources and are not necessarily dependent on the aqueduct path.

5. Conclusion

Qanat, as an ancient invention, is still a good way to supply water in arid and semi-arid regions. They have played an important role in the formation of civilizations and villages, as well as the empowerment of cooperation among water users through shared water management. At the same time, qanats are the most consistent water supply systems in

nature because it balances groundwater levels and does not demand any additional requirements.

In this study, the required and effective layers on the aqueduct route were first identified, then these data were divided into three groups: geology features, vegetation features and water features. In the next step, the fuzzy inference engine was defined, and the fuzzy rules were defined in three different scenarios. These rules were then applied to the data, and the proposed aqueduct route was determined.

The study considered three scenarios to identify a possible aqueduct route that had the accuracy of 73%, 61%, and 54%, respectively, and are shown in Figure 8. As seen from these results, the method used in this study examines the existence and possible location of aqueducts in the study area which demonstrate its effectiveness. Considering the cost and time required to conduct ground operations to discover the aqueduct route in an area, it can be said that using methods that can lead to the desired result with available data is logical and cost-effective.

Finding qanat route can be beneficial for managing and using water in arid/semi-arid areas. Also, we can protect this heritage that has existed from the past in many cities such as Isfahan city, which is an ancient and cultural capital city in Iran. The pivotal point is that, by using this method to find the qanat route is more affordable, and does not take much time nor require much work force.

Various other rules can be used to improve the results. The rules defined in this study are based on the opinions of experts. As more factors increase, and more diverse rules emerge, better answers are likely to emerge. In this study, the participation of all three groups of geology features, vegetation features and water features was tried to be equal. Future research could determine the effectiveness of each of these three categories by changing their participation.

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