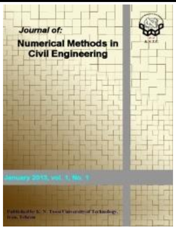

Numerical Methods in Civil Engineering

Variables Characteristics Effects on Static and Pseudo-Static Reliability-Based Design of near Slope Shallow Foundations

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ARTICLE INFO

Article history:
Received:
September 2018.
Revised:
January 2019.
Accepted:
February 2019.

Keywords:
Reliability-Based Design,
Statistical Characteristic,
Probabilistic Analysis,
Shallow Foundation.

Abstract:

Given the concept of reliability-based design (RBD) and the growing risk management trend in geotechnical engineering, proper understanding and quantification of uncertainties are very important. The complexity of methods and a large volume of calculations of probabilistic design methods are the most critical reasons for civil engineers not to be comfortable using these approaches. In this research, a practical probabilistic innovative approach is used to calculate the reliability index by applying the first-order reliability method (FORM). To analyze the bearing capacity of foundations, the Hansen method is used, and both static and seismic designs have been carried out. A scenario where the foundation is located above flat ground and another scenario where the foundation is located near the slope are both considered. Different angles of the slope are also considered. The reason for choosing two different angles for the slope is to examine the effect of slope increase on RBD. As we know, in the RBD of geotechnical structures, our knowledge of the statistical characteristics of variables is significant. That is why, in this paper, the effect of the parameters distribution type (normal or non-normal), the variables dependence, as well as the effect of coefficient of variation in the design results is evaluated. It is found that assuming normal distribution and independence of the variables yields conservative results. The coefficient of variation (COV) of variables is very influential on the results of RBD, and the effect of variation in the internal friction angle (φ) is more significant than variation in the other parameters.

1. Introduction

1.1 Reliability-Based Design

In geotechnical engineering, foundation probabilistic analyses and design methods have been used in a limited number of cases. Difficulties of the concept and applying the probabilistic methods in industrial projects are among the most important reasons for the engineers' unwillingness to use probabilistic methods [1]. Reliability-based design methods (RBD) are new approaches to quantitative uncertainties. A reliability analysis presents a more meaningful approach for geotechnical design, rather than the calculation of a factor of safety, as this can be used for risk-based analyses. For initiating a reliability-based design, some statistical properties of soil, such as the mean value and standard deviation of soil shear strength parameters, are usually created numerically.

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Subsequently, one of the techniques for calculating the reliability index should be selected out of first-order reliability method (FORM), Monte Carlo, point estimate, etc. [2,3]. In RBD, the distributions of variables are assigned to parameters instead of a particular value; hence, the distribution of the safety factor can be obtained, which can undoubtedly present a realistic perspective of design safety. Reliability analysis has been used in some geotechnical engineering problems in recent years. The types of analyses can be either practical reliability-based computations and applications as included in Phoon [4], or stochastic analysis using finite elements or other numerical methods [5,6,7,8]. Applications covered may range from complex liquefaction analysis [4] to those in retaining wall design [9], shallow foundation design and analysis [10,11,12,13], slope analysis [2,14], using random field theory for geotechnical problems analysis [15], and tunneling problems design [16].

1.2 Practical Probabilistic Approach

Low [17] proposed a practical probabilistic approach, which is a quick, accurate, and straightforward method for determining the first-order second-moment reliability index. This approach is based on the perspective of an ellipsoid that meets the failure curvature. Also, subsequent calculations will be done by an optimization tool of spreadsheet software in the original space of the variables. This perspective is mathematically equivalent to the widely adopted aspect of a sphere in the space of reduced variables, while providing a more intuitive definition of Hasofer-Lind's [18] reliability index [19]. In conventional solutions, in order to obtain the reliability index, the variables must be transferred to normal standard space. However, in the practical method, complicated calculations and transfers are not required, and the entire process is performed in the original space [19]. The reliability index obtained by the practical approach is shown in Fig. 1.

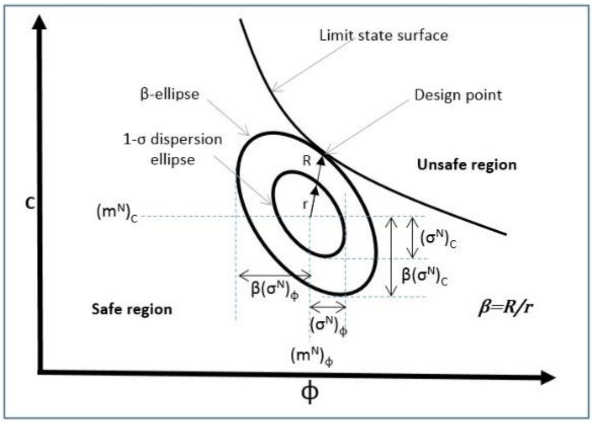


Fig. 1: In-plane equivalent normal ellipsoids and design point [17]

Hasofer-Lind reliability index β can be shown by the matrix formulation (Eq.1) [20]:

$$\beta_{HL} = \min_{x \in F} \sqrt{(x-m)^T C^{-1} (x-m)} \tag{1}$$

Or, equivalently [21]:

$$\beta = \min_{x \in F} \sqrt{\left[\frac{x_i - m_i}{\sigma_i}\right]^T [R]^{-1} \left[\frac{x_i - m_i}{\sigma_i}\right]} \tag{2}$$

Where **x** represents a vector of random variables, C is the covariance matrix, m shows the mean values, R denotes the correlation matrix, and F is the failure domain. In classical explanation, the minimum distance from the mean value point to the failure surface, in the unit of directional standard deviations, is defined as a reliability index. The procedure has been well determined in Ditlevsen [20], Ang and Tang [22], and Melchers [23], among others. The meaning of the β can be intuitively interpreted through Eq. 1; it proposes that the Hosefor-Lind index can be obtained by minimizing the ellipsoid subject to the restriction where the ellipsoid touches the surface of the failure domain [21].

In the practical approach, as shown in Fig. 2, the variables are entered along with their statistical characteristics. The bearing capacity of the foundation can be calculated concerning the variables and bearing capacity relations. The value of β should be calculated by considering the following conditions and taking into account the nonlinear optimization method [17,21]:

Minimize: $\beta = \min_{x \in F} \sqrt{\left[\frac{x_i - m_i}{\sigma_i}\right]^T [R]^{-1} \left[\frac{x_i - m_i}{\sigma_i}\right]}$

Subject to: Performance Function=0

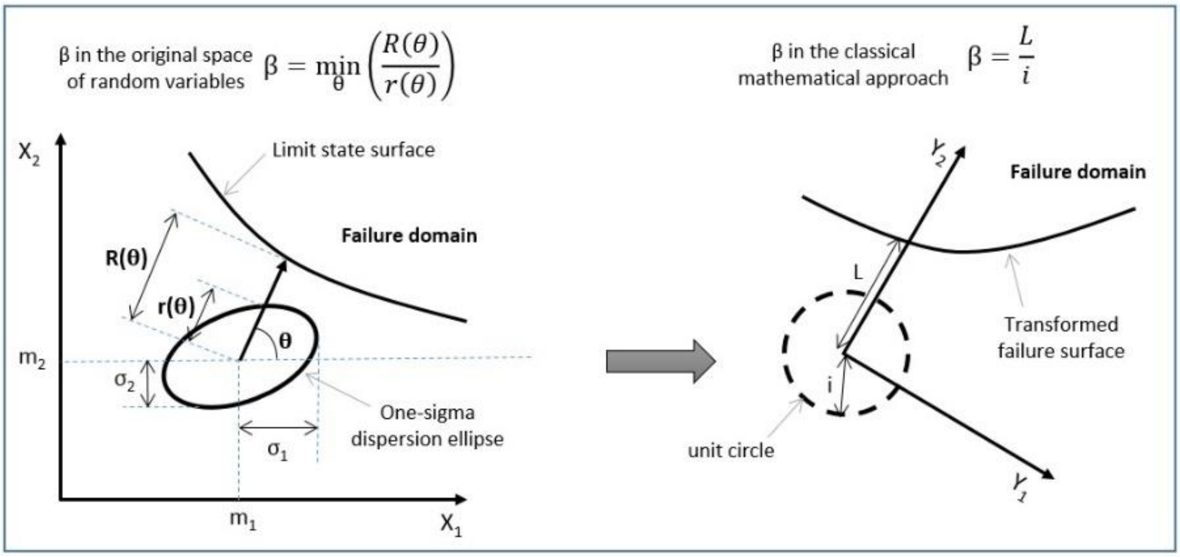


Fig. 2: Comparison of the reliability index between the original space and transformed space [21]

2. Limit Analysis Method (Upper-Bound Solution)

For the upper bound limit analysis solution, the mechanism shown in Fig. 3 is used in this paper. The mentioned mechanism is nonsymmetrical, and it will be suitable for calculating the bearing capacity in the presence of seismic loading. As is well known, an earthquake has two likely effects on a soil-foundation system. The first is an increase in the driving forces, and the second is a decrease in the shearing resistance of the soil. In this paper, only the reduction of the bearing capacity due to the increase in driving forces is considered under seismic loading conditions. The soil's shear strength is assumed to remain unaffected by the seismic loading. On the other hand, the earthquake acceleration for both the soil and the foundation is assumed to be the same, and only the horizontal seismic coefficient Kh is considered.

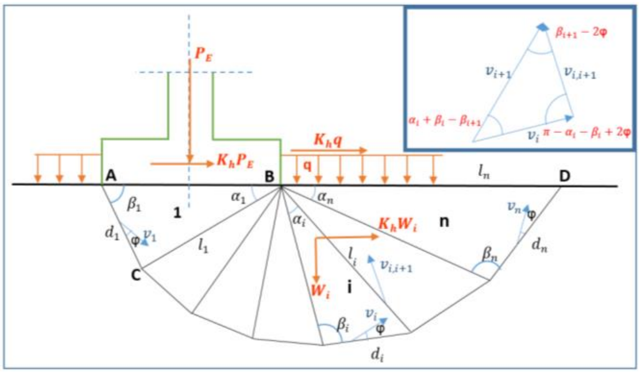


Fig. 3: The non-symmetrical multiblock failure mechanism

As shown in Fig. 3, wedge ABC is considered as a rigid body with a downward velocity V1 inclined at an angle φ to the discontinuity line AC. The foundation is assumed to move with the same velocity as the wedge ABC. The radial shear zone BCD is composed of n triangular rigid blocks which move in directions that make an angle φ with the discontinuity lines di (i = 1, . . . , n). As a result, the condition determines the velocity of each triangle. The velocity hodograph shown in Fig.3 was determined to constitute a kinematically admissible velocity field. External forces are contributing to the incremental external work consisting of the foundation load, the weight of the rigid block soil mass, and the surcharge q on the foundation level (Subra 1999 [24]). Eq 3 and 4 represent the parametric velocity related to rigid block i, and Eq 5 shows the relative velocity between adjacent blocks (i and i+1).

$$V_1 = \frac{1}{\sin(\beta_1 - \varphi)} \tag{3}$$

$$V_{i+1} = V_i \frac{\sin(\pi - \alpha_i - \beta_i + 2\varphi)}{\sin(\beta_{i+1} - 2\varphi)} \tag{4}$$

$$V_{i.i+1} = V_i \frac{\sin(\alpha_i - \beta_i - \beta_{i+1})}{\sin(\beta_{i+1} - 2\varphi)} \tag{5}$$

Eq 6, 7, and 8 show the geometry parameters of triangular block i.

$$l_i = B_0 \frac{\sin\beta_1}{\sin(\alpha_1 + \beta_1)} \prod_{j=2}^{i} \frac{\sin\beta_j}{\sin(\alpha_j + \beta_j)} \tag{6}$$

$$d_i = B_0 \frac{\sin\beta_1}{\sin(\alpha_1 + \beta_1)} \frac{\sin\alpha_i}{\sin\beta_i} \prod_{j=2}^{i} \frac{\sin\beta_j}{\sin(\alpha_j + \beta_j)} \tag{7}$$

$$S_i = \frac{{B_0}^2}{2} \frac{\sin^2\beta_1}{\sin^2(\alpha_1 + \beta_1)} \frac{\sin\alpha_i \sin(\alpha_i + \beta_i)}{\sin\beta_i} \prod_{j=2}^{i} \frac{\sin^2\beta_j}{\sin^2(\alpha_j + \beta_j)} \tag{8}$$

Now, according to the upper bound limit analysis theorem, the amount of bearing capacity can be determined by the equalization of internal and external works, which are introduced in Eqs 9 to 12. Therefore, the bearing capacity can be calculated by using Eq 13, 14, and 15. The external work consists of the work of imposed load to the foundation (W_P), work of soil weight of the block i plus the work due to the inertial force acting on the block (W_{w_i}) and the surcharge work (W_q). Internal work includes the work dissipated in velocity discontinuities di and li (W_{d_i} and W_{l_i}). Energy is dissipated along the lines l_i (i = 1 . . . n-1) and d_i (i = 1 . . . n). Assuming the velocity of the first block, V_1, equal to unity (δ = 1), the calculated load is the bearing capacity of the foundation.

$$W_P = P(1 + K_h V_1 \cos(\lambda_1 - \varphi)) \tag{9}$$

$$W_{w_i} = (\gamma S_i V_i \sin(\lambda_i - \varphi)) + (K_h \gamma S_i V_i \cos(\lambda_i - \varphi)) \tag{10}$$

$$W_{d_i} = c d_i V_i \cos(\varphi) \tag{11}$$

$$W_{l_i} = c l_i V_{i.i+1} \cos(\varphi) \tag{12}$$

W_P : Work of imposed load to the foundation
W_{w_i} : Work of soil weight of block I plus work due to inertial force acting on the block
W_{d_i} : Work dissipated on velocity discontinuity d_i
W_{l_i} : Work dissipated on velocity discontinuity l_i
C : Soil cohesion
φ : Internal friction angle of the soil
λ_i : The angle between l_i and horizon
W_q : Surcharge work

$$\overbrace{W_P + W_{w_i} + W_q}^{External\ work} = \overbrace{W_{d_i} + W_{l_i}}^{Internal\ work} \tag{13}$$

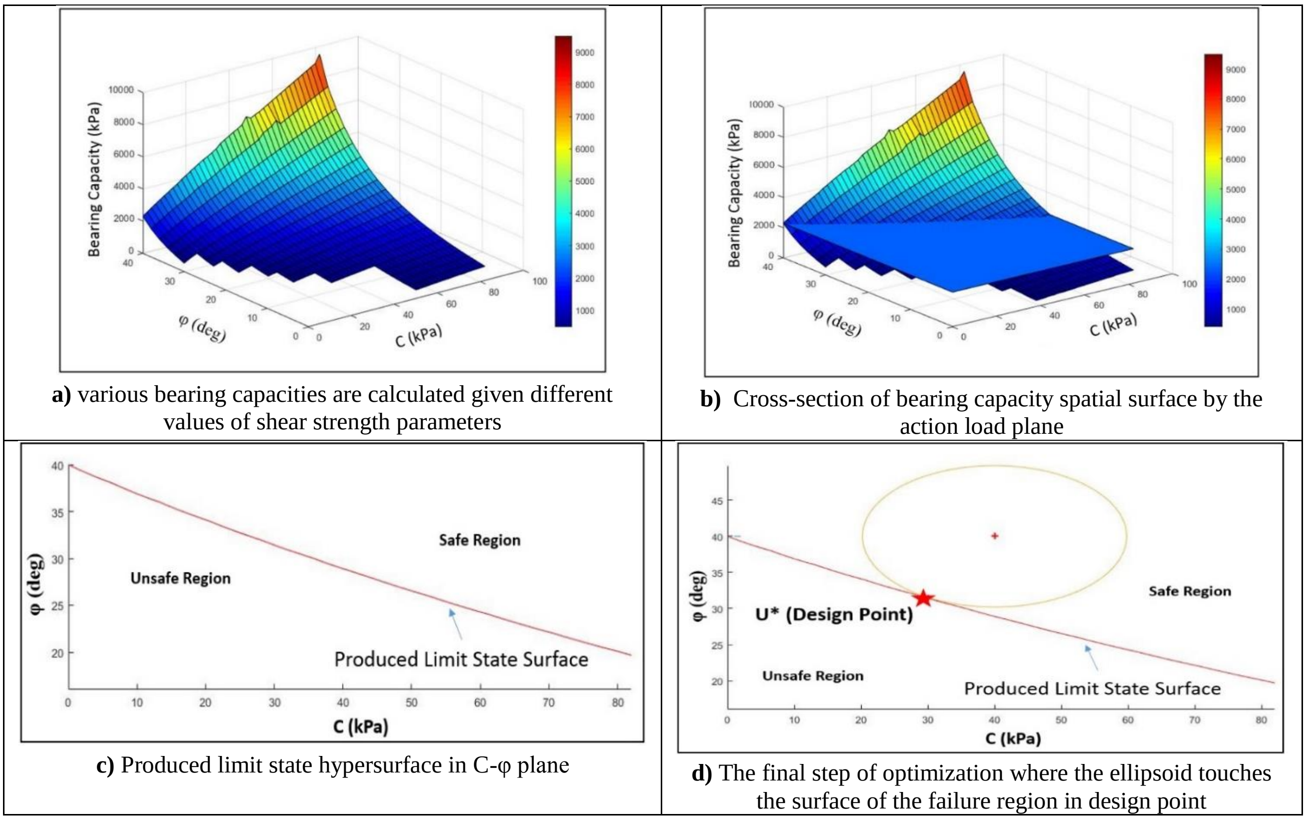
$$P(1 + K_h V_1 \cos(\lambda_1 - \varphi)) + \sum_{i=1}^{n} (\gamma S_i V_i \sin(\lambda_i - \varphi)) + (K_h \gamma S_i V_i \cos(\lambda_i - \varphi)) + W_q = \sum_{i=1}^{n} c d_i V_i \cos(\varphi) + \sum_{i=1}^{n-1} l_i V_{i.i+1} \cos(\varphi) \tag{14}$$

$$q_u = P = \frac{1}{(1 + K_h V_1 \cos(\lambda_1 - \varphi))}(\sum_{i=1}^{n} cd_i V_i \cos(\varphi) + \sum_{i=1}^{n-1} l_i V_{i.i+1} \cos(\varphi) - \sum_{i=1}^{n} (\gamma S_i V_i \sin(\lambda_i - \varphi)) + (K_h \gamma S_i V_i \cos(\lambda_i - \varphi)) + W_q) \quad (15)$$

Given the upper-bound solution of the limit analysis method, the different combinations of soil shear strength parameters are considered and the various bearing capacities are calculated as shown in Fig. 4a. To obtain the limit state surface, a plane, which is representative of the action load of the foundation, is crossed by the spatial surface as presented in Fig. 4b. The curve, which is produced by the mentioned intercross, shown in Fig. 4c can be considered as the limit state surface related to the action load considered. The practical approach that tried to minimize the distance from the mean-value point to produced limit state curve can be employed to evaluate the reliability index as shown in Fig. 4d. Also, Eq. 16 presents the limit state function.

$$q - q_u = 0 \quad (16)$$

Where q is the action stress, and q_u is the bearing capacity of the foundation.



a) various bearing capacities are calculated given different values of shear strength parameters

b) Cross-section of bearing capacity spatial surface by the action load plane

c) Produced limit state hypersurface in C-φ plane

d) The final step of optimization where the ellipsoid touches the surface of the failure region in design point

Fig. 4: MATLAB output: The process of reliability calculation

3. Case Study

A foundation that is studied in this paper is assumed to be located under a 5-story building pillar bearing a near-actual load equal to 200 tons: 50 tons of live load and 150 tons of dead load as shown in Fig. 5.

Dead Load = 150 tons

Live Load = 50 tons

Soil specifications: Ɣ= 18 kN/m^3, C = 20 kPa, Φ = 30°

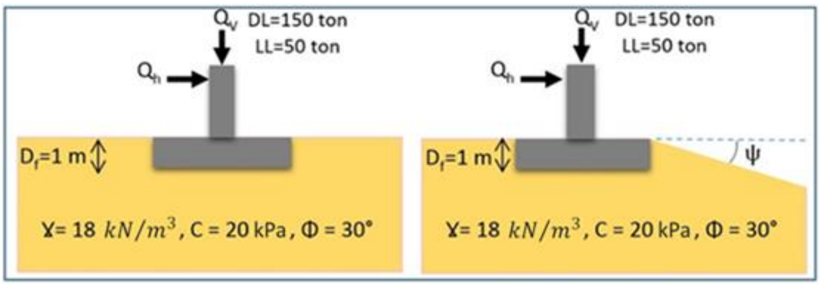


Fig. 5: Foundation conditions in various scenarios

According to the recommendation of the European Code (Eurocode 7. [25]), the target reliability index (βtarget = 3.8) is considered for the reliability-based design of shallow foundations. Various values of the coefficient of variation of the internal friction angle and cohesion are presented in the literature. Within the range of internal friction, the corresponding coefficient of variation as proposed by Phoon and Kulhawy [26] is essentially between 5 and 15%. For effective cohesion, the coefficient of variation varies between 10 and 70% (Cherubini [27]). For the coefficient of correlation, Har [28] has shown that a correlation exists between the effective internal friction angle φ and the effective cohesion C. The result of Wolf [29] ($\rho_{c,\varphi}$=-0.47), Yucemen et al [30] (-0.49≤$\rho_{c,\varphi}$≤-0.24), Lumb [31] (-0.7≤$\rho_{c,\varphi}$≤-0.37), and Cherubini [27] ($\rho_{c,\varphi}$=-0.61) are cited in the literature. In this paper, the illustrative values used for the statistical moments of the shear strength parameters and their coefficients of correlation $\rho_{c,\varphi}$ are shown in Table 1.

Table 1: Variable characteristics

Variable	Distribution	Mean value	Coefficient of Variation
Cohesion(C)	Normal	20 kPa	20%
Internal friction angle (φ)	Normal	30°	10%
Horizontal seismic coefficient (kh)	Normal	0.1 & 0.2	15%
Live load(LL)	Normal	50 ton	15%
Dead load(DL)	Normal	150 ton	10%

4. Effect of the Variables Distribution

The type of distribution of different design parameters can have a significant effect on design results in the reliability-based design method. So, in the first case, all analyses are based on normal variables. In the second case, the suggestions regarding non-normal distributions given in the technical literature are used. In this paper, the way the statistical information about the distribution of various parameters is obtained will not be discussed, because it is not the subject of this study. In this research, referred to as non-normal variables, c is assumed to be lognormal distributed while φ is assumed to be bounded and beta distribution is used (Fenton and Griffiths [32]). The parameters of the beta distribution are determined from the mean value and standard deviation of φ (Mahadevan [33]). For the gravity load and horizontal seismic coefficient, an extreme value type II distribution (EVD) is used. The types of distributions are shown in Table 2.

Table 2: Suggested non-normal distributions in technical literature

Variable	Non-normal distribution
Cohesion (C)	Lognormal
Internal friction angle (φ)	Beta Distribution
Horizontal seismic (kh) coefficient	EVD (Extreme Value Distribution)
Live load (LL)	EVD (Extreme Value Distribution)
Dead load (DL)	EVD (Extreme Value Distribution)

When the random variables are non-normal, the approach to be presented below uses the Rackwitz-Fiessler equivalent normal transformation without the need to diagonalizing the correlation matrix (Rackwitz and Fiessler [34]). Also, the equivalent normal mean value (m^N) and equivalent normal standard deviation value (σ^N) for each trial design point are automatic during the constrained optimization search. Equations 17 and 18 are used to transform the non-normal variable into an equivalent normal distribution, based on Rackwitz-Fiessler transformation:

Equivalent normal standard deviation:

$$\sigma^N = \frac{\varphi\{\Phi^{-1}[F(x)]\}}{f(x)} \tag{17}$$

Equivalent normal mean:

$$m^N = x - \sigma^N \times \Phi^{-1}[F(x)] \tag{18}$$

The Reliability-based Design has been carried out taking into account variable parameters consisting of cohesion, internal friction angle, dead load, live load and earthquake load for the case of the non-normal variable, and the results are presented in Table 3.

As shown in Table 3, the values of Designed Foundation Width in a state that the distribution of variables is considered non-normal, are 17% less than the designed values based on the normal distribution of variables. In other words, normal variables assumed are reliable and conservative. Therefore, if we do not have accurate information about the distribution of variables, the distribution of these variables can be assumed as normal variables. On the other hand, if it is possible to carry out a comprehensive study to determine the distribution and statistical parameters related to the variables, a more optimal and economical design can be carried out.

The normal or non-normal assumption of different variables can cause different effects on the results of RBD; which is why all the previous RBDs have been repeated with two specific approaches. In the first, only the soil shear strength parameters have been regarded, and in the second, only the loads have been considered as normal variables. In each approach, remnant variables have been considered as non-normal variables according to the previous assumption of the distribution. Following Table 4, it can be concluded that the distribution of soil shear parameters has a more significant effect than the loads' distribution. Consequently, recognition of soil shear strength parameters helps us design more affordably.

Table 3: Reliability-based design results comparison between normal and non-normal distribution cases

Slope Status	Type of design	kh	Designed Foundation Width (m) (Normal Variables)	Designed Foundation Width (m) (Non-normal Variables)	Difference percentage
Flat	Static	0	4.02	3.45	17
	Seismic	0.1	4.94	4.18	18
	Seismic	0.2	6.07	5.40	13
ψ = 10	Static	0	5.00	4.25	18
	Seismic	0.1	6.05	5.12	18
	Seismic	0.2	7.28	6.37	14
ψ = 20	Static	0	6.28	5.24	20
	Seismic	0.1	7.48	6.26	19
	Seismic	0.2	8.81	7.57	16

Table 4: Reliability-based design results comparison between different scenarios

Slope Status	Type of design	kh	Designed Foundation Width (m) (Normal Variables)	Designed Foundation Width (m) **c, φ**: Normal Loads: Non-Normal	Designed Foundation Width (m) **c, φ**: Non-Normal Loads: Normal
Flat	Static	0	4.02	3.85	3.25
	Seismic	0.1	4.94	4.85	4.17
	Seismic	0.2	6.07	5.97	5.35
ψ = 10	Static	0	5.00	4.81	4.06
	Seismic	0.1	6.05	5.95	5.13
	Seismic	0.2	7.28	7.15	6.37
ψ = 20	Static	0	6.28	6.08	5.08
	Seismic	0.1	7.48	7.35	6.28
	Seismic	0.2	8.81	8.65	7.59

As shown in Table 4, it seems that the amount of conservatism in different scenarios are extremely disparate. Hence, the abundant number of RBD in the wide range of slope angle (ψ), and four different values of Kh (Static and seismic cases) have been carried out to determine the effect of slope angle on RBD results by regarding normal and non-normal variables. Fig 6 shows that the difference of designed foundation width between normal and non-normal variables is increased by increasing the slope angle. Also in the static design, the RBD results regarding the normal variables are more conservative than the seismic cases. Finally, it can be concluded that in the case of the near slope shallow foundation design, the normal assumption of variables will attain excessively conservative results which would no longer be economical.

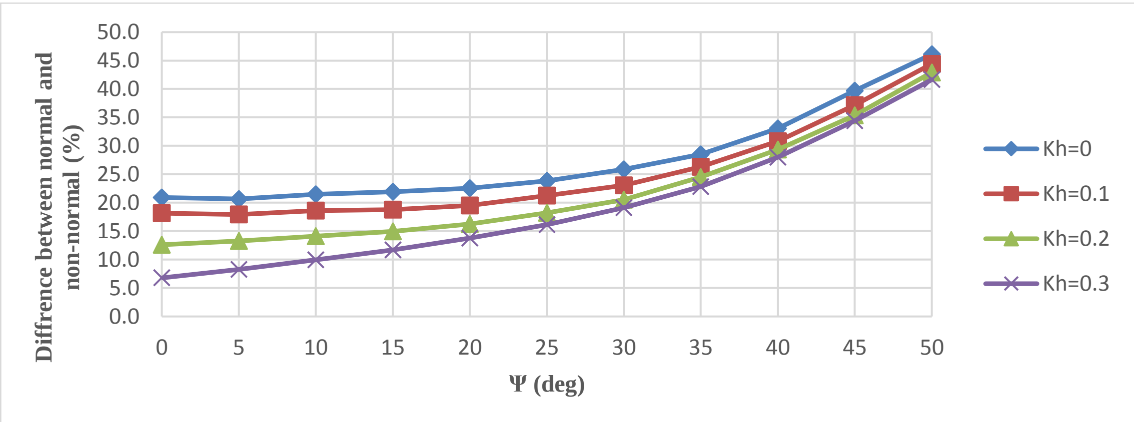


Fig. 6: Difference between the designed foundation width regarding Normal and Non-normal Variables assumption as the slope angle increases in static and seismic cases

5. Effects of the Variables Dependence

As observed in the previous section, various values of the coefficient of correlation between the effective internal friction angle (φ) and the effective cohesion (c) are presented in the technical literature. These can make the variables positively or negatively dependent or independent. According to these studies, the best value for $\rho_{c,\varphi}$ is -0.5. Also, a value of 0.5 was used to correlate gravity and seismic loads [3]. Now, to determine the correlation effect, the previous reliability-based design of the foundation has been performed taking into account these correlation coefficients. The results are laid out in Table 5.

As shown in Table 5, considering the correlation of variables, the average 25% of the designed foundation width is reduced in both static and seismic conditions. According to the results, the difference in the static state is more than seismic, and there is also a more significant difference with the increase of the slope rate. In other words, it can be concluded that if we do not have exhaustive knowledge about the correlation of variables, independent assumptions are in a conservative and trustworthy direction. To comprehend this issue completely, a foundation was designed regarding the large amounts of various correlation coefficients. The results are presented in Fig 7.

Table 5: Reliability-based design results comparison between dependence and independence variables

Slope Status	Type of design	kh	Designed Foundation Width (m) $\rho_{c.\varphi=0}$	Designed Foundation Width (m) $\rho_{c.\varphi=-0.5}$	Difference percentage (%)
Flat	Static	0	4.02	2.96	26
	Seismic	0.1	4.94	3.82	23
	Seismic	0.2	6.07	4.84	20
ψ = 10	Static	0	5.00	3.58	28
	Seismic	0.1	6.05	4.55	25
	Seismic	0.2	7.28	5.64	23
ψ = 20	Static	0	6.28	4.34	31
	Seismic	0.1	7.48	5.43	27
	Seismic	0.2	8.81	6.63	25

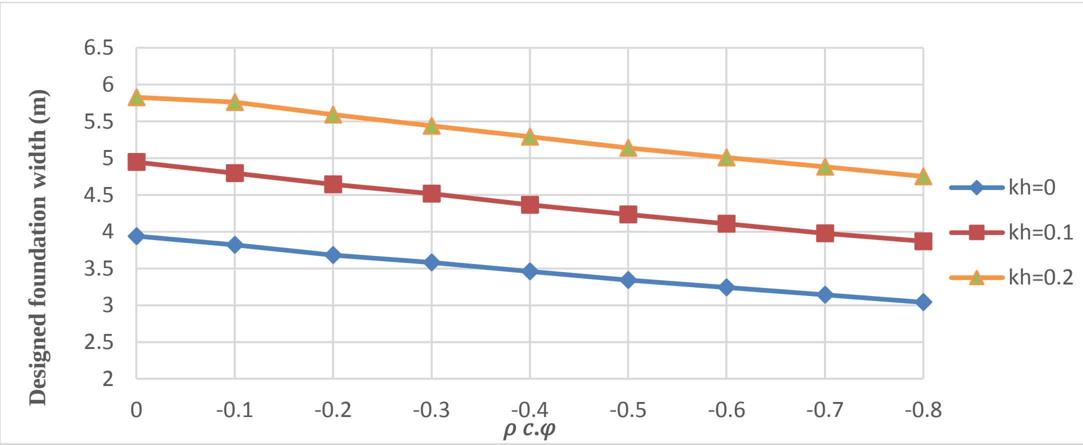


Fig. 7: Designed foundation width (B) by considering different correlation coefficients

As shown by the results in Table 7, it seems that the amount of conservatism in different scenarios are overly disparate. Hence, the abundant number of RBD in the wide range of slope angle (ψ), and four different values of Kh (Static and seismic cases) have been carried out to determine the effect of slope angle on RBD's results by taking into account dependent and independent variables. Fig 8 shows that the difference in the designed foundation width between dependent and independent variable increases when the slope angle increases. Also in static design, the results of RBD regarding the dependent variables are more conservative than the seismic cases. By comparison between static and different scenarios of seismic cases, it can be concluded that the difference in conservatism will increase by increasing the slope angle. Finally, it can be inferred that, when designing a near slope shallow foundation, assuming normally distributed variables will result in highly conservative results that are very costly.

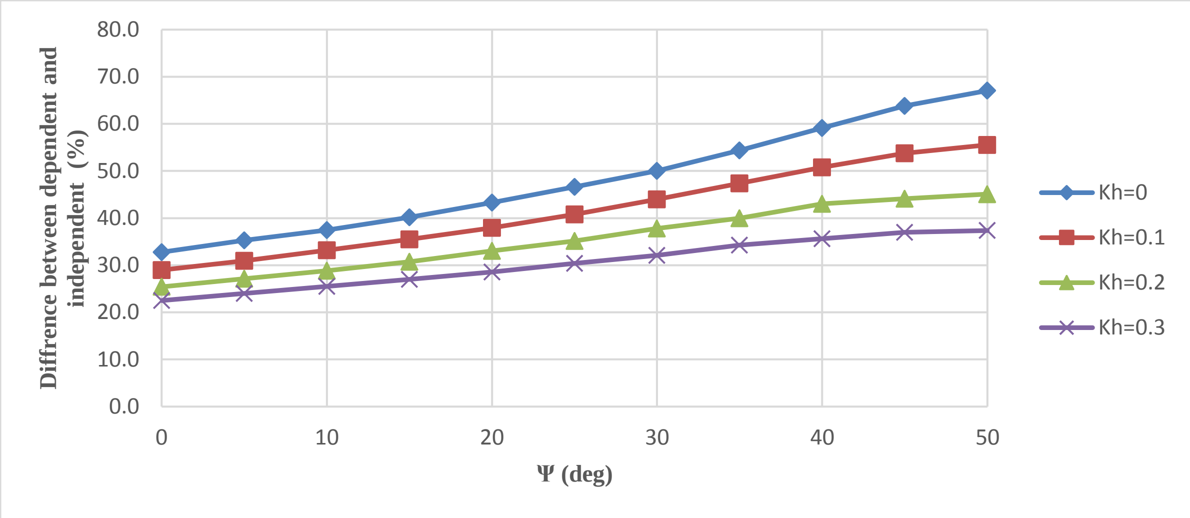


Fig. 8: Difference of the designed foundation width between dependent and independent variables assumption in a various range of slope angles in static and seismic cases

The amounts by which the dependence and distribution affect the design results are shown in Fig 9. As can be seen, the effect of the dependence of variables is greater than the effect of the distribution type of variables. Now, assume a situation where both conservative assumptions (independent and normal distribution) have been taken into account at the same time, and the importance of recognition of statistical characteristics of variables that will be evaluated. For this, two possible assumptions were considered and numerous RBDs were carried out; the first assumption with normal and independent variables, and the second with non-normal and dependent variables. As shown in Fig 10, the width of the designed foundation increases with increasing slope angle. According to Fig 11, given the comparison between 1st and 2nd assumption's effect on RBD of near slope foundation regarding static and seismic design cases, it can be seen that the differences between these two assumptions increase as the slope angle increases. Since the variables are non-normal and dependent in reality, by regarding the accurate information about the variables' statistical characteristics which display the truth of variables, especially in near slope foundation design cases with high slope angle, more economical designs can be obtained using RBD.

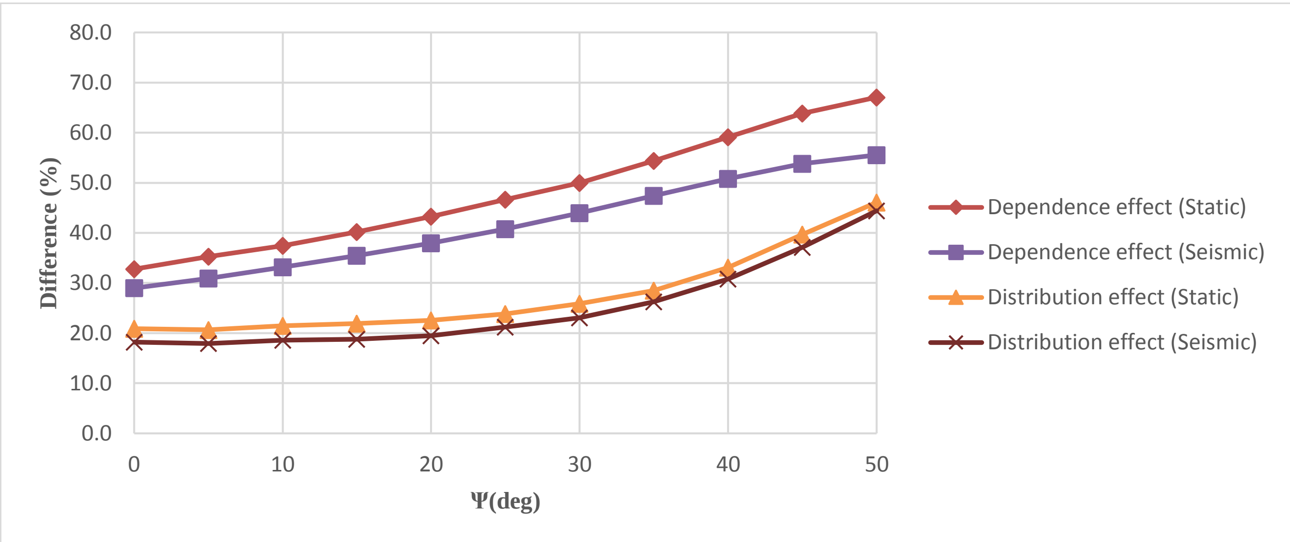


Fig. 9: Comparison between the dependence and distribution effect on RBD of near slope shallow foundation

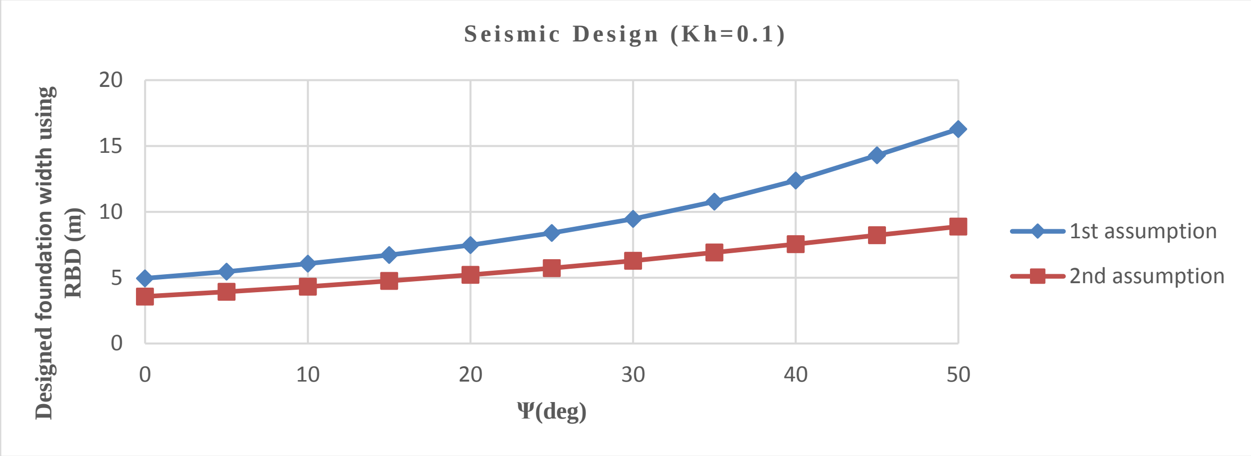


Fig. 10: Comparison between the widths of the foundation designed using RBD under 1st and 2nd assumption

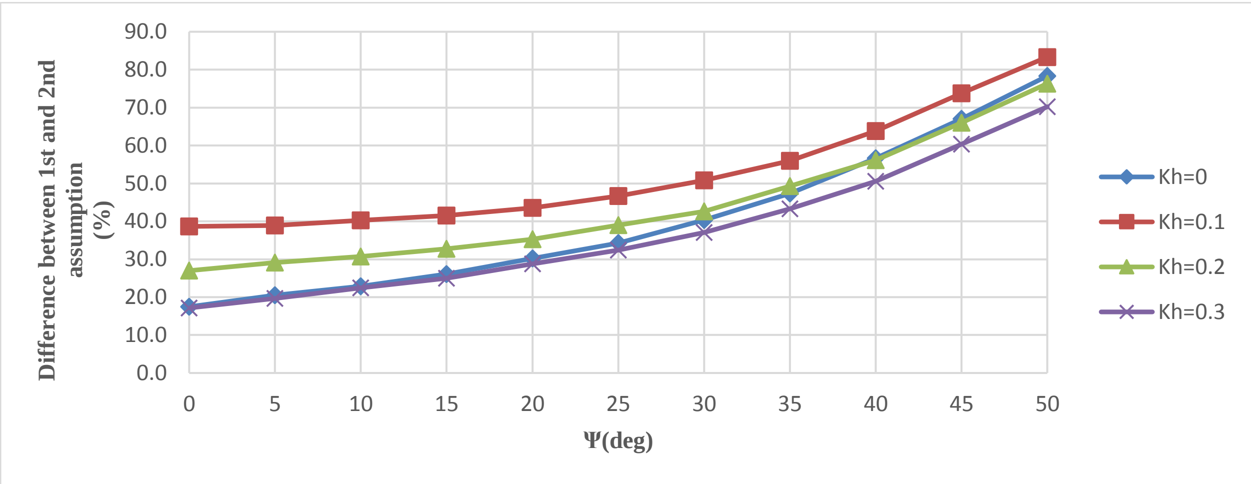


Fig. 11: Comparison between 1st and 2nd assumption's effect on RBD of near slope shallow foundation by regarding static and seismic design cases

6. Effect of the Coefficient of Variation (COV)

As we know, the coefficient of variation in statistical analyses has a large impact on the results. Indeed, when the COV of a variables increase, it means that the uncertainties in that variable are further increased. This issue will reduce reliability and ultimately affect the design results. In this section, how this effect will change by changing the coefficient of variation of variables is investigated. This investigation can also determine the importance of each variable. Thus, the variable whose change in its coefficient of variation leads to more significant changes in the design results has a more important role in reliability-based design. The variable that brings about more changes in the reliability index is more important and effective. According to table 2, the coefficient of variation has been considered as a COV_{Ref}, and as shown in Table 6, a different reliability-based design is obtained by the different values of COV to investigate the effect of changing COV on reliability. For example in Fig 12, the marked point indicates that if the $COV_{\text{Current}}/COV_{Ref}$ of internal friction angle is equal to 0.4, it means $COV_{Current} = 0.4 \times COV_{Ref} = 0.4 \times 20 = 8\%$ and the other variables have the primary value of COV according to Table 2, the reliability index will be equal to 4.52.

Increasing COV of various parameters has different effects on reliability. These effects are shown in Fig.13. It is seen that the sensitivity of the internal friction angle is higher than the other variables. As a consequence, it can be concluded that large dispersions in the internal friction angle can significantly affect the design results. Following the internal friction angle, cohesion has the most effect. Among the loads, the dead load will be more impressive than the others. Given the results of the analyses of the foundations' seismic design using RBD above, it can be deduced that our knowledge about the distribution and the amount of dispersion of internal friction angle can help us design perfectly and optimally. Consequently, the accurate determination of the internal friction angle is critical in the RBD of the foundation.

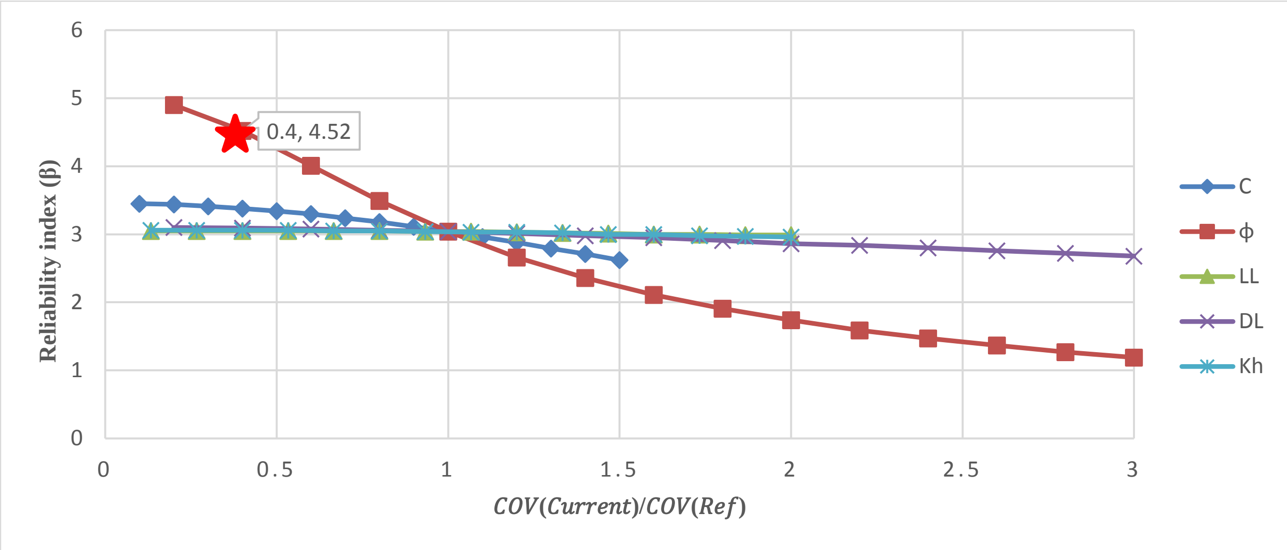


Fig. 12: Design reliability index vs. $\frac{COV_{Current}}{COV_{Ref}}$

Table 6: Reliability index (β) by regarding different values of COV for the case: Flat, kh=0.1, B=2 m, normal variables

COV (%) / Variable	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30
Cohesion (C)	3.45	3.44	3.41	3.38	3.34	3.30	3.24	3.18	3.11	3.04	2.96	2.88	2.79	2.71	2.62
Internal friction angle (φ)	4.90	4.52	4.01	3.49	3.04	2.66	2.36	2.11	1.91	1.74	1.59	1.47	1.37	1.27	1.19
Live load (LL)	3.05	3.05	3.05	3.05	3.05	3.05	3.04	3.04	3.03	3.02	3.01	3.00	3.00	2.99	2.99
Dead load (LL)	3.10	3.09	3.08	3.06	3.04	3.01	2.98	2.95	2.91	2.86	2.84	2.80	2.76	2.72	2.68
Horizontal seismic coefficient (kh)	3.06	3.06	3.06	3.06	3.05	3.05	3.04	3.03	3.03	3.02	3.00	3.00	2.98	2.97	2.96

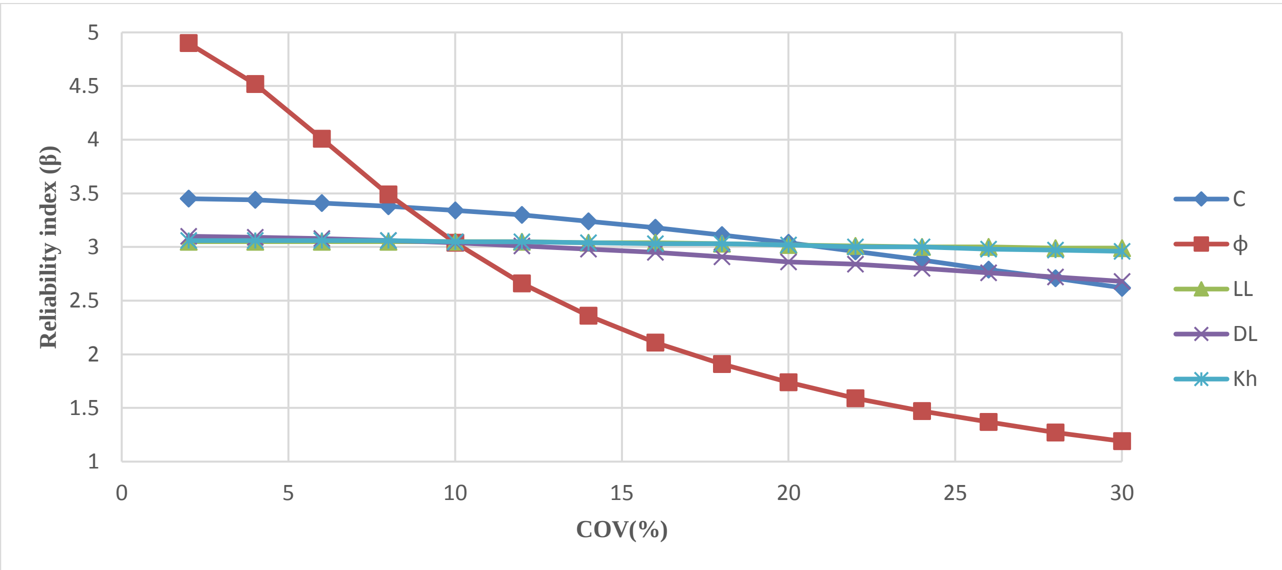


Fig. 13: Design reliability index changes vs. COV changes

7. Conclusion

Investigation of the effect of the statistical characteristics of the variable parameters on the RBD of near slope shallow foundations required a sufficient number of RBD analyses which had to be performed for different scenarios. These RBDs were carried out, and significant results were obtained, all of which have been reported in this paper and briefly explained below.

Normal distribution and independent variables can be assumed to yield conservative design results. In case there is no clear information about the statistical characteristic of variables, normal distributions and independence can be assumed. If one can obtain the exact statistical properties of the variables that are generally non-normal and dependent, a more economical design could be achieved. In reality, c and φ are dependent variables with non-normal distribution. As a result, the normal and independent assumption that is considered because of our lack of knowledge about the statistical characteristics of parameters will yield an overdesigned value of the foundation width. This issue will become more critical in a near slope foundation as the slope angle increases. Therefore, in this situation, designing a shallow foundation near a slope with a high slope angle using RBD will be more reasonable and economical, on the condition that the accurate information about the variable is available. The coefficient of variation (COV) of the variables is very influential on the results of RBD. The effect of the internal friction angle (φ) is more significant than the other parameters. Considering the aforementioned results indicates the importance of having accurate statistical information about the variables; including the distribution type, the coefficient of variation, and degree of correlation.

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