



# New expressions to estimate damping in direct displacement baseddesign for special concentrically-braced frames

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ARTICLE INFO

#### Abstract:

Article history: Received: August 2014 Revised: October 2014 Accepted: January 2015

Keywords: Ductility, Damping, Slenderness Ratio, SpecialCon-Centrically braced-frame

# 1. Introduction

Damping is one of the important parameters in Direct Displacement based-Design (DDBD) developed by Priestley [1]. In DDBD procedure, the structure is characterized by the secant stiffness ( $K_{eff}$ ) at the maximum displacement ( $\Delta_m$ ), and a level of damping ( $\xi$ ) that combines the elastic damping ( $\xi_{el}$ ) and the hysteretic damping ( $\xi_{hys}$ ) (equation 1). with knowledge of the design displacement and the system damping, the effective period ( $T_{eff}$ ) can be obtained from the displacement spectrum for the damping level. The effective stiffness of the equivalent SDOF system ( $K_{eff}$ ) at the maximum displacement and the design base shear can be obtained by equations 2 and 3.

$$\xi = \xi_{el} + \xi_{hys} \tag{1}$$

$$K_{eff} = \frac{4\pi^2 m_{eff}}{T_{eff}^2} \tag{2}$$

frame. Using Jacobsen method, damping is determined and then modified by means of nonlinear time history analysis for special inverted V and X braced frames. A simplified methodology is proposed by using revised effective mass to modify the hysteretic damping. Using the simplified methodology, expressions are proposed to estimate damping for special concentrically-braced frames.

Different expressions have been developed to determine damping of structures based on

various hysteretic models and ductility levels. Since Slenderness ratio is an important parameter

on the hysteretic behavior of special concentrically- braced frames (SCBF), in this paper new

expressions for determination of damping of such frames are developed based on slenderness

ratio and ductility. Two types of SCBF are considered: Inverted V braced frame and X braced

$$V_{Base} = K_{eff} \Delta_m \tag{3}$$

Where  $m_{eff}$  is the effective mass of the system .In the most of design standards, it is assumed that the structural systems inherently contain 5% elastic damping. Since hysteretic damping is related to yielding mechanism of energy absorber members subjected to strong ground motions, it is impossible to obtain a constant value for hysteretic damping.

In the past, expressions were developed by the researchers for different structural systems to estimate damping based on the hysteretic models. Golkan andSozen 1974 [2], Kowalsky et al 1994 [3] have determined damping based on Takeda model. Judi et al 2002 [4], Iwan 1980 [5], Kwan and Billington 2003 [6], Priestley 2003 [7], Dwairi and Kowalsky 2004 [8], Dwairi et al 2007 [9] have obtained damping based on EPP model. Harris 2004 [10], Blandon and Priestley 2005 [11] have used bilinear model to determine damping. All of the expressions developed by the above researchers are used for concrete structures. However Harris provided the equation to calculate damping for steel structures. Since yielding mechanisms

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for moment resisting frames (MRF) and special concentrically- braced frames (SCBF) are different, the expression given by Harris cannot be used for SCBF. Recently, Della Corte and Mazzolani 2008 [12] have also developed DDBD procedure for concentrically- braced frame (CBF) using Takeda-Thin model. Takeda-Thin model represents the response of ductile reinforced concrete wall or column and hence, cannot be used for steel structures [1]. Due to complexity of the hysteretic behavior of special concentrically-braced frames; damping expression cannot be represented by a simple hysteretic model such as EPP, bilinear or Takeda-Thin model. Since, an expression for damping of SCBF's has not been developed to specifically account for the pinching and lowcycle fatigue effects; a new expression should be provided to estimate damping of SCBF's considering both the effects.

Jacobsen 1960 [13] first proposed the concept of equivalent linearization based on Steady-State harmonic excitation approach by replacing a nonlinearly damped elastic SDOF with an elastic SDOF with equivalent damping. In this process Jacobsen solved the damped equation of motion by applying a hysteretic steady-state cyclic force to an elastic SDOF that had the same natural period of the nonlinearly damped SDOF. The solution was integrated to determine the energy dissipated for one cycle of response. The hysteretic damping was calculated by equating the energy absorbed by the hysteretic steady-state cyclic response at a given displacement level, as following Equation:

$$\xi_{\rm hys} = \frac{A_{\rm h}}{2\pi F_{\rm m} \Delta_{\rm m}} \tag{4}$$

Where  $A_h$  is the area of a complete cycle of forcedisplacement response and  $F_m$  and  $\Delta_m$  are the maximum force and displacement occurred in the complete cycle, respectively. Figure 1 shows the graphical representation of the parameters.



Fig. 1: Hysteretic area for damping calculation

The studies carried out by Iwan and Gates 1979 [14], Judi et al 2000 [4], Kowalsky and Ayers 2002 [15], Grant et al 2005 [16], Dwairi et al 2007 [9], have indicated that the hysteretic damping calculated by Jacobsen method can be inaccurate in some cases such as predicting of the peak response during earthquakes. Blandon, Priestley 2005 [11], Priestley, Grant 2005 [17] have developed a method to modify the hysteretic damping defined by Jacobsen method using time history analysis. Dwairi et al 2007 [9] used this method to provide a hysteretic damping relationship for a given hysteretic rule, ductility and period.

In this article for simplification, a simplified methodology is proposed to estimate the hysteretic damping using revised effective mass. This methodology is used to estimate the damping of special concentrically braced- frames (SCBFs). In this regard, sixteen SCBF's consist of eight X braced and eight inverted V (IV) braced frames are designed according to AISC 2010 [18]. These frames are analyzed in order to evaluate the hysteretic damping through Jacobsen method which is then modified using nonlinear time history analysis (NTHA) under 20 ground motions. This study proposes two expressions to estimate the damping based on the ductility level and the slenderness ratio for IV-braced and X- braced frames, separately.

## 2. Structural Model

In order to determine an expression for damping of SCBF for a given ductility, sixteen single storey SCBFs including eight IV-braced frames and eight X-braced frames are designed according to AISC 2010 [18] called SIV, SX, respectively. In order to consider a range of the slenderness ratio, two types of frames are considered. Type A consists of five frames with 4m storey height and 7m width of the bay and type B consists of three frames with 3m storey height and 3m width of the bay. Notations SXA and SXB are used for X-braced frames and SIVA and SIVB for IV-braced frames, as shown in Figure 2. All the braces are hollow square section (HSS) shapes. It should be noted that all the brace cross sections satisfy seismic compact sections as defined in the AISC 2010 [18] specification.



Fig. 2: Definition of various lengths of IV and X braced frame

The braces are designed and detailed to dissipate energy through inelastic buckling in compression and yielding in tension, while the beams and the columns are designed to remain in the elastic range under maximum expected axial forces induced from the braces. The gravity loads are not considered in this study. The brace to beam-column connections are detailed such that the braces are prone to buckle in the direction out-of-plane of the frame, by providing a clear space (two times the gusset plate thickness) in gusset plates at the each end of the brace. The beam-column and column-base connections are assumed to be pinned. The sizes of the braces, the columns and the beams are summarized in Table 1.

X braces			
Frames	HSS-X(mm)	Column (mm × kg/m)	Beam (mm × kg/m)
SXA1	HSS 77×6	W 305×52	W 356×122
SXA2	HSS 89×8	W 305×52	W 356×122
SXA3	HSS 102×12	W 305×52	W14×109
SXA4	HSS 127×12	W 305×52	W14×109
SXA5	HSS 152×15	W 305×66	W14×109
SXB1	HSS 102×12	W 305×66	W 356×122
SXB2	HSS 127×12	W 356×101	W 356×162
SXB3	HSS 178×12	W 356×196	W 356×162
IV braces			
Frames	HSS-X(mm)	Column (mm × kg/m)	Beam (mm × kg/m)
SIVA1	HSS 77×6	W 305×52	W 356×162
SIVA2	HSS 89×8	W 305×52	W 356×236
SIVA3	HSS 102×12	W 305×52	W 356×422
SIVA4	HSS 127×12	W 356×101	W 356×463
SIVA5	HSS 152×15	W 356×196	W 356×592
SIVB1	HSS 127×12	W 356×196	W 356×262
SIVB2	HSS 178×12	W 356×236	W 356×422
SIVB3	HSS 203×15	W 356×262	W 356×552

The properties of the braces indicated in table 2, are the cross-section area (A), the width to thickness ratio( $b_0/t$ ), the theoretical length( $L_0$ ), the actual length( $L_H$ ), the length between the edge of the gusset plates( $L_B$ ), the slenderness ratio( $\lambda$ ), the initial buckling force ( $P_b$ ) and the yielding force of the gross section( $T_y$ ).

The nominal yield stress ( $F_y$ ) and the Young's modulus (E) of the steel are taken as 350MPa and 200GPa, respectively. The effective length factors specified by Wakabayashi et al 1977 [19] for X, IV braced frames are taken as 0.7 and 1.0 for design purpose, respectively.

The parameters in Table 2 are given by AISC 2010 [18] as following:

$$b_0/t = (H - 3t)/t$$
 (5)

$$\lambda = K L_H / r \tag{6}$$

$$P_b = F_{cr}A \tag{7}$$

$$T_{y} = F_{y}A \tag{8}$$

In the above expressions H and t are the depth and the thickness of the brace section, respectively and  $F_{cr}$  is the critical stress defined as follows:

$$\lambda \le 4.71 \sqrt{\frac{E}{F_y}} \qquad for \quad F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right) F_y \tag{9}$$

$$\lambda > 4.71 \sqrt{\frac{E}{F_y}} \qquad for \qquad F_{cr} = 0.877 F_e \tag{10}$$

$$F_e = \frac{\pi^2 E}{\left(\lambda\right)^2} \tag{11}$$

#### 3. Numerical Modeling

In order to obtain the base shear- lateral displacement hysteretic response, the designed frames are modeled in the OpenSees 2010 [20].The study's by Aguero et al 2006 [21], Uriz et al 2008 [22], Yang et al 2009 [23] and Hsiao et al 2012 [24] have shown that, SCBF's can be modeled by OpenSees program efficiently and accurately. Since all the braces are designed to permit the out-of-plane buckling, the frames are modeled in 3-dimensional geometry. The out-of-plane displacement of the frames except braces are limited by restraining the translational degree of freedom in Z direction (perpendicular direction to the plane of the frame) and the rotational degrees of freedom about X and Y global axis at the beam-column connection.



Table.2: Properties of braces

X braces								
Frames	A(mm <sup>2</sup> )	b <sub>0</sub> /t	λ	$P_b(N)$	$T_y(N)$	$L_0(mm)$	L <sub>H</sub> (mm)	L <sub>B</sub> (mm)
SXA1	1574	9.88	166.41	98394	550900	8062	6718	6318
SXA2	2410	8.18	144.93	198619	843500	8062	6708	6268
SXA3	3884	5.60	128.01	410328	1359400	8062	6530	6030
SXA4	5084	7.75	97.10	883875	1779400	8062	6400	5760
SXA5	7548	7.33	76.82	1704953	2641800	8062	6054	5214
SXB1	3884	5.60	62.31	1019096	1359400	4243	3179	2739
SXB2	5084	7.75	45.61	1524797	1779400	4243	3007	2447
SXB3	7484	12.05	29.20	2458772	2619400	4243	2792	2052
IV braces	5							
Frames	A(mm <sup>2</sup> )	b <sub>0</sub> /t	λ	$P_b(N)$	$T_y(N)$	L <sub>0</sub> (mm)	L <sub>H</sub> (mm)	L <sub>B</sub> (mm)
SIVA1	1574	9.88	146.43	127081	550900	5315	4138	3738
SIVA2	2410	8.18	126.88	259135	843500	5315	4111	3711
SIVA3	3884	5.60	114.12	516318	1359400	5315	4075	3575
SIVA4	5084	7.75	86.54	1020652	1779400	5315	3993	3353
SIVA5	7548	7.33	68.70	1861184	2641800	5315	3790	2950
SIVB1	5084	7.75	41.51	1565840	1779400	3354	1915	1355
SIVB2	7484	12.05	24.96	2501079	2619400	3354	1670	910
SIVB3	10581	10.77	18.71	3608368	3703350	3354	1418	558

Table.3: Parameters used to simulate the gusset plate.

X braces												
Frames		t <sub>g</sub> (mm)	a (mm)	b (mm)	L <sub>ave</sub> (mm)	W <sub>w</sub> (mm)		t <sub>g</sub> (mm)	a (mm)	b (mm)	L <sub>ave</sub> (mm)	W <sub>w</sub> (mm)
SXA1		10	660	190	417	370		10	375	390	263	397
SXA2	ion	15	660	190	413	386		15	375	390	259	399
SXA3	nect	20	835	290	557	479	ion	20	375	390	246	420
SXA4	n Cor	20	835	290	516	544	nnect	20	515	470	335	541
SXA5	Ium	25	950	350	563	670	Co	22	805	630	532	744
SXB1	m-Cc	22	345	290	183	367	Base	22	185	490	256	378
SXB2	Beau	25	400	390	197	483		22	275	630	318	501
SXB3		30	540	550	256	679		25	380	750	355	655
					IV	/ braces						
Frames		t <sub>g</sub> (mm)	1 (mm)	h (mm)	L <sub>ave</sub> (mm)	W <sub>w</sub> (mm)		t <sub>g</sub> (mm)	a (mm)	b (mm)	L <sub>ave</sub> (mm)	W <sub>w</sub> (mm)
SIVA1		15	1000	300	265	399		15	295	700	407	451
SIVA2		18	1000	300	270	390		15	295	700	413	446
SIVA3	ction	20	1000	300	267	400	tion	20	295	700	399	484
SIVA4	onnee	22	1200	400	330	538	nnec	20	325	720	381	563
SIVA5	lle C	28	1300	500	359	699	° Co	28	435	900	464	730
SIVB1	Midd	30	955	310	231	518	Base	30	90	900	536	450
SIVB2		35	1240	400	310	696		35	175	1100	601	617
SIVB3		40	1600	500	445	984		40	225	1200	667	700

The column to base and beam to column connections are modeled as pinned. The aforesaid researches showed that accurate simulation of brace buckling behavior is achieved with ten or more nonlinear beam-column elements along length of brace. Thus, sixteen fiber elements are considered. Five integration points are assigned to each element. Accurate prediction of the AISC buckling curves are achieved by an initial displaced shape using a sine function  $(Z = z_0 sin\pi u/L_H)$  with the maximum amplitude equal to 1/350 of the length of the brace  $(z_0 = L_H/350)$ . the HSS is discretized using 20 fibers along the depth and width, and 10 fibers across the thickness of the cross section. This discretization can accurately predict the brace performance for the sizes considered here. The corotational theory is used to represent the large deformation effects of inelastic buckling of the braces. The Menegotto-Pinto model with kinematic and isotropic hardening is used to simulate Basuchinger effect under cyclic loading. The strain hardening ratio is assumed to be 1%. In order to consider low-cycle fatigue effect and the damage due to the large deformations, the model presented by Uriz et al 2008 [22] is used. The beams and the columns are also modeled using the nonlinear beam-column elements. The numerical simulations of both IV and X braced frames are also shown in Figure 3.

Gusset plate connections in actual structures are neither pinned nor fixed joints, and have a significant effect on stiffness, resistance and inelastic deformation capacity of SCBF's. Therefore, accurate simulation of this connection is required. To simulate the nonlinear out-of-plane rotational behavior of the gusset plate connections, the model shown in Figure 4 is employed. The rotational nonlinear spring is located at the physical end of the gusset for considering out-of-plane rotation of the brace about the fold line. The zero-length nonlinear rotational spring element at the end of the brace simulates the out-of-plane deformational stiffness of the connection. The elastic stiffness and the flexural strength of the nonlinear rotational spring are provided in equations 12 and 13, respectively. (Hsiao et al. 2012 [24])

$$K_{el}^{s} = \frac{E}{L_{ave}} \left( \frac{W_w t_g^3}{12} \right) \tag{12}$$

$$M_y^s = F_{yg} \left(\frac{W_w t_g^2}{6}\right) \tag{13}$$

Where  $W_w$  is the Whitmore width defined by a 45° projection angle,  $L_{ave}$  is the average of  $L_1, L_2, L_3$  as shown in Figure 3, and  $t_g$  is the thickness of the gusset plate. The post-yield stiffness is taken 1% of the initial rotational stiffness. Three rigid links are employed to simulate the

remaining of the gusset Plate as shown in Figure 4. Table 3 presents the parameters used to simulate the gusset plate.



Fig.4: model of the gusset plate

#### 4. Verification of the Model Using Test Results

Three experimental studies are used for verification of the numerical model in this study. These experiments are: a single storey X-braced frame from the experimental program by Archambault et al 1995 [25], a single storey IV-braced frame by Yang et al 2009 [23] and a two storey IV-braced frame tested by Uriz et al 2008 [22]. The details of adapted specimens are summarized in table 4. The hysteretic responses of the numerical and experimental studies of the specimens are shown in Figure 5. As shown in this Figure, different parameters such as initial buckling load, loading and unloading stiffness, degradation of the strength in the post buckling phase, the tensile strength of the brace and the overall hysteretic response are well predicted by the numerical model.

#### 5. Hysteretic Damping

As illustrated before, Jacobsen method considers the area enclosed by a full cycle of the force-displacement response representing the energy dissipated during cyclic loading. In SCBF's, the energy is dissipated by yielding in the tension braces, buckling in the compression braces and inelastic rotation of the gusset plate about the fold line. To determine the hysteretic damping, ten ductility levels are considered. For each ductility level, the maximum displacement ( $\Delta_m$ ) is calculated by equation 14 and applied to every frame as a lateral cyclic loading. The hysteretic response of the base shear versus the top lateral displacement is drawn. The base shear force  $(F_m)$  corresponding to the maximum displacement and the lateral yield displacement  $(\Delta_{yn})$  from the hysteretic curve are obtained.

The area enclosed by a complete cycle of the forcedisplacement response  $(A_h)$  is determined. The hysteretic damping based Jacobsen method is calculated by equation 4.

$$\Delta_m = \mu \Delta_y \tag{14}$$

In the above equation,  $\Delta_y$  is the lateral yield displacement of the frame related to the tension and compression capacity of the brace (Tremblay et al 2003 [26]). In the literature the horizontal component of the axial elongation of the tension brace at the yield ( $\Delta_{yt}$ ) has been used instead of  $\Delta_y$  (Tremblay et al. 2003 [26]).

The results of the nonlinear analysis of SCBF's indicated that  $\Delta_y$  was less than  $\Delta_{yt}$ . Especially for the slender braces, this difference can be large. In order to avoid the nonlinear analysis at the design stage, the lateral stiffness approach is recommended to determine  $\Delta_y$  as follows:

$$\Delta_{\rm y} = V_{\rm y} / K_{\rm el} \tag{15}$$

$$K_{el} = 2EA/L_{\rm H}\cos^2\theta \tag{16}$$

$$V_{\rm y} = (T_{\rm y} + P_{\rm b})\cos\theta \tag{17}$$

Where,  $K_{el}$  and  $V_y$  is the lateral stiffness and the lateral yield force of SCBF's, respectively.  $\theta$  is the angle between center line of the brace with the horizontal axis. The values of lateral yield displacements are calculated by two approaches, (the horizontal component of the axial elongation of the tension brace at the yield and the lateral stiffness approach), for sixteen frames and compared with the yield displacement obtained by the nonlinear analysis ( $\Delta_{yn}$ ). The results are presented in Table 5.



(a)One Story X Braced frame Tested Specimen by Archambault et al(1995)

0.00

1.00

2.00

-1.00

Exprimental
Numerical

3.00 <u>v</u> 4.00

Fig.5: Comparisons of hysteretic responses of the numerical and experimental results

Frames	Туре	Bracing	A (mm2)	F <sub>y</sub> (Mpa)	E (Gpa)	L <sub>0</sub> (mm)	L <sub>H</sub> (mm)	t <sub>g</sub> (mm)	F <sub>yg</sub> (Mpa)
Archambault 1995 [25] X2-A	X-1 Story	HSS 76×5	1574	350	200	6096	4619	9.8	300
Yang et al 2009 [23]	IV-1 Story	HSS 152 ×10	4890	412	175	2434	2020	9.5	392
Uriz et al 2008 [22]	IV-2 Story	HSS 152×10	4890	420	203	6815	5368	22	392

Table.4: Characteristics of the adopted specimens

<sup>Δ</sup><sub>y</sub>
1.50
1.00
0.50
0.00
-0.50
-1.00

-1.50

-2.00

-4.00

-3.00

-2.00

Table.5: Comparison of lateral yield displacements

			X braces			
Frames	Ty / Pb	Δ <sub>yt</sub> (mm)	Δ <sub>y</sub> (mm)	Δ <sub>yn</sub> (mm)	$\Delta_y/\Delta_{yn}$	$\Delta_{yt}/\Delta_{yn}$
SXA1	5.60	13.54	7.98	8.97	0.89	1.50
SXA2	4.25	13.52	8.35	8.87	0.94	1.52
SXA3	3.31	13.16	8.57	9.92	0.96	1.41
SXA4	2.01	12.90	9.65	9.17	1.05	1.40
SXA5	1.55	12.20	10.04	9.65	1.04	1.26
SXB1	1.33	7.87	6.88	6.77	1.02	1.16
SXB2	1.17	7.44	6.80	6.73	1.01	1.08
SXB3	1.07	6.91	6.70	6.70	1.00	1.03
			IV brace	8		
Frames	T <sub>y</sub> / P <sub>b</sub>	Δ <sub>yt</sub> (mm)	Δ <sub>y</sub> (mm)	Δ <sub>yn</sub> (mm)	$\Delta_y/\Delta_{yn}$	$\Delta_{yt}/\Delta_{yn}$
SIVA1	4.34	11.00	6.77	7.52	0.90	1.46
SIVA2	3.26	10.93	7.14	7.76	0.92	1.41
SIVA3	2.63	10.83	7.47	7.86	0.95	1.38
SIVA4	1.74	10.61	8.35	8.44	0.99	1.26
SIVA5	1.42	10.07	8.58	8.33	1.03	1.21
SIVB1	1.14	7.49	7.04	6.58	1.06	1.14
SIVB2	1.05	6.54	6.39	6.14	1.04	1.07
SIVB3	1.03	5.55	5.48	5.43	1.01	1.02

It can be interpreted from table 5 that the yielding force of the gross section  $(T_y)$  will be close to the initial buckling force  $(P_b)$  when the slenderness ratio decreases in SXA1 to SXB3 frame. With decreasing the slenderness ratio,  $\Delta_{yt}$ reaches to  $\Delta_{yn}$ . Although for smaller slenderness ratio  $\Delta_{yt}$ and  $\Delta_{yn}$  show nearly same values, in the entire slenderness ratio considered in this study  $\Delta_y$  is closer to the values of the displacements obtained from the nonlinear analysis ( $\Delta_{yn}$ ). Similarly, from frame SIVA1 to SIVB3 same results are identified. Hence,  $\Delta_y$  would be more appropriate value to present realistic lateral yield displacement of SCBF's in design process. It should be noted that  $\Delta_{yn}$  is obtained by means of the force-displacement response during cyclic loading.

The hysteretic damping based on Jacobsen method is determined for all the frames. The variation of the hysteretic damping against the ductility and slenderness ratio is shown in Fig 6. It is realized from this figure that, the hysteretic damping strongly depends on the slenderness ratio and the ductility levels. The hysteretic damping increases as the ductility increases and decreases as the slenderness ratio increases. It should be noted that the hysteretic damping of X- braced frames is larger than IVbraced frames because of unbalanced force at the middle of the beam resulting from the full yield capacity of the tensile brace and a degraded buckling capacity of the compression brace.

In order to use the hysteretic damping at design stage, it is required to modify damping by nonlinear time history analysis.

### 6. Nonlinear Time History Analysis

A simplified methodology based on revised effective mass (REM) to estimate the modified hysteretic damping by means of nonlinear time history analysis for SCBF's is proposed as follows:

**Step 1:** select twenty ground motions and generate average displacement spectrum for different damping values. (Given in next section)

**Step 2:** For each frame, calculate the lateral yield displacement( $\Delta_{\nu}$ ) by equation 15.

**Step 3:** Select a ductility level ( $\mu$ ) and obtain the maximum lateral displacement ( $\Delta_m$ ), the force corresponding to  $\Delta_m$  ( $F_m$ ) and the hysteretic damping ( $\xi_{hys}$ ) from prior section.

**Step 4:** The effective stiffness  $(K_{eff})$  and damping  $(\xi)$  at the selected lateral displacement level  $(\Delta_m)$  are calculated by equations 18 and 1, respectively. Note that, the elastic viscos damping  $(\xi_{el})$  is taken as 5%.

$$K_{eff} = \frac{F_m}{\Delta_m} \tag{18}$$

**Step 5:** With  $\Delta_m$  and  $\xi$ , the effective period can be read from the average elastic response spectra at the damping level.

**Step 6:** To perform a nonlinear time history analysis, it is required to assign a mass determined by equation 19.

$$m_{eff} = \frac{K_{eff} T_{eff}^2}{4\pi^2} \tag{19}$$

It should be noted that the mass is assigned as lumped to the corner nodes at the storey level.

**Step 7:** Nonlinear time history analysis is performed using OpenSees software for the twenty selected records. Newmark's average acceleration method and tangent-stiffness proportional damping model with 5% of critical damping are utilized in these analyses. The average maximum lateral displacement " $\Delta_{m,NTHA}$ " of the selected frame is obtained from the results of the nonlinear time history analysis with the twenty ground motions.

**Step 8:**  $\Delta_{m,NTHA}$  determined in step 7 is compared with  $\Delta_m$  in step 3. If the difference is within a 3% tolerance, then assigned mass is adopted for the given ductility and the slenderness ratio. If the difference is above 3%

tolerance, then the effective mass is revised and the procedure is repeated from step 7 to 8

**Step 9:** With the effective mass, the effective period is determined by equation 20. From the average elastic response spectra, the damping value can be read in terms of both  $T_{eff}$  and  $\Delta_m$ . It should be noted that the modified hysteretic damping is determined by equation 21.

$$T_{eff} = 2\pi \sqrt{\frac{m_{eff}}{K_{eff}}}$$
(20)

$$\xi_{hys} = \xi - \xi_{el} \tag{21}$$

**Step 10:** Step 3 to 9 are repeated at different ductility levels for the same frame. Step 2 to 10 is also repeated for other frames.



Fig. 6: variation of the hysteretic damping against ductility

## 7. Selection of the Ground Motions

In order to determine the effective period and the damping used in REM methodology for a given displacement level, it is required to obtain a set of displacement spectrum based on different damping levels. To achieve this goal, twenty real ground motions are selected for a given site class. In this study, Site class B is assumed according to ASCE-7 2010 [27]. To scale the selected ground motions, the design displacement spectrum with 5% damping with the PGA 0.33g is assumed. The corner period of the aforesaid spectrum for Los Angeles is determined as 8 seconds. Both the average displacement spectrum obtained by the selected ground motions and displacement design spectrum with 5% damping is shown in Figure 7.

As it can be seen from this Figure, the average and design displacement spectrums are very close up to 6 seconds, covering most structural period of vibration. The Characteristics of the selected ground motions are presented in Table 6.

To assess the accuracy of the average spectrum, the design Displacement spectrum for different damping levels is required. The value of the damping correction factor  $R_{\xi}$  determined by equation 22 which was given by Priestley 2007 [1] is utilized to obtain the design spectrum

for different damping levels. The average and design displacement spectrums are compared and shown in Figure 8. This Figure illustrates that the average spectrum is also very close to the design spectrum for different damping levels.



Fig. 7: Comparison of the average displacement spectrum and design displacement spectrum with 5% damping

Event	Vear	Station	Мад	PGA	PGV	PGD
Event	Tear	Station	iviag	(g)	(mm/s)	(mm)
Tabas, Iran	1978	Tabas	7.35	0.811	791	392
Kocaeli, Turkey	1999	Izmit	7.51	0.22	298	171
Irpinia, Italy-01	1980	BagnoliIrpinio	6.9	0.131	234	95
Irpinia, Italy-01	1980	Bisaccia	6.9	0.122	179	109
Denali, Alaska	2002	Carlo (temp)	7.9	0.099	76	37
Northridge-01	1994	Sandberg - Bald Mtn	6.69	0.086	98	34
Chi-Chi, Taiwan	1999	ILA063	7.62	0.081	132	82
Denali, Alaska	2002	R109 (temp)	7.9	0.06	57	35
Irpinia, Italy-02	1980	Bisaccia	6.2	0.096	164	53
Loma Prieta	1989	San Francisco, Sierra Pt.	6.93	0.111	97	52
Loma Prieta	1989	SF - Rincon Hill	6.93	0.052	67	35
Loma Prieta	1989	Point Bonita	6.93	0.08	89	28
Chi-Chi, Taiwan	1999	TCU085	7.62	0.52	73	130
Irpinia, Italy-01	1980	Auletta	6.9	0.067	50	23
Morgan Hill	1984	Gilroy Array #1	6.19	0.061	32	10
San Fernando	1971	Cedar Springs	6.61	0.015	14	7.6
Loma Prieta	1989	Piedmont Jr High	6.93	0.084	82	31
Northridge-01	1994	Antelope Buttes	6.69	0.046	36	22
Kocaeli, Turkey	1999	Izmit	7.51	0.22	198	170
Chi-Chi, Taiwan	1999	TTN042	7.62	0.0585	67	50

Table.6: Characteristics of the selected ground motions



Fig. 8: Comparison of average and design displacement spectra at different damping levels

## 8. Results of Time History Analysis

The modified hysteretic damping of sixteen one-story SCBF systems were obtained from nonlinear time history analyses performed using OpenSees. The variation of the modified hysteretic damping against ductility for IV and X-braced frames is shown in Figure 9.

Comparison of these curves with those of Figure 6 shows that the hysteretic damping is overestimated by Jacobsen method. The variation of the modification factor against the ductility for the slenderness ratio levels is shown in Figure 10. The modification factor is defined as the ratio of the modified hysteretic damping to the hysteretic damping based Jacobsen method.

As shown in Figure 10, the hysteretic damping based Jacobsen method cannot accurately estimate the hysteretic damping of SCBF's. Since the slenderness ratio strongly affects the hysteretic damping, the new expression should be provided in terms of both the slenderness ratio and the ductility. Referring to Figure 9, it is realized that the new expression can be defined by standard expression. The standard expression is as follows:

$$\xi = 0.05 + A \frac{\mu^B - 1}{\mu} \tag{23}$$

In order to determine the coefficients A and B in equation 23, the linear regression is used. The variations of

these coefficients with slenderness ratio for the IV and Xbraced frames are presented in Figures 11 and 12. Using the linear regression, the coefficients A and B are determined by equations 24 and 25, respectively.

 $A_{SCBF-IV} = -0.001\lambda + 0.2699,$  $A_{SCBF-X} = -0.0013\lambda + 0.3053$ (24)  $B_{SCBF-IV} = 0.0014\lambda + 1.109,$  $B_{SCBF-X} = 0.0023\lambda + 1.0546$ (25)To show the accuracy of the proposed expressions, the

damping modified by nonlinear time history analysis and determined by the proposed expressions is compared in Figure 13 for the two slenderness ratios. As it can be seen from this Figure, the proposed expressions can approximately estimate the damping.



Fig. 9: Variation of the modified hysteretic damping against ductility



(a) IV brace

Fig. 10: Variation of the modification factor against ductility



Fig. 11: Variation of the coefficient A with slenderness ratio



Fig. 12: Variation of the coefficient B with slenderness ratio



Fig. 13: Compression of the damping modified by NTHA and determined by the proposed expression

## 9. Conclusion

The hysteretic damping was determined in terms of the slenderness ratio and the ductility for two types of SCBF's (Inverted V and X braced frame) using Jacobsen Method.

Nonlinear time history analysis was performed to modify the hysteretic damping obtained by Jacobsen method. A simplified methodology was proposed to modify the hysteretic damping using revised effective mass. In this regard, twenty real ground motion were selected and applied to the sixteen SCBF's.

Results showed that the hysteretic damping strongly depends on the slenderness ratio and the ductility. The hysteretic damping of inverted V braced frames were smaller than X braced frames because the unbalanced force at the beam mid span resulting from yield capacity of the tensile brace and a reduced buckling capacity of the compression brace. The lateral stiffness approach could be used to estimate the lateral yield displacement of SCBF's. Based on the proposed methodology, it was concluded that Jacobsen method overestimates the hysteretic damping. Using results of nonlinear time history analysis, two practical expressions were proposed to determine the damping for the design purposes. It was also shown that the proposed expressions can approximately estimate the damping.

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