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Numerical solution of base shear in high tensioned cable antenna

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Abstract:

A finite element solution based on equevalent elements is proposed for the static and dynamic analysis of tallhigh tensioned cable antennas. To reduce high number of degrees of freedom in space frame body of a structure, a simple equivalent beam element is defined for each simulative substructure. This numerical procedure is applicable to analyze complex three dimensional assemblies of substructures of such similar complex structures. In this analysis wind pressure effects accompanied by change of postentioning loads in nonlinear cable elements, earthquake effects, and any other arbitrary loads on the substructures. Accordingly, the restriction of the loads on the cable elements to gravity and thermal loads can be applied.

The algorithm is developed upon an efficient cable elements depending on the given position and curved geometry of the cable, its end forces, and its tangent stiffness matrix. The employed formulation scheme permits any magnitude of deformation for straight or curved elements. The postensioning stresses in cables were considered as initial stresses.

To simulate the equevalent elements, both ends stiffness, damping and mass components are calibrated to present the same static and dynamic responses as the selected substructure. To simulate dynamic responses, the equevalent single mass matrix and its adjusted position are carried out to obtain the same frequencies in equevalent elements. The static solution of a complete structure compared well with the results presented by simulated model.

This paper proposes an alternative structural analysis modeling strategy for guyed steel towers design, considering all the equivalent structural forces and moments, by using three-dimensional beam finite elements. Comparisons of the above mentioned design models with an alternative, that models the main structure and the bracing system with 3D beam finite elements, are made for existing guyed steel telecommunication towers (325m high). The comparisons are initially based on the towers static and dynamic structural behavior later to be followed by a linear buckling analysis to determine the influence of the various modeling strategies on the tower stability behaviour.

1. Introduction

Guyed towers are special structures widely used in communication industry as antenna-supporting structures. A guyed tower is a nonlinear structural system in which the mast, consisting of single beam-column or multiple members (trusses) is supported elastically at various points along its height by inclined pretension cables with their ends anchored to the ground. As a result of the overall slenderness and flexibility of the system, guyed tower exhibit a high degree of nonlinearity and dynamic sensitivity to turbulent wind excitation. Thus to obtain reliable structural responses of the vibrating tower, a 3-D nonlinear dynamic analysis is mandatory.Leonard et al [1] used a two noded elements to study the static and dynamic response of lightly stressed cables. Argyris et al [2] introduced a spatial line element for the analysis of postensioned cable networks. Henghold et al [3] introduced multi node curved elements. A two noded curved element polynomial shape function. A two noded curved element type using Lagrangian shape function was presented by Gambhir [4] and Ozdemir [5].

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Another method named as condensation of structure or substructuring is proposed by Tiv [6]. In this method, to reduce the high number of degrees of freedom in complex structures, a repeatable substructure is selected and replaced by equivalent elements. The closer the response of the equivalent elements to original substructure, the more applicable the method is and the more accurate the results will be.

Despite these facts, most of the traditional structural analysis Nomenclature methods for telecommunication and transmission steel towers still assume a simple truss behavior, where trough all connections σ_{max} not exceed the maximum stress of the tower and all members are considered hinged. On the other hand, upon structural mechanisms, u_{max} = Maximum horizontal displacements of the tower could control and compromise the assumed structural response, and can be presented by f_{01} = First natural frequency of the tower in various commonly used tower geometries, where for truss type f_{02} = Second natural frequency of the tower, maney of such models are adopted as (Policani 2000, 2000a[7]; Silva et al. 2000, 2002[8, 9]).

Furthermore, f_{03} is equal to Third natural frequency of the tower. A usual solution to overcome this problem is the use of dummy f_{04} = Fourth natural frequency of the tow, structural bars to prevent the occurrence of the unwanted degrees of f_{05} = Fifth natural frequency of the tower, freedom. These bars, possessing a small axial stiffness, are generally L_{01} = First buckling load of the tower, employed to prevent the occurrence of structural mechanisms, L_{02} = Second buckling load of the tower , enabling the use of standard finite element programs. A possible L_{03} = Third buckling load of the tower , explanation for the structure stability is related to the semi-rigid, L_{04} = Fourth buckling load of the tower, instead of the assumed hinged joint behaviour. In fact, most major L_{05} = Fifth buckling load of the tower , steel tower constructors still rely on full-scale tests to determinewhich design and fabrication details can provide a good testcorrelation with the assumed simple truss model results. The Structural Modeling by Several authors have contributed with theoretical and experimental investigations to access the best modeling strategy for steel transmission and telecommunication towers. It is fair to mention the investigations made by: Albermani et al 2004[10]; Carril Junior 2000[11]; El-Ghazaly and Al-Khaiat 1995[12]; Kahla 1994[13] and 2000[14], Kitipornchaiand Albermani 1992[15]; Madugula and Wahba 1998[16]; Menin 2002[17], Raoand Kalyanaraman 2001[18]; Saxena et al 1989[19]; Wahba et al 1996[20] andWahba et al 1998[21].

Kahla 1994[13], numerically modelled the dynamical effects present in guyed steel towers including the cable galloping effects. Later the same author, Kahla 2000[14], dynamically modelled the rupture of acable present in guyed steel towers. The analysis indicated that the guyed steel towers cable rupture, disregarding the wind actions, was one of the most severe critical load hypotheses for the investigatedstructures .Wahba et al 1996[20], considered the dynamical nature of the load acting in guyed steel towers like wind, earthquakes and cable gallop.

El-Ghazaly and Khaiatz 1995[12], evaluated telecommunication guyed steel tower designs based on discussions of the various non-linear aspects involved on their numerical modelling. This paper also contemplated the development and comparisons of the results of a 3D model for a 600 meter height guyed steel tower . Wahba et al 1998[21], performed an investigation of the numerical models used in telecommunication guyed steel towers. The authors stressed the relevance of considering the non-linear effects present even at service load levels. In a subsequent paper, Madugula andWahba 1998[21], described two different finite element models for thedynamical simulation of guyed steel towers.

Menin 2002[17], evaluated telecommunication guyed steel towers from their static and dynamical structural responses. The static analysis compared with linear and non-linear mathematical models. The dynamical analysis employed the Monte Carlo simulation method including the wind load floating parcel producing interesting results .

Albermani and Kitipornchai 2003[22], used the finite element method by means of a geometrical and physical non-linear analysisto simulate the structural response of telecommunication and transmission steel towers. This was followed by the work of Albermani et al 2003[22], that investigated the possibility of strengthening steel truss towers from a restructure and rearrangement of their bracing systems. The adopted solution consisted on the addition of axially rigid systems to intermediate transverse planes of the tower panels .

Veletsos and Darbre, 1983[23] provided elucidation of certain aspects of the free vibration of inclined parabolic cables, providing physical insight and interpretations to major trends. They also introduced simple approximate expressions with the aid of which the complete spectrum of the natural frequencies can be sketched readily. In addition they presented simple, closed-form expressions for certain infinite series involving integrals of the natural modes of the cable.

Starossek, 1991[24] investigated the dynamic behavior of an extensible sagging cable. He presented a dynamic stiffness matrix whose coefficients were functions of the frequency of motion, an approach suitable for dynamic direct-stiffness analysis of composed systems such as cable-stayed bridges and guyed masts. His study was restricted to small displacements (linear theory) and considers motion within the vertical cable plane only. He considered viscous damping due to external fluid. He

utilized trigono-metrical solution functions with complex arguments, which implied a substantial simplification in the analysis of damped vibrations. Using example calculations, he discussed stiffness functions and compared it to other available solutions. Starossek 1993[25] derived a frequency-dependent closed form analytic functions for the dynamic stiffness of a sagging cable subject to harmonic boundary displacements. He avoided the troublesome fact that the stiffness functions were frequency-dependent especially in regards to the eigenvalue problem, by using a method whereby an analytic dynamic stiffness function was reduced to a linear matrix polynomial. This reduction corresponds to a mathematically performed transition from a continuum to a discrete coordinate vibrating system. In his work, he did not account for cable damping and assumed that the dynamic stiffness functions were real.

Yamaguchi and Adhikari, 1995[26] investigated analytically the modal damping characteristics of single structural cables. They derived an energy-based representation of modal damping in structural cables in the form of the product of modal strain energy ratio and loss factor. They calculated the modal strain energy to total potential energy ratio numerically for both axial and bending deformations using a finite element method. They deduced from their analysis that the modal damping in a structural cable is generally very low because of the very large contribution of the initial cable stress to the total potential energy, causing very small modal strain energy ratios.

This paper also contemplated a numerically conformed equevalent simple element compromising with a set of elements included in a repeated substructure system that categorized upon configuration of elements assembly through the whole structure. To achieve the compromisation, equevelent nodal stiffnesses and certain single concentrated mass matrix acts at a certain location across the equevalent element and also an average damping coefficient are considered for the single equevalent element. The method of balancing element dynamic responces is based on equal frequency modes. Accordingly, the numerical analysis of reduced-scale guyed steel towers models that produced results in consonance with the developed real numerical model.

2. Non-Linear Behavior of Cables

Structural cables constitute an important part of modem structural engineering applications involving large spans. Pre-tensioned guy cables are implemented in radio and communication guyed towers to provide stability and support to flexible masts reaching high elevations as they are subjected to the effect of wind-induced forces. Stay cables are utilized in cable-stayed and suspension bridges to support their decks and allow for longer navigational clear spans. To achieve larger free spans, cables are used to provide support for large span roofs used to cover sport arenas and exhibition halls. Recently, space structures are designed to benefit from the advantageous properties of cables in terms of strength, light self-weight and economy where they are used as structural tension elements in space stations.

The cable has a self-weight $\tilde{q}$ per unit chord length, modulus of elasticity E, uniform cross sectional area A, chord length L_{ch} with inclination angle θ with respect to the X -axis. The deformed geometry of any point on the cable is represented by the Cartesian coordinates space vector (x, y,z), or (x_1,x_2,x_3) using index notations. A 3-D uniform loading vector $\tilde{q}=\{\tilde{q}_x,\ \tilde{q}_y.\ \tilde{q}_z\}^T$ and concentrated loading vector $\tilde{P}=\{\tilde{P}_x,\ \tilde{P}_y.\ \tilde{P}_z\}^T$ are applied to the self-weight deformed cable in a varying way along its chord length. Due to 3-D loading, the internal cable tension $T=\{T_x,\ T_y,\ T_z\}^T$ at any point $(x,\ y\ ,z)$ will vary from one point to another along the cable.

The main Assumptions to simplify the derivation of the governing equilibrium equations of a cable are as follows:

• Elastic cable material with finite strain (Lagrangian nonlinear strain).

• Long and pre-tensioned cables having dominating axial stiffness (negligible bending and torsional stiffness).

• The tension in the cable varies only along the X -axis (X is the only considered independent variable).

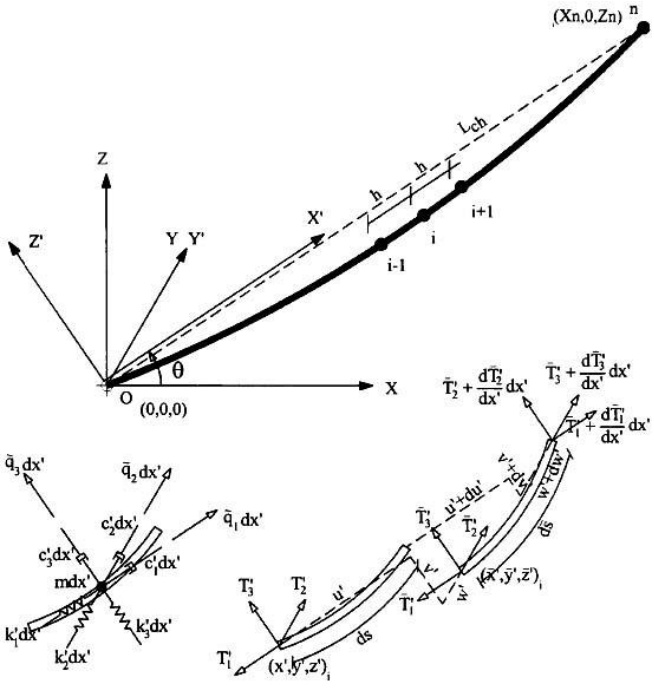

Fig.1: Cable element

Equilibrium conditions for element of length *ds* of the self-weight deformed cable initially in the *X-Z* plane in x' direction is written as follows:

$$\frac{\partial}{\partial \bar{s}}(\bar{T}\frac{\partial \bar{x}'}{\partial \bar{s}}) + (\tilde{q}_{x'} + \tilde{q}_{dx'})dx' = m dx' \frac{\partial^2 u'}{\partial t^2} + c_{x'} dx' \frac{\partial u'}{\partial t} + k_{x'} dx' u' \quad (1)$$

Assuming, $\bar{H}' = (\bar{T}\frac{\partial \bar{x}'}{\partial \bar{s}})$ the motion equation in three directions are presented as:

$$\bar{H}'(0 + \frac{\partial^2 u'}{\partial x'^2}) + \frac{\partial \bar{H}'}{\partial x'}(1 + \frac{\partial u'}{\partial x'}) + (\tilde{q}_{x'} + \tilde{q}_{dx'}) = m(\frac{\partial^2 u'}{\partial t^2}) + c_{x'}(\frac{\partial u'}{\partial t}) + k_{x'} u' \quad (2)$$

$$\bar{H}'(0 + \frac{\partial^2 v'}{\partial x'^2}) + \frac{\partial \bar{H}'}{\partial x'}(1 + \frac{\partial v'}{\partial x'}) + (\tilde{q}_{y'} + 0) = m(\frac{\partial^2 v'}{\partial t^2}) + c_{x'}(\frac{\partial v'}{\partial t}) + k_{x'} v' \quad (3)$$

$$\bar{H}'(\frac{\partial^2 z'}{\partial x'^2} + \frac{\partial^2 w'}{\partial x'^2}) + \frac{\partial \bar{H}'}{\partial x'}(\frac{\partial^2 z'}{\partial x'^2} + \frac{\partial w'}{\partial x'}) + (\tilde{q}_{z'} + \tilde{q}_{dz'}) = m(\frac{\partial^2 w'}{\partial t^2}) + c_{z'}(\frac{\partial w'}{\partial t}) + k_{z'} w' \quad (4)$$

The mechanical behavior of a postensioned cable hanging by the weightis nonlinear despite of linear elastic behavior of material due to lateral displacement. Any geometry of cable element, represent a different stiffness.

The general equilibrium at time *t* yields:

$$R^t - F^t = 0 \quad (5)$$

where R^t is external nodal forces at time *t* and F^t is nodal forces due to stresses. R^t may be composed of equivalent nodal forces due to body forces, surface tractions, and any concentrated forces as follows:

$$R^t = Rb^t + Rs + Rc^t \quad (6)$$

where *Rb* is equivalent nodal body forces, *Rs* is surface traction, and *Rc* is concentrated nodal forces.

The command "Member Cable" is assigned for cable analysis. The cable is modeled as a tensile member, taking into account the axial stresses of such a member and its sag. The cable (non-straight and pre-stressed) stiffness in a simple form can be determined via stiffness parameters as follows:

$$K_{cable} = \frac{EA}{L}[1 + \frac{\tilde{q}^2 L^2 EA\cos\theta}{12T^3}] \quad (7)$$

EA stands for axial member stiffness, *L* for member length, $\tilde{q}$ for member dead load intensity, *T* for internal force of pre-stressing, θ for angle between member axis and horizontal axis.

It is noted that such modeling practically excludes the lateral loading. The behavior of such a member corresponds to a certain stiffness spring behavior. It evaluates two effects, namely, elastic elongation and geometry (sag) change.

The guy is modeled by a cable with small primary sag, subjected by distributed or concentrated loads, acting in the plane of sag. Cable calculation evaluates primary mounting pre-stressing, support flexibility and temperature gradient. A geometrically non-linear behavior of guy is evaluated by employing the following expression for calculating the flexible space cable:

$$\Delta = L_1 - L_2 = \frac{Hl}{EA} - \frac{H_0 l}{EA} + \alpha \Delta t l + \delta - \int \frac{[Q_y(x)]^2 - [Q_z(x)]^2}{[H + N(x)]^2} dx + \int \frac{[Q_{y0}(x)]^2 - [Q_{z0}(x)]^2}{[H_0 + N_0(x)]^2} dx \quad (8)$$

Where, L_1 ,L_2 – cable span before and after deformation Δ – cable span change; H_0 , H – cable thrusting (tensile)force before and after deformation; l – cable primary length; Δt – temperature change; δ – cable primary extension/shortening; $Q(x)$ – shearing force function, analogous to a simple beam; $N(x)$ – axial force function.

In order to incorporate mechanical damping of cables into the formulation, and as reported by Kahla, 1994[13], fictitious linear viscous dampers are inserted in parallel with each cable as shown in Figure 1. These damping elements can efficiently reproduce the energy dissipation in the material and the friction due to inner-strand rubbing. The vector of fictitious damping force F_{Dcable} at the top end of the cable is computed as:

$$\left|F_{Dcable}\right|_j = c_c \frac{\Delta v . u_{ij}}{\left\|u_{ij}\right\|^2} u_{ij}\,,\ c_c = 2\zeta_c \sqrt{\frac{EA\tilde{q}}{g}}\,,\ \Delta v = v_j - v_i \quad (9)$$

c_c is the damping constant. ζ_c is a fraction of critical damping by the same stiffness and mass. u_{ij} is the cable chord vector and v_{ij} is the relative velocity of cable ends.

One must note that in the program the loading component acting along the line connecting supports, is evaluated when modeling the guy. This influences obtaining more exact mast analysis results.

While the analysis technique described herein applies to any structure including cable components, it may be suffice to look at the overall system from the point of view of the cables attachment points and assume that loads and stiffness of the structure or substructures have been condensed to the translational degrees of freedom of the attachment points of the cables. Therefore, the cable structure can be viewed as a collection of nodes interconnected by substructures and cables. The displacement force equation for a predefined equivalent element is written as follows:

$$F_e = K_e \delta_e \quad (10)$$

where F_e is equivalent nodal force, K_e is equivalent stiffness matrix and δ_e is displacements at selected degrees of freedom for equivalentelement. This equation must conform with the substructure displacement force equation participated in displacement force equation of the whole structure. The substructure displacement force equation is written as follows:

$$F_s = K_s \delta_s \tag{11}$$

F_s is the substructure nodal force, K_s is substructure stiffness matrix and δ_s is displacements of whole degrees of freedom for substructure elements.

To conform nine degrees of freedom belonging to three nodes at the corners of the triangle section where a substructure connected to the next, with six degrees of freedom belonging to an equivalent beam element ,three additional redundant elements are considered to joint mid-point of the above triangle to three corners. This is to facilitate to carry out the equivalent displacements and rotation at the mid-section of substructure junctions. Also, these deformations later must be compared well with the six deformation of equivalent element nodes .To avoid any unnecessary change in substructure stiffness the three added redundant element stiffness must be in such a way that the deformations in mid-point to be the same as what will exist at similar node in equivalent element. Figure 2 shows the substructure and forces at top mid-point that are compared with the equivalent element and similar forces at top node. To conform the stiffness components of equivalent element with the substructure, the section properties as section area against axial force, shear force, second moment of inertia, and twisting moment of inertia must be calculated separately.

In dynamic point of view, the mass matrix component presenting dynamic behavior of equivalent element, including their positions must be calculated as an element lumped mass matrix producing same frequency of modes. The stiffness components were affixed in static similarity, are employed to find equivalent mass in six position separately, to represent equivalent frequencies to the substructure components of frequency.

To ease the solution procedure, Rayleigh or proportional structural damping was assumed. The damping matrix of the element [c_c] can be expressed as:

$$[c_c] = \alpha[M] + \beta[K] \tag{12}$$

α and β are Rayleigh damping constants.

The comparison of equivalent model and the main structure deformation is shown in Figure 3. This result shows that the similarity is done well.

In finite element frame work, the variation of deformation and strain of element are functions of nodal displacements as follows:

$$\delta^e(x,y,z) = N^e(x,y,z)\ \delta^e \tag{13}$$

$$\varepsilon(x,y,z) = B^e(x,y,z)\ \hat{\delta}^e \tag{14}$$

where $\hat{\delta}^e$ is a nodal displacement, $N^e(x,y,z)$ is shape functions and $B^e(x,y,z)$ is strain displacement matrix for element *e*. Minimizing the potential energy variation at any increment, and using equation 5 and 6, the stiffness matrix of element and equivalent nodal forces are obtained as follows:

$$[K] = \int_v [B]^T [D][B]\ dv \tag{15}$$

$$R^t = \int_v [N]\ \{f_b\} dv + \int_A [N]\ \{f_s\} dA \tag{16}$$

$$F^t = \int_v [B]^T [\ o] dv \tag{17}$$

where f_b is body force, f_s is surface traction, σ is the existing stress in element.

Geometrical nonlinearity makes *B* matrix to be composed of two parts as follows:

$$B = B_\ell + B_n \tag{18}$$

$$B_n = \{ \frac{\partial \delta x}{\partial S}\frac{\partial N}{\partial S} + \frac{\partial \delta y}{\partial S}\frac{\partial N}{\partial S} + \frac{\partial \delta z}{\partial S}\frac{\partial N}{\partial S} \} \tag{19}$$

B is linear part, B_n is the part due to geometrical nonlinearity and *dS* is small length of element. To ease the solution, a set of equivalent nodal force as the effect of geometrical nonlinearity is to calculate at each increment to be applied at the next increment. This nodal force is as follows:

$$F_n^t = \int_v [B_n]^T [o] dv \tag{20}$$

These equations are solved in incremental scheme.

The strain and potential energy for an elastic cable element of undeformed length ℓe and mass per unit length ρ_e are considered. The position of any material point in the deformed configuration is writted as follows:

$$\delta_e = \sum_{i=1}^{n} N_i \hat{\delta}_i \tag{21}$$

The Lagrangian strain ε in the deformed axial direction is presented in terms of the deformed arc-length S and undeformed one as follows:

$$\varepsilon = \frac{1}{2}[(\frac{\partial s}{\partial S})^2 - 1] = \frac{1}{2}[(\frac{\partial \delta}{\partial S})^T (\frac{\partial \delta}{\partial S}) - 1] = \frac{1}{2}[(\hat{\delta})^T (\frac{\partial N}{\partial S})^T (\frac{\partial N}{\partial S})(\hat{\delta}) - 1] \quad (22)$$

S is curved axis of coordinate along element length s.

The elastic stress-strain relation leads to write the tensile force, f_t in the cable as follows:

$$\sigma = E\varepsilon \quad (23)$$

$$f_t = EA(\frac{\partial s}{\partial S})\varepsilon = \frac{1}{2}EA[(\hat{\delta})^T (\frac{\partial N}{\partial S})^T (\frac{\partial N}{\partial S})(\hat{\delta}) - 1][(\hat{\delta})^T (\frac{\partial N}{\partial S})^T (\frac{\partial N}{\partial S})(\hat{\delta})]^{\frac{1}{2}} \quad (24)$$

A and E are undeformed cross section area of the element and the elastic modulus respectively.

The potential energy for the cable element can be expressed upon dynamic condition as follows:

$$\pi = \int_0^{\ell_e} \frac{1}{2} A\sigma\varepsilon dS - \int_0^{\ell_e} \delta^{eT} f dS - \int_0^{\ell_e} \frac{1}{2} m^e (\overset{\circ}{\delta})^T (\overset{\circ}{\delta}) dS \quad (25)$$

$$\pi = \int_0^{\ell_e} \frac{1}{8} DA[(\hat{\delta})^T (\frac{\partial N}{\partial S})^T (\frac{\partial N}{\partial S})(\hat{\delta}) - 1]^2 dS - \int_0^{\ell_e} \hat{\delta}^T N^T f^e dS - \quad (26)$$

$$\int_0^{\ell_e} \frac{1}{2} m^e (\overset{\circ}{\hat{\delta}})^T N^T N(\overset{\circ}{\hat{\delta}}) dS$$

f^e is static load vector and (^{o}) means d/dt.

The deformed configuration of cable element can be the summation of static $\hat{\delta}_s$ and dynamic deformation $\hat{\delta}_d$ as follows:

$$\hat{\delta} = \hat{\delta}_s + \hat{\delta}_d \quad (27)$$

Therefore, the potenial energy for element e is find as follows:

$$\pi^e = \pi_0^e + \frac{1}{2}(\overset{\circ}{\hat{\delta}}_d)^T \hat{m}^e (\overset{\circ}{\hat{\delta}}) - \hat{\delta}_d^T (\hat{f}^e - D\hat{\delta}_s) - \frac{1}{2}\hat{\delta}_d^T (D + G^e(\hat{\delta}_s, \hat{\delta}_s)]\hat{\delta}_d^s -$$

$$-\frac{1}{2}\hat{\delta}_d^T \hat{G}^e(\hat{\delta}_s, \hat{\delta}_d)]\hat{\delta}_d^s - \frac{1}{2}\hat{\delta}_d^T \hat{G}^e(\hat{\delta}_d, \hat{\delta}_d)]\hat{\delta}_d^s \quad (28)$$

$$\pi_0^e = \hat{\delta}_s^T \hat{f}^e - \frac{1}{8}\int_0^{\ell_e} DA[(\hat{\delta}_s)^T (\frac{\partial N}{\partial S})^T (\frac{\partial N}{\partial S})(\hat{\delta}_s) - 1]^2 dS \quad (29)$$

where, $\hat{f}^e = \int_0^{\ell_e} N^T f^e dS \quad (30)$

$$D = \frac{1}{2}\int_0^{\ell_e} EA[(\hat{\delta}_s)^T (\frac{\partial N}{\partial S})^T (\frac{\partial N}{\partial S})(\hat{\delta}_s) - 1](\frac{\partial N}{\partial S})^T N dS \quad (31)$$

$$G^e(\hat{\delta}_s, \hat{\delta}_s) = \int_0^{\ell_e} EA(\frac{\partial N}{\partial S})^T (\frac{\partial N}{\partial S})(\hat{\delta}_s)^T (\frac{\partial N}{\partial S})^T (\frac{\partial N}{\partial S}) dS \quad (32)$$

According to minimum energy level, the variation of equaion 28 yields:

$$K\hat{\delta} = F \quad (33)$$

K is equevalent matrix of material behaviour that also includes dynamic and geometrical non-linearityeffects. F is equevalent nodal forces affected by different loads.

Applying Hamilton's principle to Lagrangial equation 28, leads to the equation of motion as follows:

$$m^e(\overset{\circ\circ}{\hat{\delta}}_d) + \{D + G^e(\hat{\delta}_s, \hat{\delta}_s) + \frac{1}{2}[G^e(\hat{\delta}_s, \hat{\delta}_s) + 2G^e(\hat{\delta}_d, \hat{\delta}_s)] \quad (34)$$

$$+\frac{1}{2}G^e(\hat{\delta}_d, \hat{\delta}_d)]\hat{\delta}_d = 0$$

This equation can be numerically solved in finite element framework with the consideration of non-linear cable effects on the satisfaction of equilibrium, compatibility, and any proposed mechanical behgaviour of material.

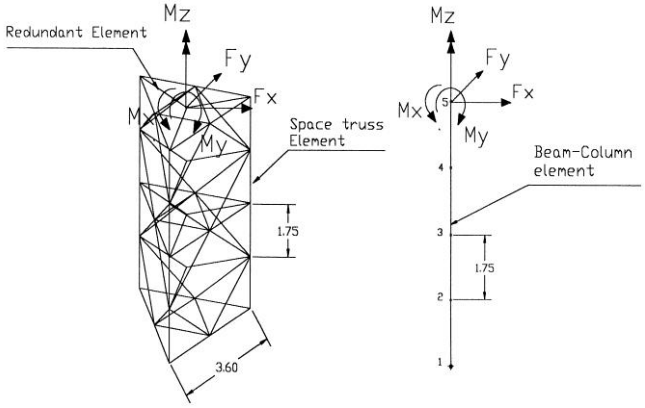

Fig.2: *Substructure and equevalent element*

In the dynamic analysis of cable systems, super position of loads and displacements is not strictly valid due to the effects of loading nonlinearities and large amplitude vibrations. There are three sources of nonlinearity in cable behavior:

• Nonlinearities associated with the deformation of spirally wound strands forming the cable cross section are of important as they affect the calculation of the instantaneous stiffness of the system. In addition, hysteretic damping can be involved with large amplitudes.

• Nonlinearities associated with large deformations cause changes in the cable stiffness and, hence, affect the natural frequencies of the entire system.

• Nonlinearities associated with aerodynamic forces that depend upon the continuously varying response and hence

geometry of the cables. The dynamic pressure changes direction according to the moving and rotating cable axis.

In general, the governing equations for pre-tensioned cable dynamics are coupled and highly nonlinear. General analytical solutions for these systems are not available, and therefore, numerical techniques have to be used. The finite element method, FEM, is the most well-known of all numerical techniques. It is capable of modeling such a three dimensional geometry of such complex structure, loading and both geometric and material nonlinearities with different prescribed boundary conditions.

3. The Cable Antenna Specifications

The selected antenna is 325m high that is composed of main truss tower and cables constraining the tower at five levels. The main tower is pyramid up to level 7 m and jointed on base. A triangle section (3.6 m members) space truss is between 7 m up to 245 m elevation. A transition space truss changes the cross section diameter from 3.6 m at elevation 245 to 3 m at elevation 248.5 m. This section continues up to elevation 294 m. At this elevation the cross section of space truss is square with 1.45 m edges and continues up to 325 m elevation.

Figure 3-a show the tower and truss cross sections, respectively. The cables constrained the tower at elevations 59.5, 122.5, 185.5, 248.5, and 294 m. The total weight of tower is approximately 186.84 ton. Figure 3-b shows the number of cable elements with beam-column equivalent elements for main tower.

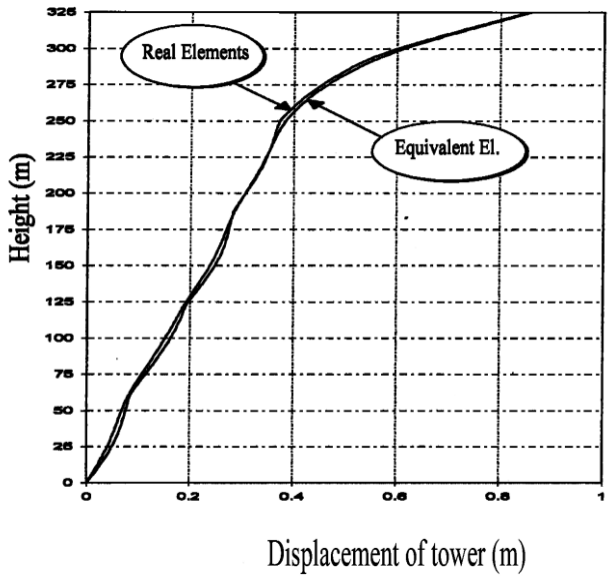

Fig.3: *Typical comparison of model and structure deformation*

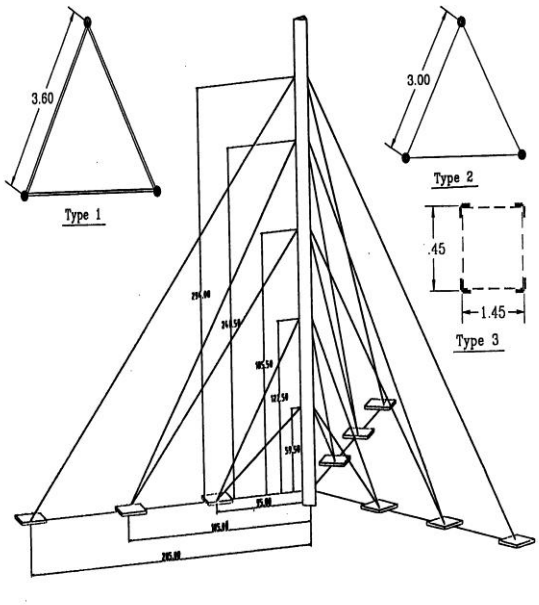

Fig. 4-a: *Tower and space truss cross sections*

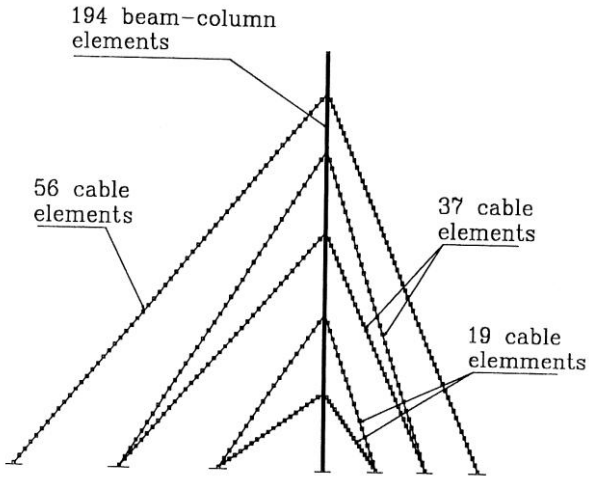

Fig. 4-b: *Cables & beam-column Equevalent elements*

4. Results

Figure 4-a shows the effect of postensioning on tower vertical deformation. The change of bending moments along tower in equivalent beam-column elements due to postensioning forces is shown in Figure 4-b. These effects on axial and shear forces are shown in Figures 4-c and 4-d respectively.

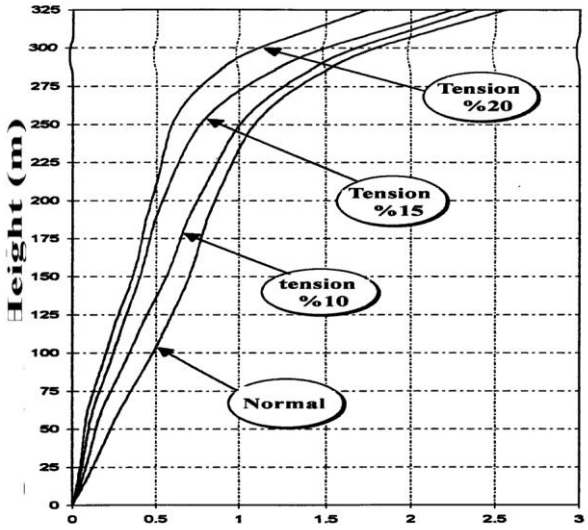

Fig.5-a: *Max. displacement along tower(m)*

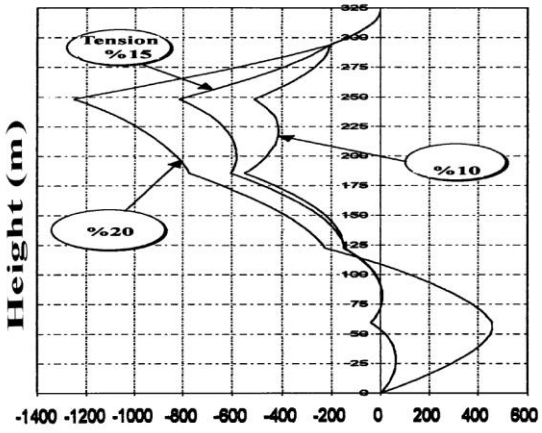

Fig.5-b: *Moment along tower(ton-m)*

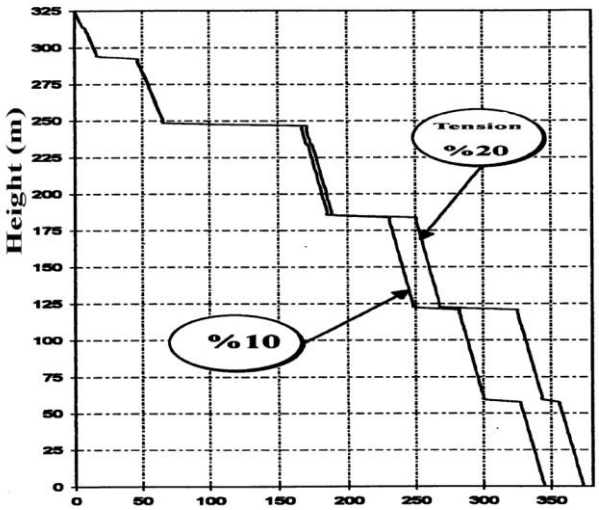

Fig.5-c: *Axial force due to postensioning(ton)*

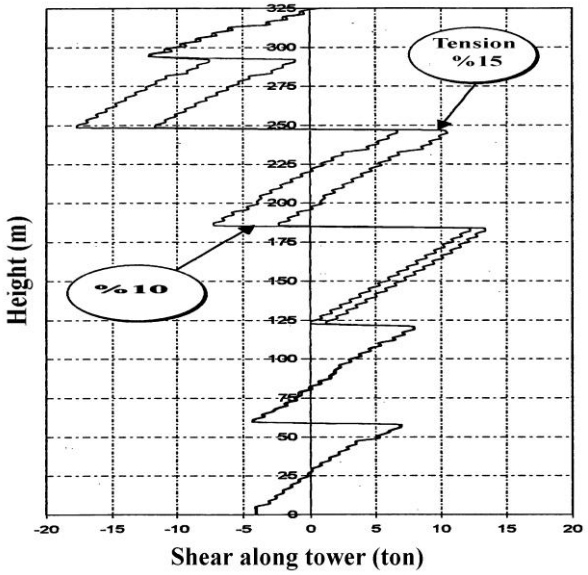

Fig.5-d: *Shear force along tower(ton)*

In dynamic analysis of antenna upon the proposed equevalent beam-column and cable elements, the number of elements is reduced for more speed in multi-member complex dynamic analysis. Different acceleration time histories were applied on the introduced antenna and the maximum base shear force in each earthquake carried out and compared. Figure 5-a shows these comparison. The comparison of antenna deformations upon different earthquake effects are shown in Figure 5-b. The effects of acceleration time history amplitude on antenna deformation is presented in Figure 5-c for Tabas earthquake.

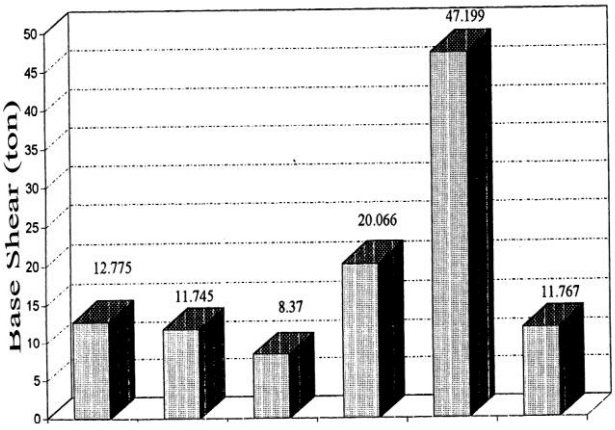

Fig.6-a: *Tabas, Noghan, Manjil, Park-field, Morgani-hill*

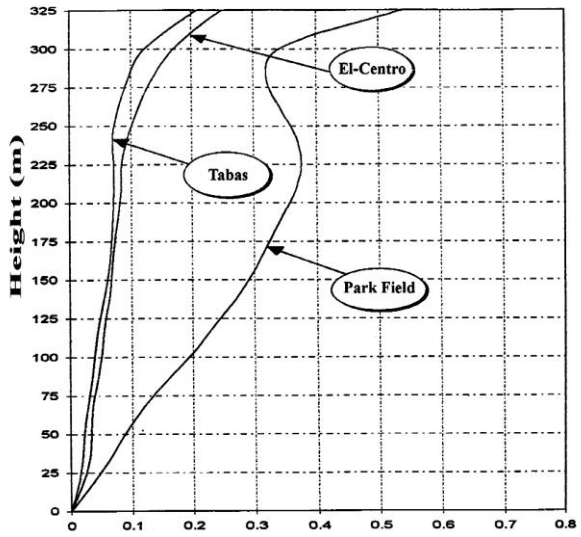

Fig 6-b: *Max. displacement along tower (m), (0.25g)*

Table 1 presents the angular and transversal frequencies, and period of structure for different modes.

5. Conclusion

A tall postensioned cable tower is analyzed upon simulated equivalent simple elements under static and dynamic loading. This method of introducing equivalent elements, not only increase the speed of computing, however, make possibility to analyze complex structures. The effects of increase in postensioning of the cables on the behavior of such a tall structure as moment, axial and shear forces are presented. The presented results are in general agreement with what is expected in behavior of this kind of structure.

The methodology adopted to model the nonlinear dynamic interaction between the guy cables and the mast of a guyed tower can be extended to model the interaction between stay cables and decks and towers of a cable-stayed bridges.

The extension to bridge applications will require consideration of support flexibilities and movements, in addition to consideration of different loading combinations (dead load, traffic load, wind load, seismic load, rain/ice, etc.). Other important issues in modeling stay cables, both flexural and torsion stiffness matrices will have to be included in the cable analysis in addition to axial stiffness.

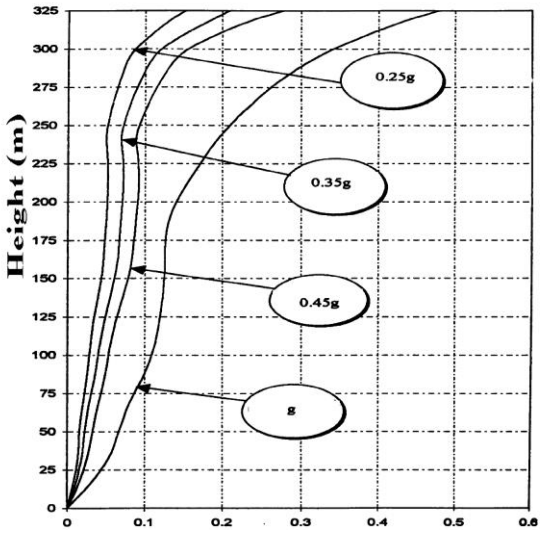

Fig.6-c: *Maximum displacement along tower (m) (Tabas)*

Table.1: *Angular and transversal frequencies, and period of structure*

Mode No.	1	2	3	4	5	6	7	8	9	10
Frequency(R./S.)	1.19024	1.19244	1.19258	1.19264	1.1931	1.19018	1.35112	1.35242	1.35352	1.35404
Frequency(Hz.)	0.18943	0.18978	0.18980	0.18981	0.1899	0.18942	0.21504	0.21524	0.21542	0.21550
Period (S.)	5.27891	5.26919	5.26857	5.26832	5.2663	5.27917	4.65036	4.64590	4.64209	4.64033

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