

Numerical Investigation of Hydraulic Characteristics Effective on Vertical Drop

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ARTICLE INFO

RESEARCH PAPER

Article history:

Received:

August 2021

Revised:

January 2022.

Accepted:

January 2022.

Keywords:

Vertical drop,

Hydraulic parameters,

Relative length of the

drop,

Turbulence model,

VOF

Abstract:

The drops are used to control the descents, stabilize the bed level, and control the upstream water level in sloping channels with less slope than the ground slope. The current study presents a numerical analysis of hydraulic characteristics in the vertical drop using computational fluid dynamics. At first, the laboratory models were used for verification and choosing the best model of turbulence; three types of turbulence models, $k-\epsilon$, $k-\epsilon$ RNG, and $k-\omega$ were used. The results revealed that the RNG $k-\epsilon$ turbulence model has less RE% and RMSE than other models and more efficiency in simulating hydraulic characteristics on drops. Also, it was observed that the highest rate of RE% and RMSE for this turbulence model was 6.18 and 0.109 for the relative length of the drop, whereas the lowest relative downstream depth was 5.27 and 0.003, respectively. Furthermore, by increasing the relative critical depth, the characteristics of relative downstream depth, the relative depth of the pool, and the relative length of the drop increased, whereas the relative energy dissipation decreased. For the parameter of the relative length of the drop in the range of 0.08 to 0.5, this increase was obtained to be 2.6 times. In addition, using numerical data, a series of equations have been presented to predict hydraulic parameters of the vertical drop with a high correlation coefficient.

1. Introduction

Drops are used in irrigation, drainage, and surface water collection channels. These structures are usually low-altitude and suppressed. The most important tasks of these structures are energy dissipation, prevention of bed scour due to the high slope of the ground, transferring water from the higher to the lower level, reducing the slope to optimal slope design, and creating optimal speed at the downstream of the channel. Previous studies usually focused on the hydraulic flow upstream and downstream of these structures. Providing methods to increase energy dissipation

downstream has led to presenting empirical and semi-empirical relations.

Many researchers, including Daneshfaraz et al. [1-2] and Nayebzadeh et al. [3], have conducted extensive experimental and numerical studies to estimate the hydraulic parameters of vertical drops.

The first studies on the upstream subcritical flow crossing vertical drops were carried out in 1932 by Bakhmeteff [4]. White [5] was the first researcher to present an analytical method based on the energy equation in drop structures, which Blaisdell [6] later modified. Hong et al. [7] investigated the effect of downstream slope on hydraulic performance. The results revealed that increasing the slope downstream of a drop increases the drop length and collision forces.

Daneshfaraz et al. [8] evaluated the hydraulic parameters of vertical drops equipped with horizontal screens with a supercritical upstream flow. The results revealed that the horizontal screens on the brinks of the vertical drops increased energy dissipation. Norouzi et al. [9] investigated the performance of inclined drops equipped with vertical

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screens by an adaptive neuro-fuzzy inference system. The results revealed that the use of screens caused a 407% to 903% increase in total relative energy dissipation efficiency compared to the plain inclined drop. Sadeghfam et al. [10] investigated the scour of supercritical flow jets upstream of screens and modeled scouring dimensions using artificial intelligence to combine multiple models (AIMM). The results showed that the Level 2 model improves model performances compared with the single models in terms of R^2 , RMSE, Nash–Sutcliffe coefficient (NSC), and residual errors. While Level 1 models remain fit-for-purpose, the comparative improvement from Level 1 to Level 2 can be as high as 58% in terms of NSC for the testing phase. The Level 1 model uses the experimental data and tests the models of Sugeno Fuzzy Logic (SFL) and Neuro-fuzzy (NF), and the Level 2 model uses outputs of the Level 1 model as inputs to support vector machine (SVM). Norouzi et al. [11] predicted relative energy dissipation for vertical drops equipped with a horizontal screen using soft computing techniques. The findings show that the efficiency of soft computing techniques in predicting relative energy dissipation is acceptable. Daneshfaraz et al. [12] evaluated the performance of a drop with a horizontal screen. The results revealed that using horizontal screens in vertical drops can increase the relative energy dissipation compared with a plain vertical drop.

Hydraulic parameters on the drops are challenging to investigate due to the complexity of turbulent flow in the pool and the diffusion process between the flow jets with walls, bed, and the water rolling surface [13]. Also, considering that after jet collision with the pool, two-phase turbulent flow is established, simulating this two-phase phenomenon using turbulence-like $k-\varepsilon$, RNG $k-\varepsilon$, $k-\omega$ models, and with regard to fluid volume (VOF) can lead to more accurate results. Kabiri-Samani et al. [14] evaluated the rate of improvement of energy dissipation efficiency in vertical corrugation drops using Flow-3D software. The results revealed that the use of corrugation dissipation in vertical drops increases the energy dissipation compared to plain vertical drops.

Many researchers have conducted extensive studies on computational fluid dynamics (CFD) problems, including Ferrari [15], Mahdavi and Shahkarami [16], and AbdullahMuhsin and Mohammad AliNoori [17].

Most research in this field has been done experimentally. However, in most studies, all hydraulic parameters (initial jump depth, falling sub-jet pool depth, the horizontal distance of the jet from the drop brink, and relative energy dissipation) have not been evaluated simultaneously. Therefore, in the current study, a numerical investigation of these parameters using the Flow-3D software is performed using computational fluid dynamics and the prepared equations. The numerical data will also be compared with

the experimental data and the experimental relations of previous researchers.

2. Materials and Methods

2.1 Basics of flow in vertical drops

The upstream flow of the drops in a steady-state condition enters a sub-critical state while approaching the drop brink and then becomes supercritical near the drop brink. The flow falls after passing the drop brink because of the gravity and collides with the pool bed. As the jet falls into the pool and after colliding with it, the stream is divided into two parts; one flows down to the pool and the other downstream [18]. The backflow and vortex flow make the depth of the pool greater than the downstream depth. In addition, fluctuations, weather interference, and the presence of backflow inside the pool cause the pressure distribution not to be hydrostatic, which leads to losses in the flow energy [19]. While the downstream flow is supercritical, most research has been on vertical drops with the subcritical flow.

The most important parameters studied in this study are critical depth (y_c), drop brink depth (y_b), falling sub-jet pool depth (y_p), downstream depth before the hydraulic jump (y_1), drop length (L_d), and energy dissipation (ΔE). Figure 1 shows the schematic of the flow in different parts of the vertical drop.

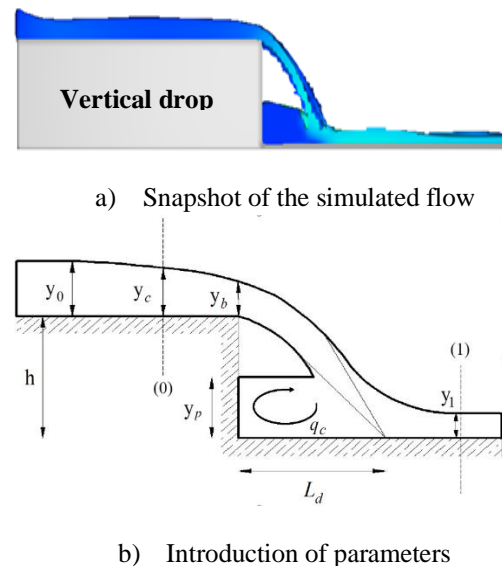


Fig. 1: Schematic view of the flow in a vertical drop

2.2 Numerical simulation

The Flow-3D software is a powerful and highly accurate tool for solving complex CFD problems and can model an extensive range of fluid flows [20]. This software uses the finite volume method to solve the governing equations with regular meshing and the volume of fluid (VOF) method to calculate the free flow in open channels. The Flow-3D

software can simulate the turbulent flows in different methods. In the current study, three types of models, k-ε, RNGk-ε, and k-ω were used to model turbulence. Experimental data of Rajaratnam and Chamani [18] were used for validation of the model. Experiments were conducted in a flume 6.55 m long, 0.46 m wide, and 0.91 m high with Plexiglas walls. The drop height used was 0.25, and the range of experiments for the relative critical depth was between $0.06 < y_c/h < 0.35$. The channel slopes upstream and downstream are assumed to be zero.

Flow intensity boundary conditions at the inlet and outlet sections were selected to avoid the impact of the outlet boundary condition flow. The symmetry boundary conditions of the free surface of the fluid and the walls due to proximity to the Plexiglas were used as the boundary condition of the wall, which acts as a frictionless wall. A 1-cm thin layer of channel bed was selected and used for better simulation. Figure 2 shows the schematic view of boundary conditions and meshing.

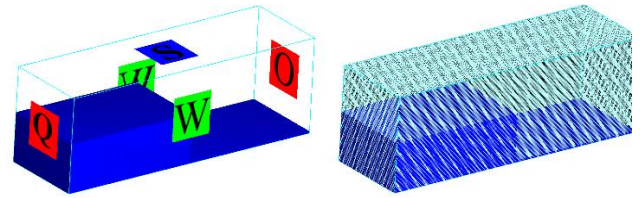


Fig. 2: Schematic view of boundary conditions and meshing

3. Results and Discussions

To select the best model, three models, namely, k-ε, RNGk-ε, k-ω were generated. A comparison of numerical results with experimental results by Rajaratnam and Chamani [18] for downstream relative depth is shown in Table 1 and Figure 3.

The amount of error between the results of the various turbulence models and the laboratory results is also given in Table 2.

Table 1: Comparison of numerical results with experimental results by Rajaratnam and Chamani [18] for downstream relative depth

$\frac{y_c}{h}$	0.06	0.08	0.1	0.12	0.14	0.155	0.165	0.2	0.25	0.3	0.35
Exp	0.017	0.023	0.026	0.038	0.047	0.053	0.057	0.078	0.094	0.116	0.14
RNG	0.0149	0.0215	0.0286	0.036	0.0438	0.05	0.054	0.069	0.091	0.116	0.14
k-ε	0.016	0.0206	0.298	0.0383	0.0449	0.0495	0.049	0.0739	0.0939	0.121	0.139
k-ω	0.0155	0.0199	0.0332	0.0432	0.0467	0.047	0.063	0.0746	0.0954	0.119	0.14

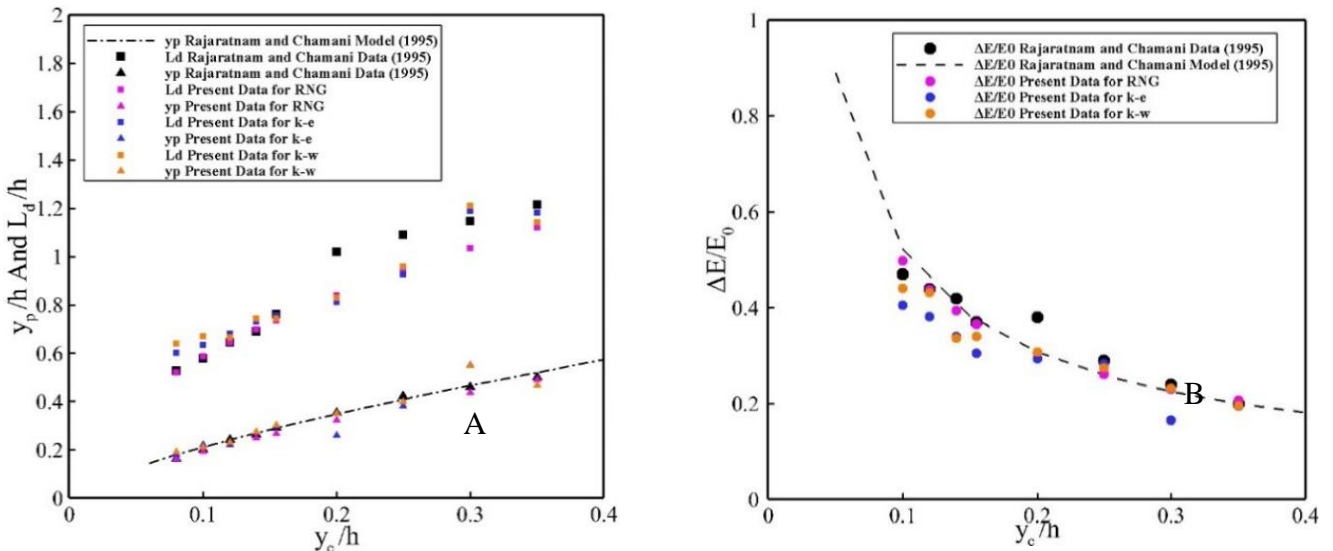


Fig. 3: A. Comparison of the results of various turbulence models with experimental results for the parameters of the relative energy loss) B. for the parameters of the relative pool depth and the relative length of the drop

Table 2: Comparison of the results of turbulence models and experimental results using relative error (RE%) and RMSE

Turbulence models	RE%			
	y_1	y_p	L_d	ΔE
$k - \varepsilon$	6.11	10.78	8.63	12.61
$RNG k - \varepsilon$	5.27	5.69	6.18	6.09
$k - \omega$	8.577	7.41	10.27	10.09

Turbulence models	RMSE			
	y_1	y_p	L_d	ΔE
$k - \varepsilon$	0.0035	0.056	0.094	0.041
$RNG k - \varepsilon$	0.003	0.023	0.109	0.025
$k - \omega$	0.003	0.044	0.096	0.032

A comparison of the results of the turbulence models with the laboratory data showed that Model RNGk-ε is better than Model k-ω and Model k-ε. Due to the good agreement of the numerical model results with the experimental results for all parameters in the current study, it is tried to increase the range and number of relative critical depths to 0.5 for predicting the parameters involved in the drops. Also, all the relations presented with the experimental results are compared with those presented by previous researchers.

3.1 Downstream depth (y_1) and pool depth (y_p)

Figure 4 illustrates the relative downstream depth changes versus the relative critical depth. y_1/h is decreased by increasing the y_c/h . As can be seen in relation $q = \sqrt{gy_c^3}$, discharge is a function of the critical depth. Therefore, with increasing critical depth, discharge increases. Also, assuming that the slope is constant and that the velocity is approximately constant in section (1) and since the flow through this section is the same increased flow, according to the relation $q = v_1 y_1$, by increasing the critical depth, y_1 also increases.

In plain vertical drops, the most important parameter affecting the flow is the drop brink height of the pool bed. As the drop height increases, the potential energy of the upstream water increases [21]. As the stream flows through the drop brink and falls to the pool, no energy dissipation occurs along this path. Therefore, according to the conservation law, the upstream potential energy is converted to kinetic energy, and water velocity rises, whereas the downstream depth (y_1) decreases.

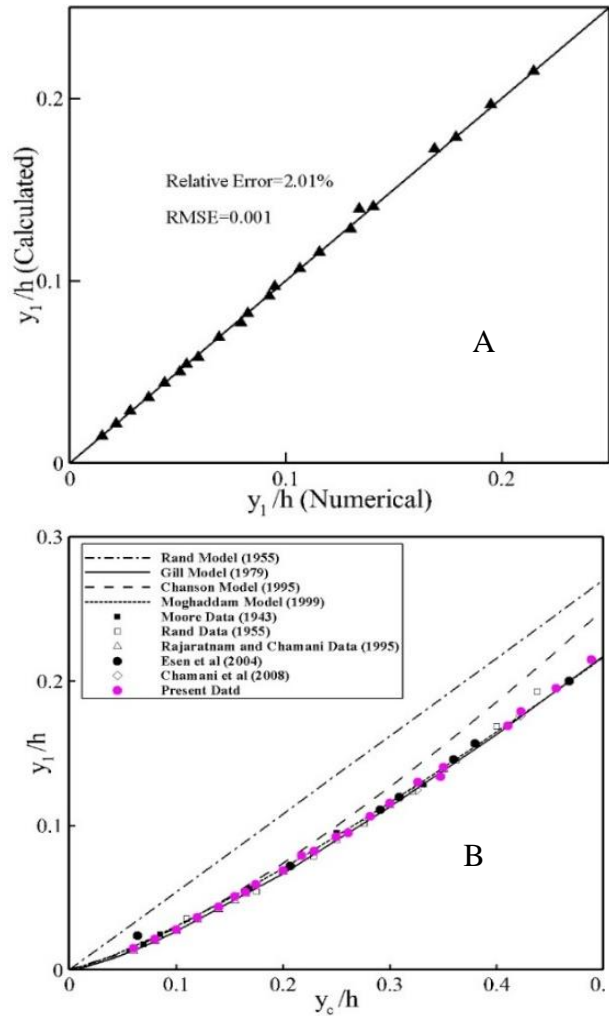


Fig. 4: A. Changes of relative downstream depth versus y_c/h
 B. Scatter plots calculation and numerical values of y_1/h

The following relation to the relative downstream depth is presented using numerical data:

$$\frac{y_1}{h} = 0.533 \left(\frac{y_c}{h}\right)^{1.27}, \quad R^2 = 0.9984 \quad (1)$$

A comparison of the numerical data with the laboratory data of other researchers revealed that the results of the present study are most correlated with the laboratory results of Esen et al. Thus, RE% and RMSE are 0.4 and 0.002, respectively. A comparison of the results of the present study with previous research is shown in Figure 5 as changes of y_p/h versus y_c/h . The pool depth depends on the backflow (q_c). Thus, by increasing y_c , the amount of backflow and turbulence in the pool increases. As a result, it increases the depth of the pool.

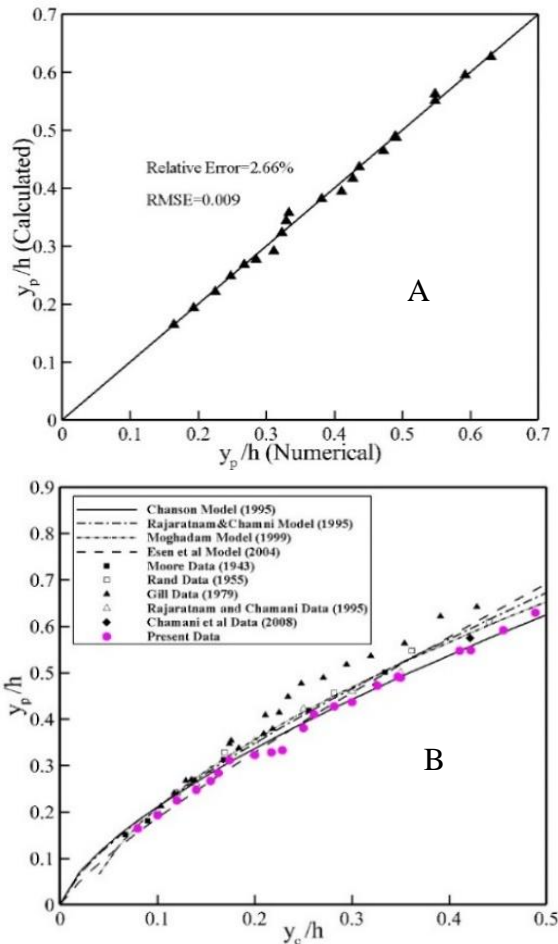


Fig. 5: A. Changes in relative pool depth with y_c/h B. Scatter plots calculation and numerical values of the parameter y_p/h

The following relation was proposed using the numerical calculations of the present study for the relative depth of the pool:

$$\frac{y_p}{h} = 1.065 \left(\frac{y_c}{h} \right)^{0.741}, \quad R^2 = 0.9928 \quad (2)$$

A comparison of the numerical data with the laboratory data of the researchers revealed that there is a good agreement between the numerical results of this research and the laboratory results of the researchers. The lowest RE% and RMSE are 2.615 and 0.0182 for the work by Chanson [22], respectively, and 2.7288 and 0.0198 for the work by Chamani et al. [19], respectively.

3.2 Falling jet collision position (L_d)

Figure 6 illustrates the changes in the relative drop length to the relative critical depth. Due to the projectile range relation $L_d = V \sqrt{\frac{2h_t}{g}}$, it is observed that the horizontal distance to the impact site depends on the two factors of velocity and total height ($h_t = h + y_b$). By increasing y_c , the path discharge from the drop brink increases. Also, considering studies of the drop brink depth by Rouse [23], Dey [24], Nabavi et al. [25],

and many other studies have shown that y_o/y_b is a constant value. Due to relation $q = V_b y_b$, the increase in discharge will have two states: A) depth at the drop brink remains constant, whereas velocity at the drop brink increases. B) Velocity at the drop brink remains constant, whereas depth at the drop brink increases.

As a result, the total height of the water level on the drop brink increases. Therefore, it can be concluded that the drop length also increases by increasing critical depth.

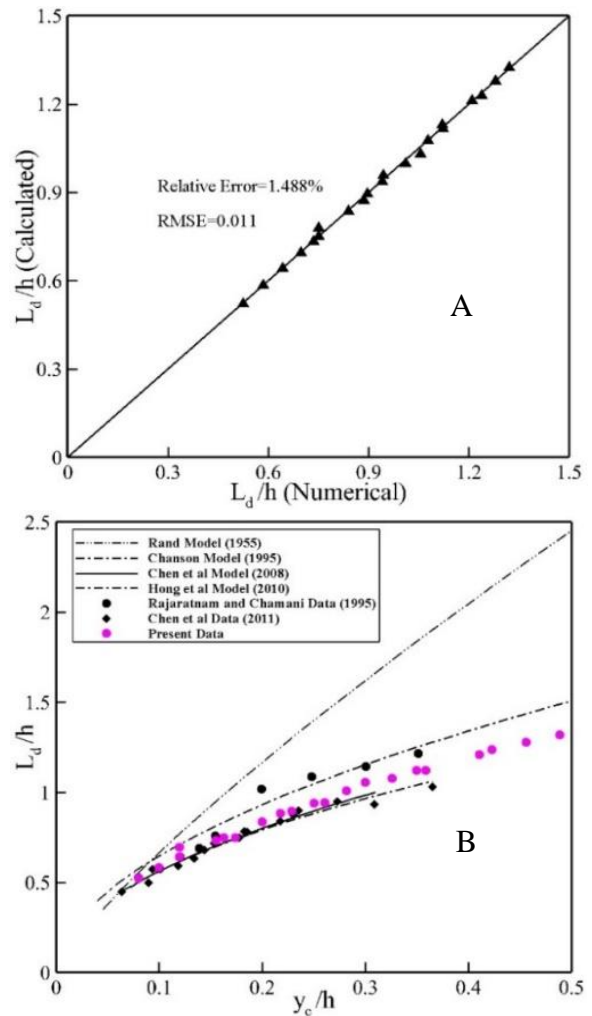


Fig. 6: A. Changes in the relative length of the drop versus y_c/h B. Scatter plots calculation and numerical values of the parameter L_d/h

Numerical results indicated that with increasing y_c/h of 0.08 to 0.5, the relative drop length increased by 2.6 times. Also, according to the numerical data, the drop length relation is presented as follows:

$$\frac{L_d}{h} = 1.9156 \left(\frac{y_c}{h} \right)^{0.516}, \quad R^2 = 0.994 \quad (3)$$

Equation (2) has the lowest RE% and RMSE of 4.697 and 0.045 for Chen et al.'s [26] experiments, respectively.

According to Figure 6, Gill's [27] model obtained the drop length value more than all the researchers.

3.3 Energy dissipation in drop

Eq. (4) shows the relative energy dissipation, where E_0 is the upstream total head, and E_1 is the downstream head after a jet collision to bed.

$$\frac{\Delta E}{E_0} = \frac{E_0 - E_1}{E_0} = \frac{(h + 1.5 * y_c) - (y_1 + \frac{V_1^2}{2g})}{(h + 1.5 * y_c)} \quad (4)$$

A comparison of the numerical results of the present study with those of other researchers for the relative energy damping dissipation to y_c/h is shown in Figure 7. For relative critical depths of less than 0.3, the Rand's [26] laboratory data and the calculated values of Gill [25] reveal less relative energy dissipation. Nonetheless, White's [5] model has consistently and significantly outperformed relative energy dissipation more than any other research and the present study. Also, when y_c is increased, energy dissipation decreases, but when h is increased, energy dissipation increases.

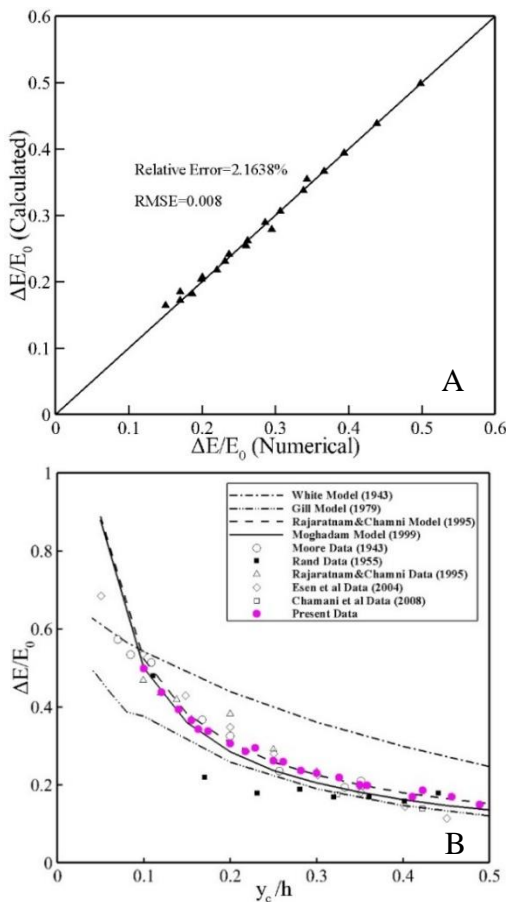


Fig. 7: Changes of relative energy dissipation versus y_c/h . B. Scatter plots calculation and numerical values of the parameter

The relation of the relative energy dissipation was obtained as follows:

$$\frac{\Delta E}{E_0} = 0.0993 \left(\frac{y_c}{h} \right)^{-0.7}, \quad R^2 = 0.994 \quad (5)$$

A comparison of the results of the present study with previous studies indicated that the results of the numerical model have the lowest RE% and RMSE with respect to the experimental equation presented by Rajaratnam and Chamani [18], with values of 3.228 and 0.0078, respectively. Also, for the relative critical depth in the range of 0.1-3.0, relative energy dissipation decreased by 0.27, and in the range 0.5-0.3, this parameter decreased by 0.07. This shows that the rate of energy dissipation changes in low discharges is greater than in higher discharges for a constant drop height. Table 3 shows the RE% and RMSE effective parameters on the vertical drop to compare the numerical results with those of other researchers.

Table 3: Comparison of numerical results with experimental results of previous researchers

Researchers	RE %			
	$\frac{y_1}{h}$	$\frac{y_p}{h}$	$\frac{L_d}{h}$	$\frac{\Delta E}{E_0}$
Moore [21]	2.26	5.47	-	6.9
Rand [28]	3.797	8.9	-	26.6 9
Gill [27]	3.163	-	-	26.8 1
Chanson [22]	8.146	2.72 7	10.21 1	-
Rajaratnam and Chamani [18]	5.27	5.69	6.18	6.09
Moghaddam [29]	1.85	6.53 1	-	11.1 2
Esen et al. [13]	0.473 9	4.32 9	-	-
Chamani, Rajaratnam and Beirami [19]	1.537	20.7 9	-	19.9 7
Chen et al. [30]	-	-	4.697	-
Chen et al. [26]	-	-	5.221	-
Hong et al. [7]	-	-	6.188	-

According to Table 3, it is observed that except for Rand's [28] proposed equation for y_1/h and the experimental Gill [27] data for parameter y_p/h , the numerical results are in good agreement with the experimental results for parameters y_1/h and y_p/h . Also, based on the studies on the relative drop length and observing Figure 6, it has been shown that Rand's [28] proposed equation shows the values of this parameter more than the results of other researchers.

4. Conclusions

Drops are among the most common structures for controlling, directing the flow, stabilizing water levels, and reducing the kinetic energy downstream of open channels. The current study presented the numerical simulation of hydraulic parameters on vertical drops. The vertical drop was simulated in the Flow-3D software with a height of 0.25 m for a range of $0.05 < y_c/h < 0.5$ with a grid of 6 mm. Investigations of turbulence models showed that the RNGk- ϵ turbulence model is a better model for simulating hydraulic parameters on drops. Also, the results of numerical models and their comparison with laboratory data, in general, showed that by increasing relative critical depth, the values of relative downstream depth parameters, relative pool depth, and relative drop length increase, while the relative energy dissipation parameter value decreases. The results also revealed that by increasing the relative critical depth from 0.08 to 0.5, the relative drop length increased by 2.6 times. Finally, using the numerical data, a series of equations for hydraulic parameters in drops with a high correlation coefficient was presented.

Acknowledgments

The authors of this article would like to thank all those who accompanied us in writing this article.

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