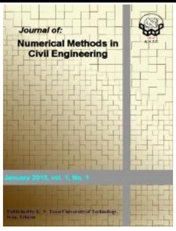

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Relevance vector machine and multivariate adaptive regression spline for modelling ultimate capacity of pile foundation

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Abstract:

This study examines the capability of the Relevance Vector Machine (RVM) and Multivariate Adaptive Regression Spline (MARS) for prediction of ultimate capacity of driven piles and drilled shafts. RVM is a sparse method for training generalized linear models, while MARS technique is basically an adaptive piece-wise regression approach. In this paper, pile capacity prediction models are developed based on data obtained from the literature and comprise in-situ pile loading tests and Cone Penetration Test (CPT) results. Equations are derived from the developed RVM and MARS models, and the prediction results are compared with those obtained from available CPT-based methods. Sensitivity has been carried out to determine the effect of each input parameter. This study confirms that the developed RVM and MARS models predict ultimate capacity of driven piles and drilled shafts reasonably well, and outperform the available methods.

1. Introduction

Piles have been used for decades as one of the most important types of structural foundations. Consequently, the determination of an accurate axial capacity of pile foundations is an important task in geotechnical engineering. Numerous traditional methods have been proposed in the geotechnical literature to predict the axial capacity of driven piles and drilled shafts; however, most available methods have failed to achieve consistent success. Among the available methods, the cone penetration test (CPT) based models have been shown to give better predictions in many situations. This can be attributed to the fact that CPT-based methods have been developed in accordance with the CPT results, which have been found to yield more reliable soil properties; hence, more accurate axial pile capacity predictions. In recent years, artificial neural networks (ANNs), which is one of the most emerging soft computing techniques, have been used with some degree of success for axial capacity prediction of pile .This study employs the Relevance Vector Machine(RVM) and Multivariate Adaptive Regression Spline (MARS) for prediction of axial capacity of driven piles and drilled shafts using data of in-situ pile load tests and Cone Penetration Test(CPT) results. RVM is a statistical learning method proposed by Tipping (2000)[5] and represents a new approach to regression that has recently attracted a great deal of interest in the machine learning community as it is highly insensitive to the curse of dimensionality. MARS, on the other hand, is a non-parametric method developed by Friedman (1991)[6] that can handle high-dimensional problem efficiently and is considered to be a modern methodology from statistical learning and performs well for both classification and regression. The advantage of RVM and MARS over ANNs is their ability to provide well-structured mathematical expressions between the model inputs and the corresponding outputs. The current study has the following aims:

1. To examine the feasibility of using the RVM and MARS for predicting axial capacity of driven piles and drilled shafts;

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2. To derive mathematical equations for determination of axial capacity of driven piles and drilled shafts that facilitate the use of the RVM and MARS models; and

3. To carry out a comparative study between the developed RVM and MARS models with other available CPT-based methods of axial capacity predictions.

2. Details of RVM Models for Pile Capacity Prediction

In RVM, the prediction $y(x, w)$ is expressed as a linear combination of basis functions $\phi(x)$ and is written as follows:

$$A = y(x,w) = \sum_{m=0}^{N} w_m \phi_m(x) = w^T \phi \tag{1}$$

where: A is the actual outputs, x is the inputs, w is the weights and N is the number of basis functions. For driven piles, 79 load tests that have been collected from the literature by Shahin (2010)[2] are used for the RVM model calibration and verification. The load tests include compression and tension loading conducted on steel and concrete piles driven statically (jacked-in) into the ground. The driven piles used had different shapes (i.e. circular, square and hexagonal) and ranged in diameter between 250 and 900 mm, with embedment lengths between 5.5 and 41.8 m. The RVM model input variables used include the pile equivalent diameter (D_{eq} = pile perimeter/π), embedment length (L), weighted average cone point resistance over pile tip failure zone ($\bar{q}_{c-tip}$), weighted average cone point resistance over pile length ($\bar{q}_{c-shaft}$), weighted average cone sleeve friction over pile length ($\bar{f}_s$) and pile material which is represented by the numeric values "1" for steel piles and "2" for concrete piles. The single output of the RVM model is the ultimate pile capacity (Q_p). So,

$x = \left[D_{eq}, L, \bar{q}_{c-tip}, \bar{q}_{c-shaft}, \bar{f}_s, 1 \text{ or } 2\right]$ and $A = \left[Q_p\right]$. It should be noted that the following aspects are applied to the input and output variables used in the RVM driven piles model:

- The ultimate pile capacity (Q_p) is taken to be at the plunging failure for the well-defined failure cases, and at 80%-criterion [7] for the cases that failure load is not clearly defined, as suggested by Eslami (1996)[8].
- The pile tip failure zone over which $\bar{q}_{c-tip}$ is calculated is taken in accordance with Eslami (1996)[8], in which when the pile toe is located in non-homogeneous soil of dense strata with a weak layer above, the influence zone extends to 4 D_{eq} below and 8 D_{eq} above pile toe. Also, in non-homogeneous soil, when the pile toe is located in weak strata with a dense layer above, the influence zone extends to 4 D_{eq} below and 2 D_{eq} above pile toe. In homogeneous soil, however, the influence zone extends to 4 D_{eq} below and 4 D_{eq} above pile toe.

For drilled shafts, 88 load tests that have been collected from the literature by Shahin (2010)[2] are used for the RVM model calibration and verification. The load tests were conducted on straight and belled concrete shafts and included compression loading (for straight and belled shafts) and tension loading (for straight shafts only). The drilled shafts used had stem diameters ranging from 305 to 1798 mm, with embedment lengths from 4.5 to 27.4 m. The RVM model input variables used include the shaft stem diameter (D_{stem}), shaft base diameter (D_{base}), embedment length (L), weighted average cone point resistance over shaft base failure zone ($\bar{q}_{c-base}$), weighted average cone point resistance over shaft length ($\bar{q}_{c-shaft}$) and weighted average cone sleeve friction over shaft length ($\bar{f}_{ds}$). The single output is the ultimate capacity of dilled shafts (Q_s). So, $x = \left[D_{stem}, D_{base}, L, \bar{q}_{c-base}, \bar{q}_{c-shaft}, \bar{f}_s\right]$ and $A = \left[Q_s\right]$. It should be noted that the following issues are applied to the input and output variables used in the RVM drilled shafts model:

- The ultimate bearing capacity (Q_s) for drilled shafts under compression is taken as the axial load measured at a displacement equal to 5% of shaft base diameter plus the elastic compression of the shaft (i.e. PL/Ea, where: P is the applied load, L is the shaft length, a is the shaft cross-sectional area and E is the shaft elastic modulus). On the other hand, (Q_s) for drilled shafts under tension is defined as the axial load at 12 mm (0.5") of displacement. The above criteria for determination of ultimate load are as suggested by Alsamman (1995)[9] and recommended by Reese and O'Neill (1988)[10].
- The shaft base failure zone over which $\bar{q}_{c-base}$ is taken in accordance with Alsamman (1995)[9]to be equal to one diameter depth beneath the shaft base.

The RVM modeling technique uses the conditional distribution of the actual output as a zero-mean Gaussian with variance σ^2, so that:

$$p(A_m/x) = N\left(A_m / y(x_m; w), \sigma^2\right) \tag{2}$$

The likelihood of the complete data set is written as follows:

$$p\left(A/w,\sigma^2\right)=\left(2\pi\sigma^2\right)^{-N/2} exp\left\{-\frac{1}{2\sigma^2}\left\|A-\Phi w\right\|^2\right\} \quad (3)$$

where: $A=\left(A_1,...,A_N\right)^T$, $w=\left(w_0,...,w_N\right)^T$,

$$\Phi\left(x_i\right)=\left[1,K\left(x_i,x_1\right),K\left(x_i,x_2\right),...,K\left(x_i,x_N\right)\right]^T$$

iscalled the design matrix and $K(x_i,x_N)$ is a kernel function. The radial basis function $\exp\left\{-\frac{(x_i-x)(x_i-x)^T}{2\sigma^2}\right\}$, where σ is the width of radial basis function) is used as the kernel function, and the design value of σ is determined by trial-and-error. The maximum likelihood estimation of w and σ^2 from Equation (3) can cause over-fitting, therefore, Tipping (2001)[11] recommended imposition of some prior constraints by adding a complexity penalty to the likelihood of the error function. An explicit zero-mean Gaussian prior probability distribution over the weights, w, with diagonal covariance of α is proposed as follows:

$$p(w/\alpha)=\prod_{i=0}^{N} N\left(w_i/0,\alpha_i^{-1}\right) \quad (4)$$

where: α is a vector of N+1 hyperparameters. Consequently, using Baye's rule, the posterior over all unknowns could be computed given the defined non-informative prior distribution, as follows:

$$p\left(w,\alpha,\sigma^2/y\right)=\frac{p\left(y/w,\alpha,\sigma^2\right)p(w,\alpha,\sigma)}{\int p\left(y/w,\alpha,\sigma^2\right)p\left(w,\alpha,\sigma^2\right)dw\,d\alpha\,d\sigma^2} \quad (5)$$

Full analytical solution of the integral Equation (5) is obdurate the decomposition of the posterior according to $p\left(w,\alpha,\sigma^2/y\right)=p\left(w/y,\alpha,\sigma^2\right)p\left(\alpha,\sigma^2/y\right)$ is used to facilitate the solution [11]. The posterior distribution over the weights is thus can be given by:

$$p\left(w/y,\alpha,\sigma^2\right)=\frac{p\left(y/w,\sigma^2\right)p(w/\alpha)}{p\left(y/\alpha,\sigma^2\right)}$$
$$=(2\pi)^{-(N+1)/2}|\Sigma|^{-1/2}exp\left\{-\frac{1}{2}(w-\mu)^T\Sigma^{-1}(w-\mu)\right\} \quad (6)$$

where the posterior covariance and mean are given, respectively, as follows:

$$\left.\begin{aligned}\Sigma&=\left(\sigma^{-2}\Phi^T\Phi+C\right)^{-1}\\ \mu&=\sigma^{-2}\Sigma\Phi^T y\end{aligned}\right\} \quad (7)$$

where: $C=diag\left(\alpha_0,\alpha_1,...,\alpha_N\right)$. Therefore, learning becomes a search for the hyperparameter posterior most probable, i.e. the maximization of

$$p\left(\alpha,\sigma^2/y\right)\propto p\left(y/\alpha,\sigma^2\right)p(\alpha)p\left(\sigma^2\right)$$

with respect to α and σ^2. For uniform hyperpriors over α and σ^2, one needs only to maximize the term $p\left(y/\alpha,\sigma^2\right)$, as follows:

$$p\left(y/\alpha,\sigma^2\right)=\int p\left(y/w,\sigma^2\right)p(w/\alpha)dw$$
$$=\left((2\pi)^{\frac{-N}{2}}\Big/\sqrt{\left|\sigma^2+\Phi C^{-1}\Phi^T\right|}\right)\times exp\left\{-\frac{1}{2}y^T\left(\sigma^2+\Phi C^{-1}\Phi^T\right)^{-1}y\right\} \quad (8)$$

Maximization of this quantity is known as the type II maximum likelihood method [12,13] or the "evidence for hyperparameter" [14]. The perparameter estimation is carried out in iterative formulae, e.g., gradient descent on the objective function [11]. The outcome of this optimization is that many elements of α go to infinity such that w will have only a few non-zero weights that will be considered as relevant vectors. The relevance vector can be viewed as counterparts to support vectors in Support Vector Machine (SVM). Therefore the resulting model enjoys the properties of SVM as well as provides estimates to the uncertainty bounds. In this study, RVM is implemented using the MATLAB software.

For developing the RVM models, the available data are divided into the following two groups: (i) training set for model calibration, which comprises 55 out of 79 data records for driven piles and 61 out of 88 for drilled shafts; and (ii) testing set for verification of model performance. The testing set had 24 data records for driven piles and 27 data records for drilled shafts. Before training, all data are normalized between 0.0 and 1.0 using the following equation:

$$d_{normalized}=\frac{\left(d-d_{\min}\right)}{\left(d_{\max}-d_{\min}\right)} \quad (12)$$

where: d is the value of any input/output variable, d_{min}= minimum value of that input/output variable within the entire dataset, d_{max}= maximum value of that input/output variable within entire dataset and $d_{normalized}$ = normalized value of that input/output variable.

In this study, a sensitivity analysis has been done to extract the cause and effect relationship between the inputs and outputs of the RVM model. The basic idea is that each

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input of the model is offset slightly and the corresponding change in the output is reported. The procedure has been taken from the work of Liong et al (2000). According to Liong et al (2000), the sensitivity(S) of each input parameter has been calculated by the following formula

$$S(\%) = \frac{1}{N}\sum_{j=1}^{N}\left(\frac{\%\text{ change in ouput}}{\%\text{ change in input}}\right)_j \times 100 \tag{13}$$

Where N is the number of data points. The analysis has been carried outon the trained model by varying each of input parameter, one at a time, at a constant rate of 20%.

3. Details of Mars Models for Pile Capacity Prediction

According to MARS, which is developed by Friedman (1991)[6], the relation between an input (x) and the corresponding output (y) can be written as follows:

$$y = a_0 + \sum_{m=1}^{M} a_m B_m(x) \tag{14}$$

where: a_0 is the coefficient of the constant basis function, $B_m(x)$ is the m^{th} basis function and M is the number of basis functions. The basis functions in MARS are defined as one single spline function or the product of two (or more) spline functions for different predictors. For driven piles, $x = \left[D_{eq}, L, \bar{q}_{c-tip}, \bar{q}_{c-shaft}, \bar{f}_s, 1\text{or }2\right]$ and $y = \left[Q_p\right]$, and for drilled shafts, $x = \left[D_{stem}, D_{base}, L, \bar{q}_{c-base}, \bar{q}_{c-shaft}, \bar{f}_s\right]$ and $y = \left[Q_s\right]$. The spline function consists of two segments, i.e. truncated functions of the left-hand side of Equation (14) and right-hand side of Equation (15) separated from each other by a so-called knot location [6], as follows:

$$b_q^-(x-t) = \left[-(x-t)\right]_+^q = \begin{cases} (t-x)^q & \text{if } x>t \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

$$b_q^+(x-t) = \left[+(x-t)\right]_+^q = \begin{cases} (x-t)^q & \text{if } x>t \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

where: t is the knot location and $b_q^-(x-t)\,\&\,b_q^+(x-t)$ are the spline functions. In general, MARS contains the following three steps:

- Constructive phase;
- Pruning phase; and
- Selection of optimum MARS.

In the constructive phase, the basis functions are introduced to define Equation (9) and the selection of these basis functions is carried out using the generalized cross-validation (GCV) statistic. The value of GCV is determined by the following equation:

$$GCV(M) = \left(\frac{1}{n}\right)\frac{\sum_{m=1}^{M}\left(y_i - \hat{y}_i\right)^2}{\left(1 - \frac{C(M)}{n}\right)^2} \tag{17}$$

where: n is the number of data objects, y_i is the response value for object i, $\hat{y}_i$ is the predicted response value for object i and $C(M)$ is a penalty factor. The value of $C(M)$ is determined from the following expression:

$$C(M) = M + dM \tag{18}$$

where: d is a cost penalty factor for each basis function optimization. Over-fitting can occur due to many basis functions and to prevent over-fitting, some basis functions are deleted in the pruning phase. In the third step, the optimum MARS model is selected.

It should be noted that for model calibration and verification using MARS, the same training and testing data sets previously used for the RVM modeling are utilized and MARS is implemented using the MATLAB software. Furthermore, the same normalization technique and sensitivity analysis that is used by RVM for scaling the model input and outputs is also adopted for MARS.

4. Results and Discussion

The coefficient of correlation (R) has been adopted to assess the performance of the RVM and MARS models. The value of R is determined using the following equation:

$$R = \frac{\sum_{i=1}^{n}\left(Q_{pai} - \overline{Q}_{pa}\right)\left(Q_{pei} - \overline{Q}_{pe}\right)}{\sqrt{\sum_{i=1}^{n}\left(Q_{pai} - \overline{Q}_{pa}\right)}\sqrt{\sum_{i=1}^{n}\left(Q_{pei} - \overline{Q}_{pe}\right)}} \tag{19}$$

where: Q_{pai} and Q_{pi} are, respectively, the actual and predicted values of pile capacity, $\overline{Q}_{pa}$ and $\overline{Q}_{pe}$ are, respectively, the mean of actual and predicted values of pile capacity corresponding to n data records. Good model performance should have a value of R close to one. For the RVM driven piles model, the design value of σ is found to be equal to 0.6 and the model produced 10 relevance

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vectors for the design value of σ. The performance of the model in the training and testing sets is depicted in Figure 1.

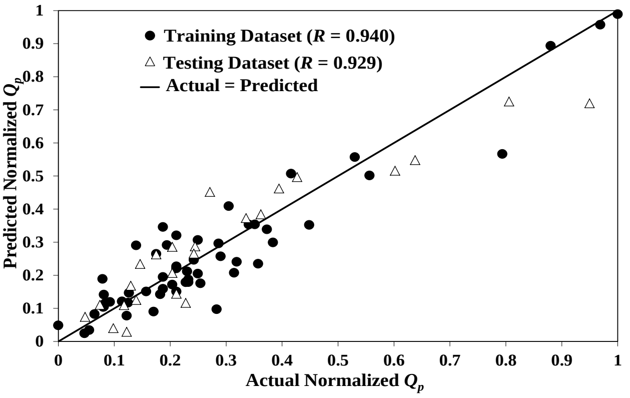

Fig.1: Performance of the RVM model for driven piles.

It can be seen that the scattering of data around the line of equality indicates good correlation between the actual and predicted pile capacities, with R = 0.940 and 0.929 for the training and testing datasets, respectively. This demonstrates that the developed RVM driven piles model has the ability to predict the axial pile capacity reasonably well. The following equation has been developed for capacity prediction of driven piles, Q_p:

$$Q_p = \sum_{i=1}^{55} w_i \exp\left\{-\frac{(x_i - x)(x_i - x)^T}{0.72}\right\} \tag{20}$$

The values of w in Equation (19) are given in Figure 2 and the variance of the predicted Q_p in the training and testing datasets are shown in Figures 3 and 4, respectively. For drilled shafts, the RVM gives best performance when σ = 0.39 and the model performance in the training and testing datasets is depicted in Figure 5. It can be observed that the model performs reasonably well with high values of R = 0.983 and 0.982 in the training and testing datasets, respectively.

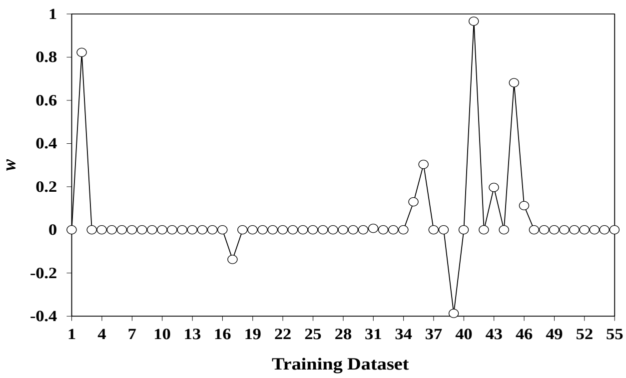

Fig.2: Values of w for the RVM driven piles model.

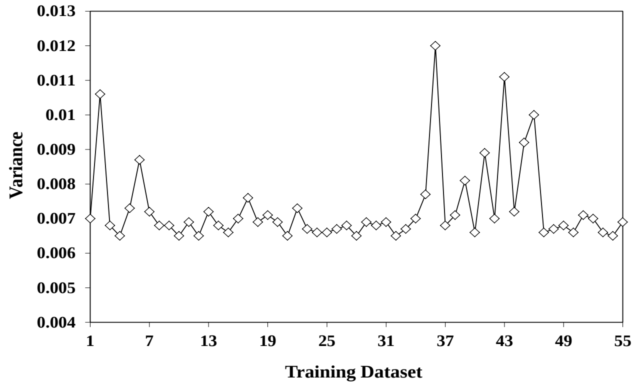

Fig.3: Variance of the training dataset for the RVM driven piles model.

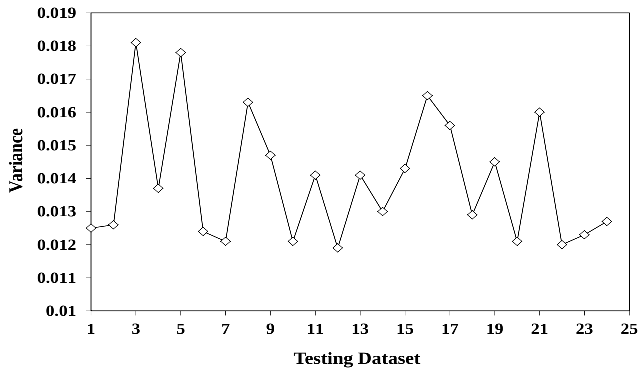

Fig.4: Variance of the testing dataset for the RVM driven piles model

The developed RVM gives the following equation for prediction of the pile capacity of drilled shafts, Q_s:

$$Q_s = \sum_{i=1}^{61} w_i \, exp\left\{-\frac{(x_i - x)(x_i - x)^T}{0.30}\right\} \tag{21}$$

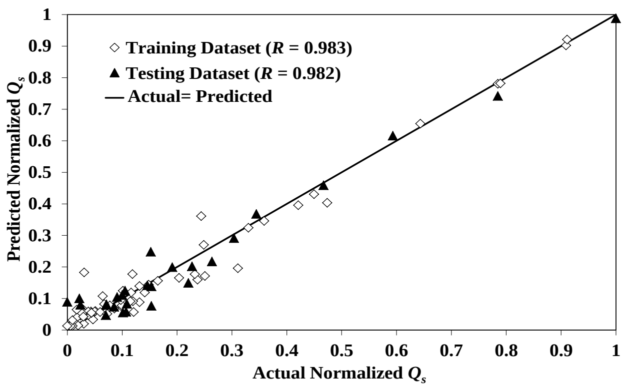

Fig.5: Performance of the RVM drilled shafts model.

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The values of w in Equation (20) are given in Figure 6 and the variance of the predicted $Q_s$in the training and testing datasets are illustrated in Figures 7 and 8, respectively.

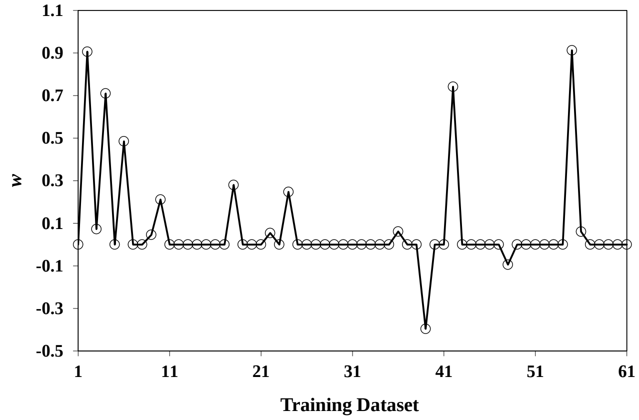

Fig.6: Values of w for the RVM drilled shafts model.

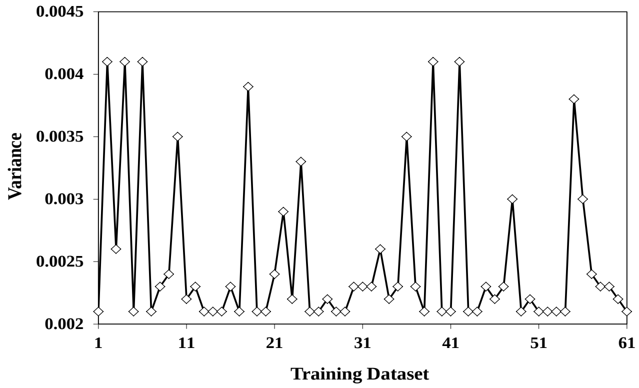

Fig.7: Variance of the training dataset for the RVM drilled shafts model.

The performance of MARS models for driven piles and drilled shafts are presented and discussed in this section. For driven piles, MARS model is developed using 20 basis functions and the final optimum model consists of nine basis functions. The different basis functions and their corresponding equations are listed in Table 1, and the final equation of the optimum MARS model is given as follows:

$$Q_p = 0.103 + 1.45 * BF1 - 0.064 * BF2 + 0.666 * BF3 + 5.025 * BF4 - 0.016 * BF5 - 94.383 * BF6 - 35.018 * BF7 + 6.558 * BF8 - 1.889 * BF9 \quad (22)$$

The performance of MARS driven piles model in the training and testing datasets is shown in Figure 9, which indicates that MARS has successfully predicted $Q_p$with high values of R = 0.958 and 0.955 in the training and testing datasets, respectively.

For drilled shafts, MARS model used 12 basis functions to predict Q_s and the final optimum model contains eight basis functions (see Table 2), and the final optimum equation of MARS model for drilled shafts is given as follows:

$$Q_s = 0.056 + 0.937 * BF1 + 0.701 * BF2 - 9.757 * BF3 + 19.332 * BF4 + 30.124 * BF5 + 0.487 * BF6 - 4.409 * BF7 - 6.001 * BF8$$

The model performance in the training and testing datasets are depicted in Figure 10, which confirms that the developed MARS drilled shafts model has predicted Q_s reasonably well.

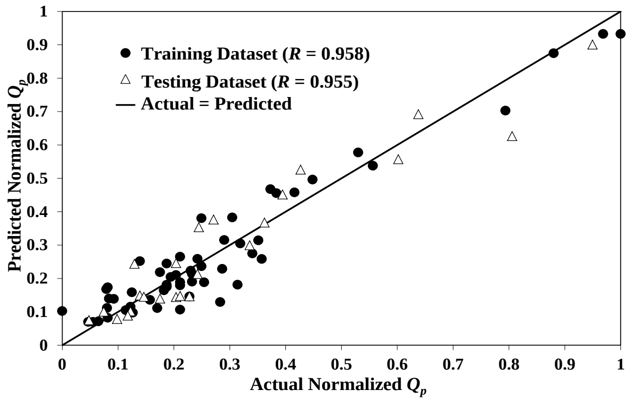

Fig.9: Performance of MARS model for driven piles.

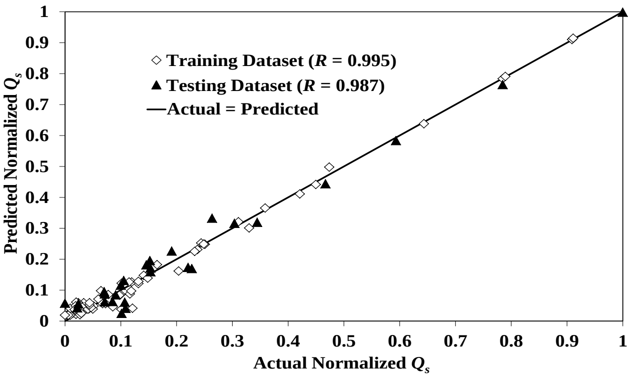

Fig.10: Performance of MARS model for drilled shafts.

5. Comparison of Rvm and Mars Models with Available Cpt-Based Methods

To examine the accuracy of the developed RVM and MARS driven piles and drilled shafts models against available methods, a comparative study has been carried out. For driven piles, the RVM and MARS are compared with the European method [15]; LCPC method; the method proposed by Eslami and Fellenius (1997)[16] and the recent ANN method developed by Shahin (2010)[2]. For drilled shafts, the RVM and MARS models are compared with the Schmertmann (1978)[17] method; LCPC method [18]; Alsamman (1995)[9] method and the recent ANN model developed Shahin (20102)[2]. The comparison is carried out for the testing dataset only (which has not been used for calibration of the RVM and MARS models) and is based on the in terms of Root Mean Square Error (RMSE) and Mean Absolute Error (MAE). The values of RMSE and MAE are computed using the following two equations:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}\left(Q_{pai} - Q_{pei}\right)^2}{n}} \tag{23}$$

$$MAE = \frac{\sum_{i=1}^{n}\left|Q_{pai} - Q_{pei}\right|}{n} \tag{24}$$

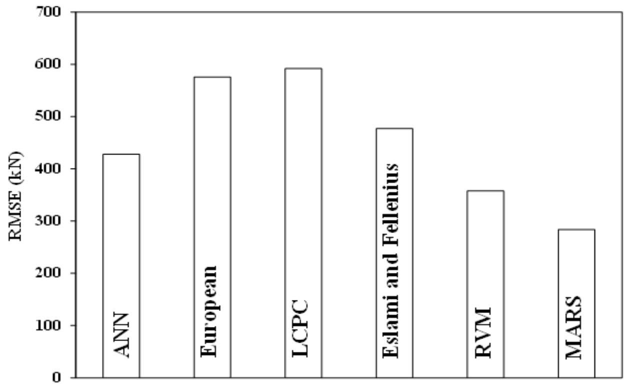

Fig.11: Bar charts of RMSE for different models of driven piles.

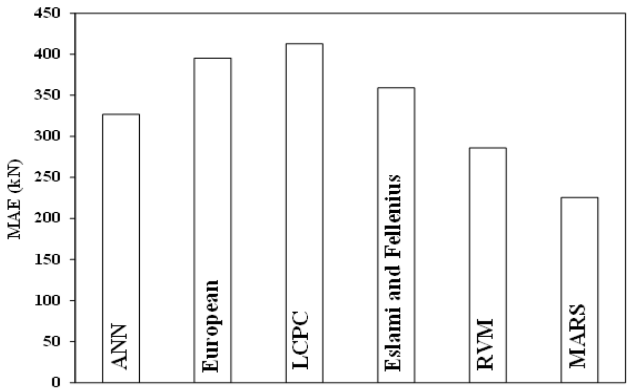

Fig.12: Bar charts of RAE for different models of driven piles.

Figures 11 and 12 show bar charts of RMSE and MAE of the different models used for comparison of driven piles methods, whereas Figures 13 and 14 show bar charts of RMSE and MAE of the different models used for comparison of drilled shafts methods. It can be observed from all figures that MARS models (for both driven piles and drilled shafts) outperform the other methods with minimum RMSE and MAE values. On the other hand, it can also be observed that the RVM models represent the second best models (after MARS models) for predicting the axial capacity of both driven piles and drilled shafts.

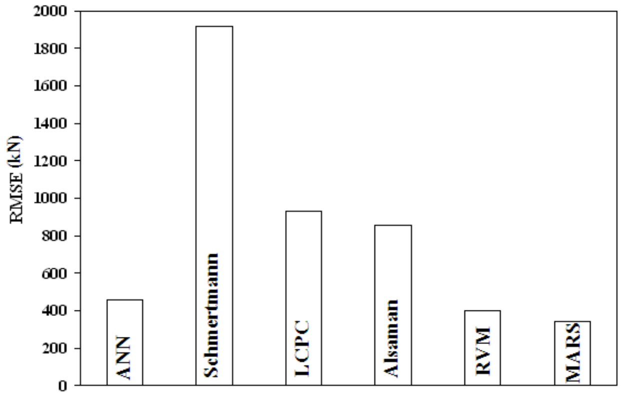

Fig.13: Bar charts of RMSE for different models of drilled shafts.

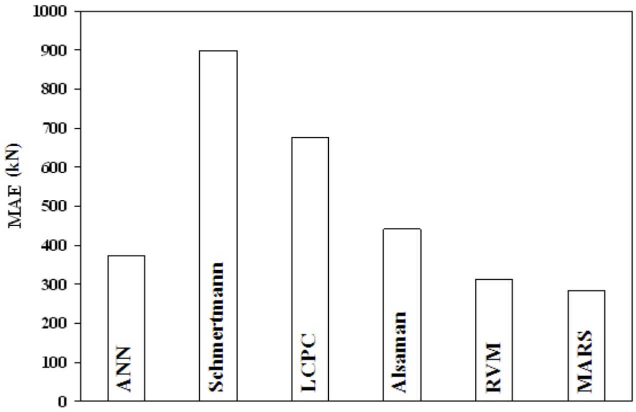

Fig.14: Bar charts of MAE for different models of drilled shafts.

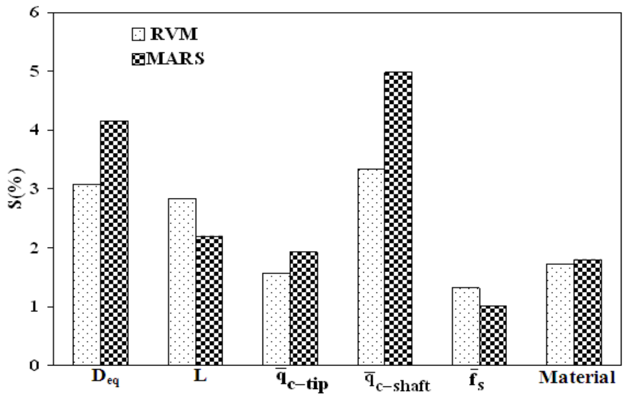

Fig.15: Sensitivity analysis of input parameters for driven piles.

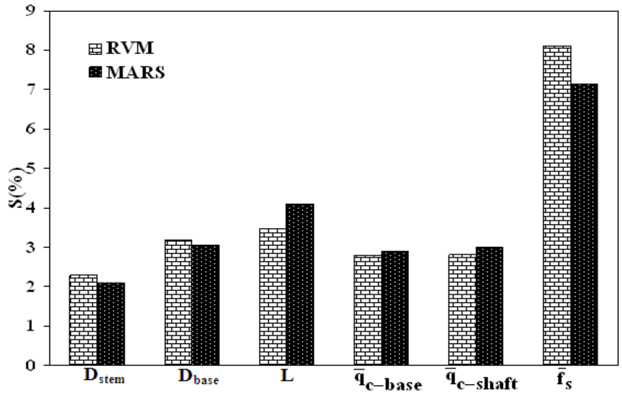

Fig.16: Sensitivity analysis of input parameters for drilled shaft.

The results of sensitivity analysis have been shown in figures 15 and 16. For driven piles, figure 15 shows that $\bar{q}_{c-shaft}$ has maximum effect on the predicted Q_p. For drilled shaft, it clear from figure 16 that $\bar{f}_s$ has maximum effect on the predicted Q_s.

6. Summary and Conclusion

This paper describes the use of RVM and MARS for predicting the ultimate capacity of driven piles and drilled shafts. A series of in-situ pile load tests and CPT results collected from the literature were used for development of the RVM and MARS models. This article shows the effect of each input parameters on the predicted Q_p and Q_s. The predictive ability of the developed RVM and MARS models was examined by comparing their predictions with those obtained from experiments as well as available CPT-based models. Tractable design equations based on the RVM and MARS models were derived and can be readily used by practicing engineers.

The results indicate that the RVM and MARS models were capable of accurately predicting the ultimate capacity of driven piles and drilled shafts. For the RVM models, the coefficients of correlation, *R*, between the actual and predicted driven pile capacities were 0.940 and 0.929 in the calibration (training) and verification (testing) sets, respectively, whereas the drilled shafts models had *R* equal to 0.983 and 0.982 for the calibration and verification sets, respectively. For MARS models, *R* for driven piles were equal to 0.958 and 0.955 in the calibration and verification sets, respectively, whereas the drilled shafts models had *R* of 0.995 and 0.987 in the calibration and verification sets, respectively. The above results indicate that the RVM and MARS models have the capability of predicting the ultimate capacity of driven piles and drilled shafts reasonably well. The comparison results of the RVM and MARS models with available CPT-based methods indicate that the RVM and MARS models outperform available CPT-based methods; however, the performance of MARS models is slightly better than that of the RVM models as they gave minimal RMSE and MAE for both driven piles and drilled shafts.

Table.1: Basis functions and their corresponding equations for driven piles.

Basis Function	Equation
BF1	$max\left(0.0, D_{eq}-0.398\right)$
BF2	$max\left(0.0, 0.398-D_{eq}\right)$
BF3	$max\left(0.0, L-0.250\right)$
BF4	$max\left(0, \bar{f}_s-0.365\right)$
BF5	$max\left(0, 0.365-\bar{f}_s\right)$
BF6	$BF4*max\left(0, L-0.250\right)$
BF7	$BF4*max\left(0, 0.250-L\right)$
BF8	$BF3*max\left(0, \bar{q}_c-0.15\right)$
BF9	$BF3*max\left(0, 0.15-\bar{q}_c\right)$

Table. 2: Basis functions and their corresponding equations for drilled shaft.

Basis Function	Equation
BF1	$max\left(0.0, L-0.172\right)$
BF2	$\max\left(0.0, \bar{f}_s-0.548\right)$
BF3	$max\left(0.0, 0.548-\bar{f}_s\right)*max\left(0.0, L-0.172\right)$
BF4	$BF3*max\left(00.0, \bar{q}_{c-base}-0.403\right)$
BF5	$BF3*max\left(0.0, 0.403-\bar{q}_{c-base}\right)$
BF6	$max\left(0.0, D_{base}-0.178\right)$
BF7	$BF2*max\left(0.0, D_{base}-0.203\right)$
BF8	$BF2*max\left(0.0, 203-D_{base}\right)$

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