Numerical Methods in Civil Engineering

Analytical estimation of natural frequency in earth dams with respecting to the foundation effects

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Abstract:

There are varieties of techniques for calculating natural frequency of the structures. Nevertheless, these equations are less developed in the geotechnical structures such as slopes, buttress group, dams, and retaining walls. In earth dams, with respecting to the dimensions and weight of dam, soil-structure interaction is of prime importance for finding seismic parameters including natural frequency, deformation, and other parameters. To accomplish this objective and the inclusion of foundation flexibility, dam foundation was simulated using transition spring. in this research an equation is presented for calculating natural frequency of earth dam using analytical methods. The advantages of this method include more accurate estimation of seismic parameters as well as considering the flexibility of earth dams' foundation. Riley approximation method and shear-free shape function were used in the mathematical solution steps and the trend of gaining the proposed equation. Finally, the results obtained from the proposed equation were compared to those of finite element software GeoStudio-2007[9] and the correlation of responses was observed.

1. Introduction

Calculating natural frequency and fundamental period is required for anticipating the dynamic behavior of dams. Thus, presenting suitable solution is important for gaining frequency. Unlike concrete dams, the earth dams are placed in the group of flexible structures. The same quality has led to the suitable behavior of these structures in the past earthquakes. Earth dams are mostly preferred to other type of dams in nearby active faults and in earthquake-bound zones.

Dams' natural frequency is one of the key parameters explaining dynamic response of dams at the time of earthquake. In this regard, numerical methods are presented in recent years for frequency calculation in addition to few analytical equations. Idriss et al. [1] applied RI parameter or the interaction ratio for studying the effect of some

parameters on dam-foundation system interaction defined as follows:

$$R_I = 100 \frac{|a_F - a_{FD}|}{a_F}$$

where, a_F is maximum horizontal acceleration on foundation layer without dam; a_{FD} is maximum horizontal acceleration on dam basis. The lower the RI is, the lower the effect of interaction is. In addition, it is possible to analyze dam without foundation. The results indicated that the effect of interaction cannot merely be considered as resulted from the ratio of foundation shear wave velocity to dam shear wave velocity $\left(\frac{V_{SD}}{V_{SH}}\right)$ and the ratio of dam period to foundation period$\left(\frac{T_{dam}}{T_{fnd}}\right)$. Rather, the effects of many factors must be taken into consideration at the same time. Also, *RI* value increases if $\frac{D}{B} \leq 1$ or the ratio of foundation layer depth at the base is less than one. On the other hand it is not possible to ignore the effect of interaction. In addition, the system acts like a dam on the rigid foundation

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if $\frac{V_{SD}}{V_{SH}}$ is larger than one or the rigidity of foundation materials is more than dam materials rigidity.

Kishi et al. [2] developed graphs for calculating dam frequency. They used earth dam bi-dimensional partial wedge model and simultaneously considered the bending moment and shear force at perpendicular section of the earth dam in a rectangular channel. With respecting to the foundation flexibility beneath dam, Dakoula [3] examined non-linear response of a heterogeneous earth dam located in a narrow valley using shear beam technique. He concluded that as the ratio of shear velocity of foundation to shear velocity of dam body $\left(\frac{C_2}{C_1}\right)$ increases, maximum horizontal acceleration of mid dam increases and maximum displacement decreases. Including the foundation will increase the natural period of system. Gazetas and Dakoulas [4] applied shear beam method and presented $T = 2.6\frac{H}{Vs}$ equation to calculate free vibration period of the earth dam on rigid bed composed of cohesive soil. Foundation flexibility and some geometric characteristics like dam slope, dam base length, and dam crown were not included in his equation.

Watanabe [5] calculated the natural frequency and its corresponding modes to the fifth movement mode of the earth dam, based on fourier spectrum and modal analysis. Zhen [6] gained the frequencies of heterogeneous earth dam existing in a triangular channel modeled in elastic form using shear wedge analysis method. Papalou and Bielak [7] conducted a finite element numerical analysis for estimating seismic response of earth and rockfill dams considering the interaction effect between valley and dam. They used La Villita dam as model to determine the effect of valley flexibility and the angle of incoming waves. Based on previous studies, in general dam response increases, when rigid valley is considered. As a result, considering the interaction between valley and dam may prevent unnecessary conservations.

Tsai et al. [10] studied the effect of clay core dimensions on the earth dam natural frequency. Using finite element software, they showed that as the width of core increases, the first mode frequency decreases. Moreover, it was concluded that the first mode frequency increases after the seepage. Parish et al. [11] gained the natural frequency of dam-foundation system by fourier analysis of dam free vibration. The results of numerical analysis demonstrated that considering the elastoplastic behavior for dam materials led to decrease of intensification phenomenon and dam natural frequency, due to the increase of energy absorption.

Soil-structure interaction in concrete dams has been considered importantly by the researchers (e.g. Saleh and Madabhushi [12]). Based on their results, dam flexibility caused significant decrease of dynamic responses. The importance of soil-structure interaction has led to the spread of such studies in various subjects. Dodagoudar and Boominathan [13] have examined the effect of soil-structure interaction on deep embedded ventilation stack. In recent years, quantitative studies regarding the soil-structure interaction have considerably increased. However, the analytical methods for calculating dynamic parameters including natural frequency in earth dams have barely been considered. In this study, a new equation is presented using analytical methods where dam foundation flexibility is included in the calculation of natural frequency.. Finally, the results of proposed method are compared to those of software.

2. The Hypotheses of Estimating Natural Frequency

To gain the analytical equation for calculating natural frequency of earth dam with respecting to the effect of soil-structure interaction, the earth dam dynamic behavior is taken similar to the behavior of a cantilever beam. Since the earth dam is so wide, the horizontal flexibility has more importance and the vertical flexibility of foundation is ignored. As observed in Fig. 1, beam with varying section restrained by a transition spring on one end can be a suitable model for the earth dam behavior regarding dam flexibility. Accordingly, natural frequency can be gained for beam model using Riley approximation method and generalized to the earth dam.
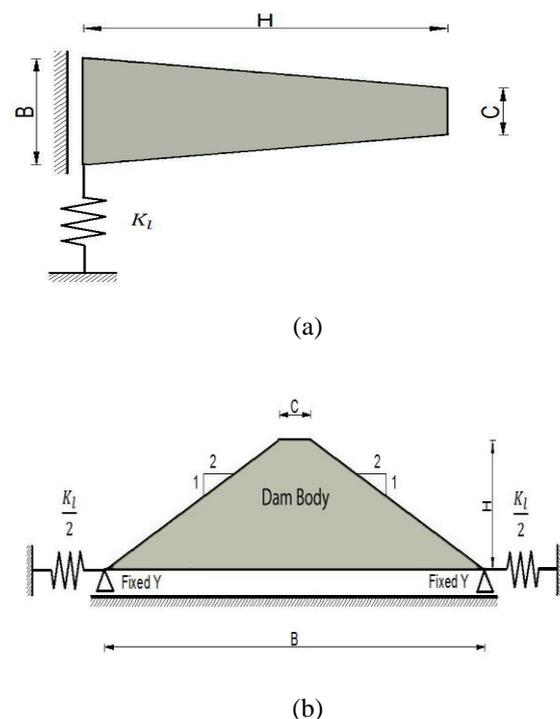

(a)

(b)

Fig.1: Generalizing the beam to earth dam

The following hypotheses are considered for solving the issue:

i. Plane-strain behavior is considered;
ii. Shear-free shape function is used in the analytical solution;
iii. Dam body is homogeneous materials;
iv. Poisson ratio is assumed constant and equal to 0.3;
v. Dam behavior is considered to be linear elastic;
vi. Dam slope is presumed as two horizontal versus one vertical.

An appropriate shape function should be employed in order to gain the analytical equation for natural frequency. In this study, the following shape function, which creates shear-free conditions, is used [8]:

$$\phi(x) = (\cos ax + \cosh ax) - \frac{\sinh al - \sin al}{\cosh al + \cos al}(\sin ax + \sinh ax) \tag{1}$$

Where a is a coefficient and l is the length of beam.
Hence, solving the equation based on the constant al and simplifying it, the following equation is gained:

$$(\tan al)(\tanh al) = 0 \tag{2}$$

Solving Eq. 2 will result as follows:
$$al_1 = 2.365$$
$$al_2 = 5.4978$$
$$al_3 = 8.6394$$

Regarding the importance of first motion mode, it is obtained by placing the first response in the shape function as follow:

$$\phi(x) = \cos(\frac{2.365}{l}x) + \cosh\left(\frac{2.365}{l}x\right) - 0.821\sin(\frac{2.365}{l}x) - 0.821\sinh(\frac{2.365}{l}x) \tag{3}$$

The shape function must be associated with a time coordinate $Y(t)$, which is the true displacement of an optional material point of dam. Doing so, all dam points will be related to the generalized coordinate $Y(t)$ using conditional equation $\phi(x)$. It is enough to set the displacements in this coordinate. The displacement of other points will be determined by the function $\phi(x)$. The following function was considered for time function,.

$$Y(t) = \cos(\omega t - \varphi) \tag{4}$$

Accordingly, total structural displacement function, which is a function of time and place, will be defined as follow:

$$v(x,t) = \phi(x).Y(t) \tag{5}$$

Natural frequency can be gained using Riley's method in case of considering the energy constant during earthquake loading and ignoring the effect of damping,. Therefore, a model is regarded for a beam with varying sections and a free-end; the other end is connected to the transition spring. If the dimensions of the beam are transmitted to the dimensions of a dam, a suitable generalized model can be proposed for a dam. Therefore, in the rest, using Riley's method, an equation will be gained for beam model, which is similar to an earth dam. Then, a modified index will be considered for dam based on the results gained from finite element software.

3. Natural Frequency in Varying Section State

The effect of soil-structure interaction is studied in the horizontal direction, considering the transition spring. Transition spring simulates foundation flexibility in the horizontal direction. In the following, the results of proposed model are presented for the dams of 10 to 50 meters high.
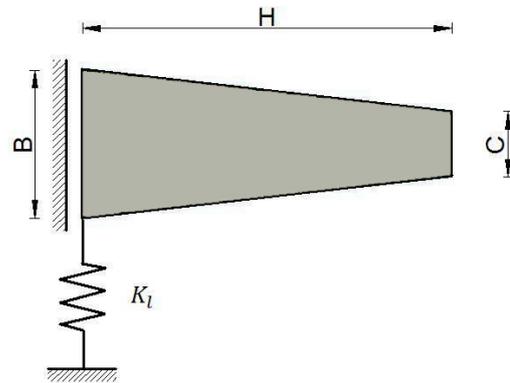

Fig.2: Beam model with transition spring and varying sections

Regarding Fig.2, the structural unit mass can be expressed as follow:

$$m(x) = \rho(C + \frac{(H-x)(B-C)}{H}) \tag{6}$$

Considering the variability of the section, the inertia moment will be obtained as follow:

$$I(x) = \frac{1}{12}(C + \frac{(H-x)(B-C)}{H})^3 \tag{7}$$

Based on Riley's method hypotheses, the energy is constant in a dynamic movement. On the other hand, the

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potential energy is completely turned into inertia. Regarding the geometry and boundary conditions of the issue, the potential energy can be taken as the inertia in maximum state and then it is possible to calculate frequency. Maximum inertia for Fig. 4 is:

$$T_{max} = \frac{1}{2}\int_0^H m(x)\left(\frac{\partial v}{\partial t}\right)^2 dx = \frac{1}{2}\omega^2 \int_0^H m(x)\phi(x)^2 dx \quad (8)$$

Also, with respecting to the transition spring, the potential energy is defined as follows:

$$V_{max} = \frac{1}{2}\int_0^H EI(x)(\frac{d^2\phi}{dx^2})^2 dx + \frac{1}{2}k_l\phi^2(0) \quad (9)$$

Equating potential energy and inertia, an angular velocity is gained as follow:

$$\omega = \left(\frac{0.255EB^2C+0.0628EB^3+1.129EC^3+0.613EBC^2+2K_lH^3}{\rho H^4(0.306B+0.062C)}\right)^{0.5} \quad (10)$$

Having the angular velocity, frequency can be gained:

$$f = \left(\frac{1}{2\pi}\right)\left(\frac{0.255EB^2C+0.0628EB^3+1.129EC^3+0.613EBC^2+2K_lH^3}{\rho H^4(0.306B+0.062C)}\right)^{0.5} \quad (11)$$

Where, E is soil elasticity module; b is earth dam width; c is dam crest width; H is dam height; ρ is soil density; and K_l is horizontal spring rigidity or soil rigidity in the horizontal direction.

4. Numerical Analysis Results

The results of equation are compared to those of dynamic analysis of earth dam using GeoStudio2007[9] to verify the accuracy of the proposed equation. The formula gained from mathematical equations was compared using finite element software GeoStudio-2007[9]. The dam is modeled in the software based on the presence of horizontal springs and considering the effects of soil-structure interaction, shown in Fig. 3.

To maintain symmetry in the model, two horizontal springs are used the sum of their rigidities equals' transition rigidity of formula. In addition, the dynamic analysis in software is considered as *0.1g* based on the ground maximum acceleration. Five dams with five heights and various scales were modeled to control formula by software The results of proposed formula obtained for natural frequency was compared with those of software. The specifications presented in Table were employed for modeling. For software modeling, the specifications of the materials are used based on Table 1.
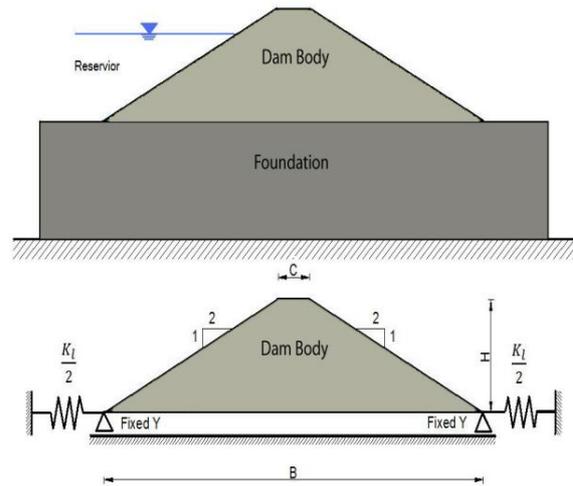

Fig.3: The models with true foundation (top) and equivalent spring (bottom) for analyzing the earth dam by software

Table.1: Specification of material used in the numerical analysis of dam

Parameter	value	unit
E	70000	kNm^{-2}
ν	0.3	-
γ	20	kNm^{-2}
K_h	40000	kNm^{-1}

Five dams with different heights and horizontal slope two to vertical one were modeled by GeoStudio-2007[9]. Fig. 4 shows contours of vertical stress from static analysis of dam. The static analyses have been conducted first, presuming the reservoir full and then empty. Fig. 5 displays the results of earth dam dynamic modeling using spring.
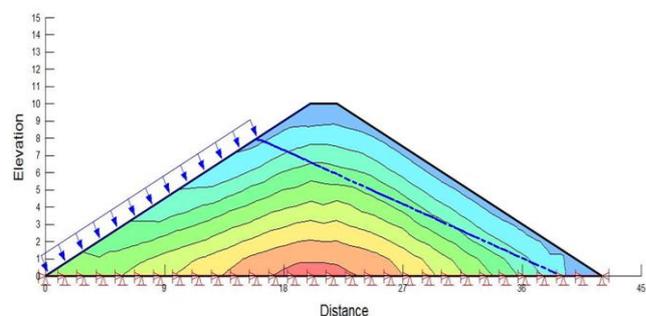

Fig. 4: Vertical stress contours resulted from dam static analysis
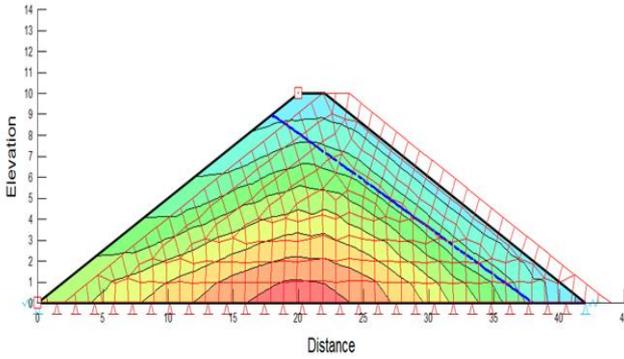
Fig. 5: Vertical stress contours resulted from dam dynamic analysis using equivalent spring model

For dynamic analysis by software, an earthquake record with $a_{max} = 0.1g$. Fig. 6 shows the accelerograph used in dynamic analysis.
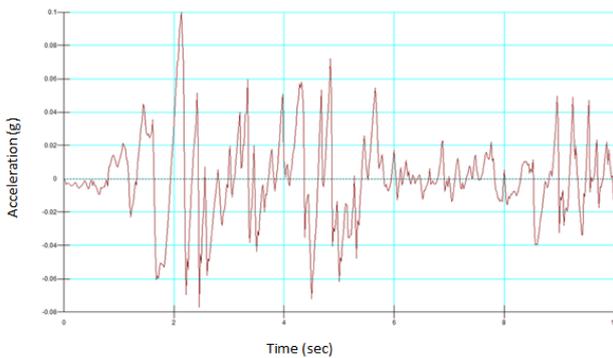

Fig. 6: Accelerograph used in analyses maximum acceleration *0.1g*

The period changes are drawn versus velocity in horizontal direction to gain natural frequency, Fig. 7. According to the figure, the period corresponded to maximum velocity reverses the natural frequency of system.
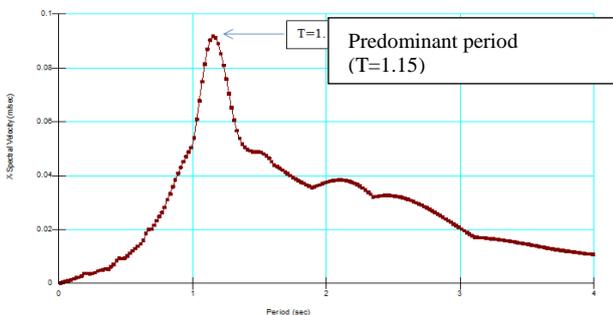

Fig. 7: An example of period changes versus spectral horizontal velocity for calculating dam natural frequency

The same trend was conducted for gaining the frequency for other four dams and results are represented in Table 2.

The results obtained from analytical equation are compared with those of software modeling, Fig. 8. Eq. (11) has no agreement correlation with the results obtained from numerical analysis. The analytical equation was gained using the modified index 0.13 as follows:

$$f = \left(\frac{0.13}{2\pi}\right)\left(\frac{0.255EB^2C+0.0628EB^3+1.129EC^3+0.613EBC^2+2K_lH^3}{\rho H^4(0.306B+0.062C)}\right)^{0.5} \quad (12)$$

Where E is the elastic modulus of dam body. Fig. 8 shows natural frequency versus dam height. As observed, the results of analytical equation have good correlation with software. Also, as dam height increases, the frequency decreases and on the other hand the structural period increases.

Table.2: Results for dams up to 50m high

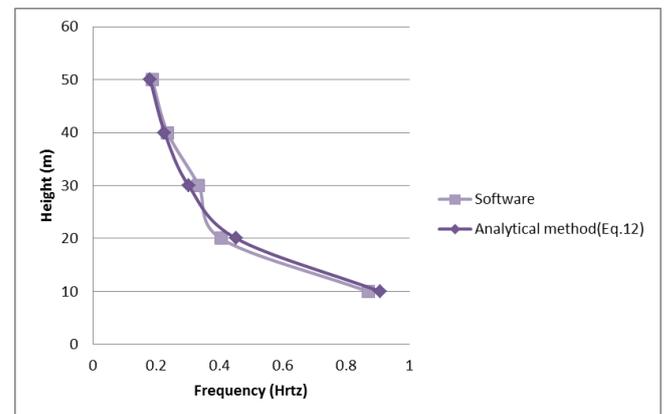| Geometric characteristics of earth dam			Natural frequency of earth dam (Hrtz)	
H(m)	B (m)	C (m)	Software	Analytical method
10	42	2	0.869	0.907
20	84	4	0.404	0.453
30	126	6	0.334	0.302
40	168	8	0.236	0.226
50	210	10	0.187	0.181

Fig.8: A comparison between the proposed equation and software results

5. Foundation Rigidity Effect

Fig. 9 shows the foundation flexibility role in natural frequency. K is the rigidity of horizontal spring that is foundation rigidity. The rigidity of springs was doubled in each step and its results were plotted. According to the figure, as the rigidity of foundation increases, natural frequency increases and the period decreases.
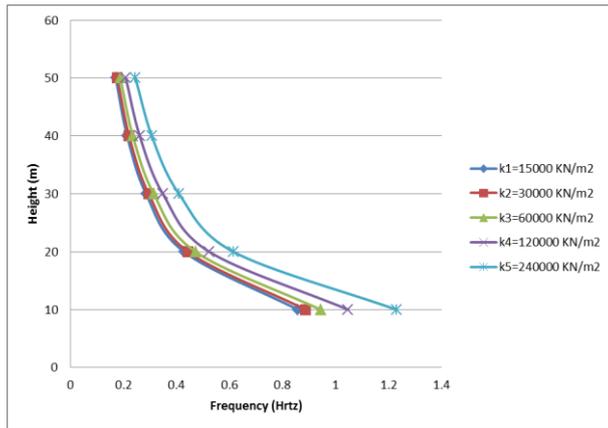
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Fig.9. The changes of dam free vibration frequency versus height for different rigidities of dam foundation

6. Comparing Natural Frequencies of Different Shear Wave Velocities

Shear wave is proportionate to the rigidity of dam materials. That is to say, the more the shear modulus of consumed material is, the further the shear wave will be. Fig.10 shows the influence of shear wave velocity in the natural frequency of earth dam. It must be noted that in the presented analytical equation, the shear wave velocity is indirectly related to frequency. To observe the effect of shear velocity for constant materials density in dam as well as the constant foundation rigidity (60000kN/m^2), shear wave velocity were changed. According to Fig. 10, as shear wave velocity increases, frequency enhances. Regarding the matter that shear wave velocity is proportionate to materials shear modulus, the rigidity of period decreases and the frequency increases in proportion with the materials shear modulus.
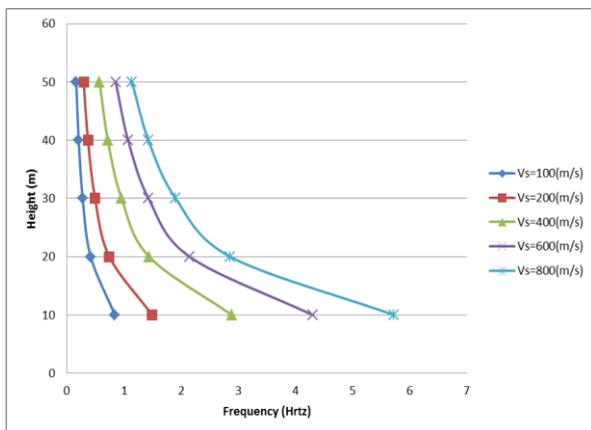

Fig.10: Natural frequency changes versus dam height for different shear wave velocities

Also, analytical Eq. (10) can be simplified further using the following presumptions:

$$B = 4H \,, C = 0.2H$$

$$f = \frac{0.168}{2\pi H}\sqrt{(4.94V_s^2(1 + v) + \frac{K_l}{\rho})} \qquad (13)$$

7. Comparing the Results of Proposed Equation and Previous Research

It is important to compare the proposed equation with the equations presented by previous researchers. Therefore, the frequencies gained from the proposed analytical equation for different foundation rigidities were compared to those of the equation presented by Gazetas and Dakoulas [4]. Dam specifications are based on Table 3.

Table.3: Dam specifications

Parameter	value	unit
E	52000	$\text{kN}m^{-2}$
v	0.3	-
γ	20	$\text{kN}m^{-2}$
V_s	100	$\text{m}s^{-1}$

Gazetas and Dakoulas [4] analytical equation presented for the structural period considering the rigidity of foundation is as follow:

$$T = 2.61\frac{H}{Vs} \qquad (14)$$

This can be changed as follow to gain Eq. (15):

$$f = \frac{1}{2.61}\frac{Vs}{H} \qquad (15)$$

The proposed equation is compared with the analytical equation of Gazetas and Dakoulas [4] and presented in Fig. 11. According to the figure, as the dam rigidity increases, the diagram related to the proposed equation (flexible foundation) further approaches Gazetas and Dakoulas [4] analytical equation, and will finally correspond to Gazetas and Dakoulas [4] diagram for the foundation rigidity K_l =380Mpa. Therefore, it is suggested that if the foundation rigidity is higher than the mentioned amount, the effect of interaction can be ignored and the foundation is considered as rigid. It is also recommended to apply Gazetas and Dakoulas [4] equation for rigidities higher than K_l =380Mpa, yet the proposed equation for the amounts lower than K_l =380Mpa.
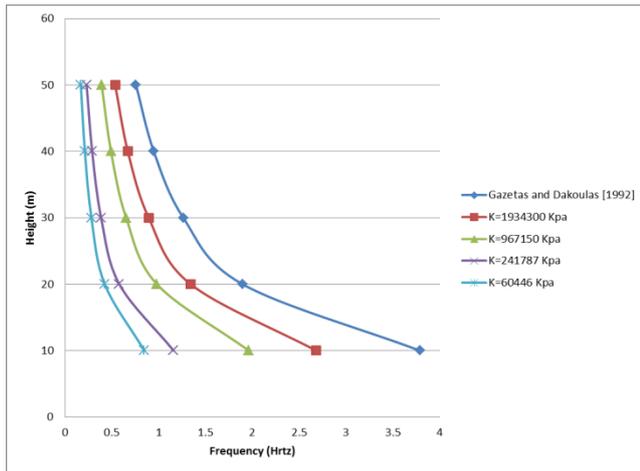
Fig.11 : Comparing the proposed equation and Gazetas and Dakoulas [4] analytical equation for different dam foundation rigidities

8. Conclusion

To calculate the natural frequency of earth dam, it was assumed that the earth dam behaves like a beam with one restrained end. Thus, a beam with varying section (one free end, another restrained by transition spring) was considered. Respective springs module the earth dam foundation flexibility. Shear-free shape function is used for solving problem. Using Riley's method maximum frequency of free vibration was gained by equating the potential energy with the inertia. The results of analytical equation obtained for five dams with the same specifications were compared to those of finite element software GeoStudio-2007[9], and the proposed equation was modified. The proposed analytical equation will lead to natural frequency, which is a function of dam materials characteristics, dam geometry, and dam foundation rigidity, and shown as follow:

$$f = (\frac{0.13}{2\pi})(\frac{0.255EB^2C+0.0628EB^3+1.129EC^3+0.613EBC^2+2K_lH^3}{\rho H^4(0.306B+0.062C)})^{0.5}$$

The proposed equation can estimate dam natural frequency with proper accuracy without doing dynamic analysis. Moreover, comparing to other equations, presented by the researchers, the presented equation included dam body characteristics more completely in the calculation of natural frequency.

For usual dimensions of the earth dams where dam width is about four time its height and also crest width of dam is about %20 of its height, the equation will be simplified as follow:

$$f = \frac{0.168}{2\pi H}\sqrt{(4.94V_s^2(1+\nu)+\frac{K_l}{\rho})}$$

It is noteworthy that the equations presented in this study are for the time when horizontal foundation rigidity (K_l) is lower than 380Mpa. In most cases, foundation rigidity is less than the amount mentioned. Therefore, the equation can be used with an appropriate accuracy. Besides, in cases where horizontal rigidity is greater than 380Mpa, the foundation can be considered as rigid and its effect on natural frequency can be ignored.

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