Numerical Methods in Civil Engineering

Frequency Analysis of Concrete Gravity Dam with Finite Element Model and LHS Method

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Abstract:

In this paper, the dynamic response of a concrete gravity dam along with reservoir domain is investigated by frequency analysis and Latin hypercube sampling (LHS) statistical methods using finite element model. In this analysis, the frequency value is assigned to the model as an input variable. Then, the effects of frequency parameter are studied on maximum horizontal displacement of the dam crest, maximum tensile stress in the heel, maximum compressive stress in the toe and hydrodynamic pressure on the heel of the dam. The ANSYS software, according to finite element method (FEM), is applied for modeling and analysis. In order to represent the effect of the dynamic loading frequency, which is an important parameter in the analysis of the structures, the maximum response values are presented as sensitivity and probability curves. According to the sensitivity diagram of the hydrodynamic pressure response vs. the input frequency, it can be concluded that in the frequency of loading near the natural frequency of reservoir, the most critical condition occurs for seismic display, which should be highly considered in designing concrete dams.

1. Introduction

In order to calculate dam responses to critical forces accurately, it is crucial to consider the effects of the interaction between dam and reservoir so as to achieve a safe design. Most engineers design structures with complex public utility packages for the analysis of structures. They often have no access to the source codes of programs and are less aware of the structural algorithm details in these software applications. Therefore, the main challenge for researchers to optimize structures is to develop appropriate methods for such software. Another major challenge is the high computational cost of analyzing several complex structures, such as concrete dams. One of the latest methods for optimization and parametric evaluation of structure behavior is the LHS statistical method and probabilistic analysis.

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Hence, this method has been applied to address effects of the dam-reservoir interaction on frequency analysis. Accordingly, the most critical condition is selected for the analysis and applied to the design and analysis of concrete dams in order to achieve safety and cost-effective results in the project. Modeling of interaction effects has a long history in seismic analysis of dams. The first study in this field was presented by Westergaard (1933) [18]. He considered the effects of interaction as a two-dimensional model of dam-reservoir, which was affected by the vibration caused by the horizontal motion of the earth. He assumed that the dam is rigid and has a semi-limited reservoir with a constant depth. Westergaard found that the force generated by the interaction is proportional to the acceleration is caused by the seismic motion of the earth and could be approximated by a parabolic mass distribution on the height of the dam.

In most studies, given the rigid nature of the dam, the forces caused by the interaction during seismic motion of the earth are considered as an external force on the dam

and the dam response is neglected. Chopra (1968) [3] studied the impact of the dam flexibility on the interaction forces by modeling a dam as a system including the mass, damper and spring. He extracted the parameters of this system through considering the first vibrational mode of a triangular dam and showed that the natural frequencies of the dam and reservoir coupled system were different from the uncoupled system. In further studies, Hall and Chopra (1982) [9] investigated the hydrodynamic effects of the reservoir on the seismic performance of a concrete gravity dam. They used one-dimensional approximation to propagate waves released from the reservoir. They showed that the approximation of finite elements could be successful in seismic analysis of the dam and reservoir system in the mode that where the dam is flexible and water is compressible. Chwang and Housner (1978) [6] determined the distribution of hydrodynamic pressure in dams with the inclined upstream based on the momentum equilibrium. Fenves and Chopra (1985) [7] investigated the interaction between reservoir water and its foundation using the energy dissipation and its effective behavior on the sediment layer of the reservoir bottom. In their method, it is assumed that the compressive waves hitting the bottom of the reservoir are dissipated before reaching the lower layers of the rock and the sediment layers' properties are only to evaluate the reflection from the boundaries. In their model, the thickness of the layer is not precisely determined and the effect of the proximity to the lower bedrock on the reflection coefficient for the bottom of the reservoir is neglected. Therefore, their simple approximation indicates a further decrease in the dam response during the absorption of the reservoir bottom. Saini et al. (1978) [13] and Chopra - Chakrabarti (1981) [5] studied the dam-reservoir interaction problem in the frequency domain using the finite element model. Finite element analyses in time-domain were performed by Sharan (1985 and 1986) [14, 15 and 16] and Tsai et al. (1992) [17]. For the reservoir with irregular geometry, numerical methods such as finite element method must be used, because an analytical solution cannot achieve results for the arbitrary boundary and geometry of the system. Jablonski and Humar (1990) [10] applied the boundary element method in frequency domain for seismic analysis of concrete dams. Alembagheri and Seyedkazemi (2014) [1] conducted a probabilistic study in which the seismic behavior of the concrete gravity dam is addressed with regard to concrete tensile behavior parameter as the sensitivity parameter. The results of their research showed that accurate examination of tensile behavior and final failure of concrete in concrete gravity dams requires a proper definition of nonlinear models of materials. Using Monte Carlo probabilistic method, Pasbani Khiavi (2015) [11] investigated the reservoir bed characteristics effect on

reducing the pressure induced in the reservoir. Results confirmed a high dependence of responses to the reservoir bottom absorption. Additionally, Pasbani Khiavi (2017) [12] investigated the influence of concrete stiffness on the seismic responses of concrete gravity dams by the Monte Carlo simulation. According to the results, the optimized value of the concrete Young Modulus to access the confident response of the structure, which was economically important, was achieved.

This study attempts to introduce the Monte Carlo method as an effective tool for investigation and safe design in the uncertainty space by the FE-ANSYS simulations. Results of this simulation show the influence of the earthquake frequency on the performance of concrete gravity dams under seismic loading. In probabilistic analysis, loading frequency is chosen as an input parameter. Then the maximum displacement of dam body along river, induced dynamic pressure at dam-reservoir interface in bottom, tensile principle stress at heel and compressive principle stress at toe of dam were selected as the output critical responses of the model and presented in this paper.

2. Monte Carlo simulation characteristics

There are several methods such as simulation methods to solve structural reliability problems. Simulation is defined as the numerical simulation of some phenomena and observation of the events that have taken place. The concept of simulation is relatively straightforward, but its process may be very challenging.

As the information and results related to the N test are available in a bag, if the results of an n-member sample are required, instead of doing n additional tests, n out of N test results can be selected randomly. This sampling method is introduced as a special technique.

The Monte Carlo method is a special technique to generate numerical results without performing a physical test. The results of previous experiments can be used to generate probabilistic distributions of important parameters in the problems. This information distribution can then be used to generate data samples. The Monte Carlo method is applicable to all types of distribution and also to check the accuracy of the results of other methods. The error associated with this type of technique is completely controlled by the number of simulations. It is confirmed that once the number of samples tends to reach infinity, the results converge to an exact value. Uncertainty in the analysis decreases by increasing the number of samples. One of the main criticisms against Monte Carlo method is the high computational time. However, variance reduction methods may make this method more efficient. In some cases, the analysis is very complex and the time required for a single analysis phase may be too long. Therefore, it

might be impossible to perform hundreds or thousands of simulations in terms of time. The Latin Hypercube Sampling Method is a technique for reducing the number of simulations required to achieve acceptable results. In this method, a range of possible values of the random input variables is partitioned in layers and a value from each layer is randomly selected as the sample value. Sample values for each random variable are combined in such a way that each value is considered only and only once. In this method, all possible values of random variables are brought into simulation. Consider the boundary condition Y with the following K random variables:

$$Y = f(X_1 \cdots X_k) \tag{1}$$

The basic stages of the Latin Hypercube Sampling Method are:

- Each X_i is partitioned at certain intervals (N intervals). This partitioning should be such that the probability of occurrence of any value of X_i in this interval is $1/N$.
- For each variable X_i and its N interval, a value is randomly selected as a sample value. In practical applications, if the number or size of intervals is large, the central point of each interval will be chosen instead of random sampling.
- Subsequent to the above steps, N sample values are obtained for each of K random variables. Generally, there are N^K possible combinations of these values.
- To obtain the first combination, a sample value is selected randomly for each input random variable K. In order to obtain the second combination, a sample value $N-1$ is selected randomly for each input random variable. This selection process proceeds to the point that combinations of the values of the input variables are generated.
- Equation (1) is evaluated for each of the above-generated N combinations of input variables. As a result, the N function is obtained as $Y_i = (i = 1,2, \ldots, N)$

This process creates simulation data and it is necessary to determine how data is used to estimate the statistical parameters for y. The most commonly used formulas are as follows:

$$Y = \tilde{Y} = \frac{1}{N} \sum_{i=1}^{N} y_i \tag{2}$$

$$Y = \frac{1}{N} \sum_{i=1}^{N} (y_i)^m \tag{3}$$

$$F_y(y) = \frac{number\ of\ times\ y_i \leq y}{N} \tag{4}$$

3. The governing conditions of the system

Considering the rules and conditions governing the system from the hydrodynamic point of view, the selected model is considered in accordance with the following conditions:

- Considering the conditions governing the behavior of the concrete gravity dam and the geometric shapes of the reservoir, a two-dimensional plane stress model is used for the dam elements.
- The behavior of the dam is linear.
- Given the governing conditions of the problem, the fluid is non-rotational, inviscid and with linear condensation and its displacement is insignificant.
- The effects of gravity waves on the free surface are neglected and the surface pressure is zero.
- The Newmark method is used to solve dynamical equations.

3.1 Modeling the dam

The dam behavior is presented as the motion equation. However, it includes the interaction between the fluid and the structure, and the applied load due to the fluid hydrodynamic pressure on the structure. The fluid contact point must be added to the structures' equations.

$$M\ddot{u} + C\dot{u} + Ku = M\ddot{u}_g + F^{Pr} \tag{5}$$

Where, M, C and K represent the mass, damping and stiffness matrices respectively. u is the relative movement vector and $\ddot{u}_g$ refers to the acceleration vector. F^{Pr} denotes the hydrodynamic force pressure vector at the contact surface.

3.2 Modeling the reservoir

The equation for the dynamics of the structure should be based on Navier-Stokes, momentum and fluid continuity equations in relation to problems related to the acoustic interaction between the structural and fluid. As the water inside the tank is inviscid and incompressible with small displacement, the equations of continuity and momentum are summed up to the wave equation. Moreover, the pressure applied to the structure by the fluid at the contact point is considered to form the interaction matrix (Chopra and Chakrabarti 1972 [5]).

$$\frac{1}{C^2} \frac{\partial^2 P}{\partial t^2} - \nabla^2 P = 0 \tag{6}$$

Where, $c = \sqrt{k/\rho_0}$ represents the velocity of sound in the fluid environment. ρ_0 is the average fluid density, k refers to the fluid bulk modulus, P refers to the acoustic pressure, and t indicates the time.

3.3 Boundary conditions

The target system includes dam- reservoir. Four boundary conditions, including the truncated boundary of the reservoir, boundary of the reservoir bottom, free surface boundary condition and the boundary condition of the interaction at the dam and reservoir contact point are defined as follows:

- At the truncated boundary of the reservoir, the Sammerfeld dissipation boundary condition is used:

$$\frac{\partial P}{\partial x} = -\frac{1}{c}\frac{\partial P}{\partial t} \tag{7}$$

- At the surface of the reservoir, the surface pressure is assumed insignificant. Thus:

$$P = 0 \tag{8}$$

At the contact points between the reservoirs, the dam and the foundation, the interaction is applied as follows:

$$\rho \vec{a}_{ns} = -\frac{\partial P}{\partial n} \tag{9}$$

Where, $\vec{a}_{ns}$ is the vector of acceleration of the dam or foundation at the common boundary with the reservoir and n is a unit vector perpendicular to the dam or foundation and inside of the fluid (Ghaemian and Ghobarah 1998 [8]). After inserting equations and boundary conditions, the model of the dam and reservoir system are analyzed using ANSYS software based on FEM (ANSYS User Manual 2007 [2]). In order to apply the effects of interaction, it is necessary to add the pressure load applied by the fluid onto the structure in interface boundary. Matrices of the reservoir components are also extracted by discretizing the wave equation. In the extraction of the matrices, velocities and accelerations are expanded as first and second order derivatives of displacements (Zienkiewicz and Bettess 1978 [19]).The discretized equations are solved using the ANSYS software and the results are extracted for the system.

3.4 Analysis

As a case study, a two-dimensional concrete gravity dam is selected with a height of 120 meters in accordance with Ghaemian and Gobarah (1998) [8]. The geometric properties along with the reservoir are given in Fig. 1, in which all dimensions are in meters. The Young modulus, density and Poisson coefficient of the concrete in the structure are 3.43GPa, 2400 kg/m^3 and 0.20 respectively. The velocity of compressive waves in water is 1438.66 m/s and its density is 1000 kg/m^3. For finite element model, the reservoir length is considered three times greater than the height of the dam. For the far-end, the Sommerfeld boundary condition is used and the time step is selected as $t = 0.02$ seconds as the input frequency. The PLANE 183 is

used to discretize the dam model and Fluid 29 element is used for water, which is a proper element for displaying the fluid compression property (ANSYS User Manual 2007 [2]). In this paper, the ANSYS standard version is used for modeling and analysis. It should be noted that the standard version that this software possesses, could apply to various boundary conditions and the effects of interaction between the dam, reservoir and foundation. To apply the interaction effect, the FSI command contained in this software has been used. The finite element discretization model is shown in Fig. 1.
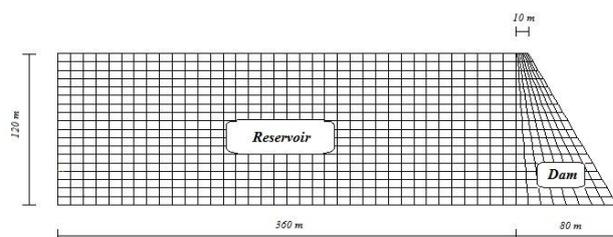

Fig. 1: dam model geometry and finite element discretization

$$a(t) = A * sin(\omega.t) \tag{10}$$

Where, $a(t)$ is the applied acceleration, A is the maximum acceleration, ω is the angular frequency and t is the time for dynamic analysis. To obtain appropriate answers in probabilistic analysis and achieve convergence in the LHS method, the analysis specifications are chosen as Table 1.

Table 1: Specifications of the probabilistic function used in the LHS method

Type of statistical distribution	Number of samples	Number of repetitions	Coefficient of dispersion
Gaussian	40	3	0.25

Regarding the mentioned cases, selecting the loading frequency is analyzed as an input variable in the LHS method and the results of probabilistic and sensitivity analyses are presented.

4. Results of analysis

After analyzing the selected model and given all of the mentioned conditions, the results of the analysis are presented in this section. In order to study the frequency effect on the concrete gravity dam, the responses of dynamic pressure on the reservoir bed, displacement of the dam crest, maximum value of tensile stress in the heel and compressive stress in the toe are studied. Then the probabilistic diagrams of four important and influential responses in the design of the dams are discussed and the probability of occurrence of each of the responses is

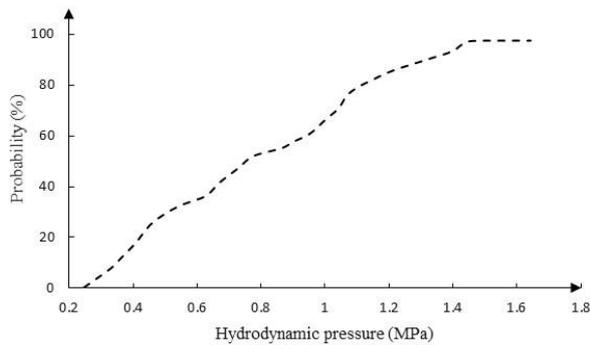presented in the form of a probabilistic graph in Figs. 2 to 5.

Fig. 2: The probabilistic distribution of the hydrodynamic pressure at the dam and reservoir interface in the bed
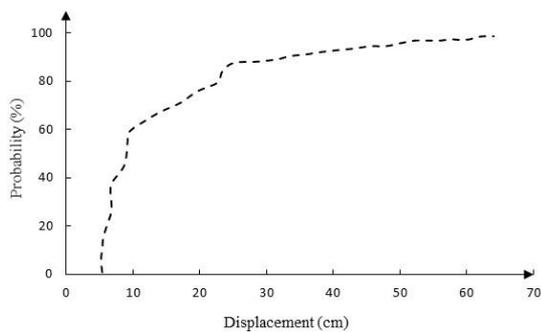

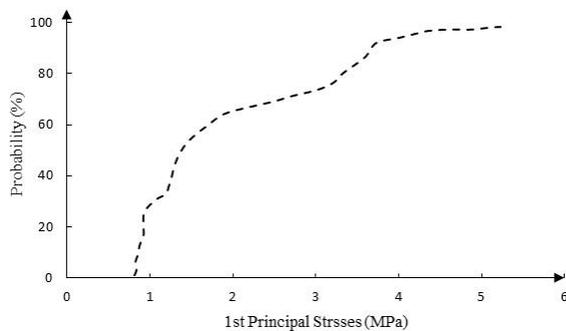**Fig. 3:** The probabilistic distribution of dam crest displacement

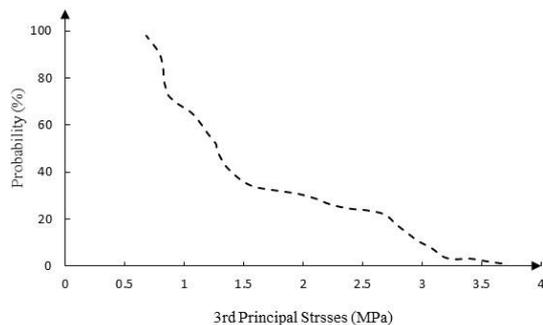**Fig. 4:** The probabilistic distribution of tensile principal stress in the heel

Fig. 5: The probabilistic distribution of compressive principal stress in the toe

Given the probabilistic diagrams displayed in the frequency range, the following results can be deduced:

- In all frequency values, the hydrodynamic pressure exceeds 200 KPa;
- The probability of dam crest displacement exceeding 5 cm is deterministic for all frequency values;
- The probability of 1.5 MPa tensile stress at the heel is about 50%;
- The probability of 1.25Mpa compressive stress at the toe is about 50%.

For precise presentation of the effect of the input frequency on dynamic analysis of the concrete gravity dam, the sensitivity diagrams of the structure vs. the input frequency are presented. Figs. 6 to 9 show the process of these changes.
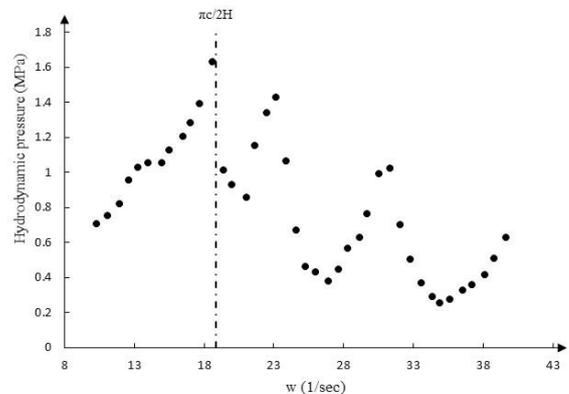

Fig. 6: Hydrodynamic pressure sensitivity at the dam and reservoir interface in the bed vs. the input frequency of the earthquake

Accordingly, the highest pressure occurs within the natural frequency range of the reservoir $(2n-1)\pi c/2H$, n=1,2,…), which should be considered in the analysis and design of concrete dams [3].
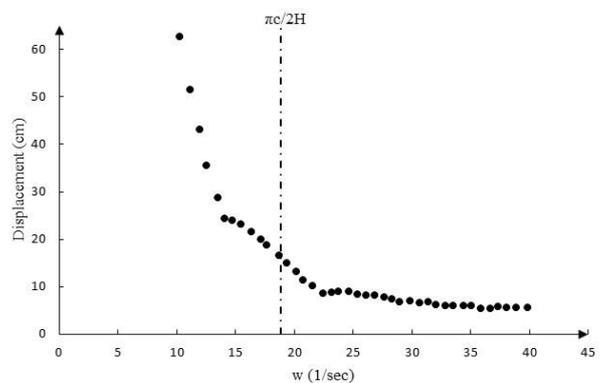

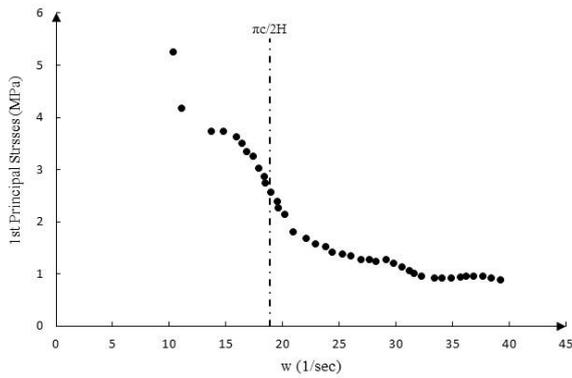**Fig. 7:** Dam crest displacement sensitivity vs. the input frequency of the earthquake

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Fig. 8: Sensitivity of tensile principal stress in the heel vs. the input frequency of the earthquake
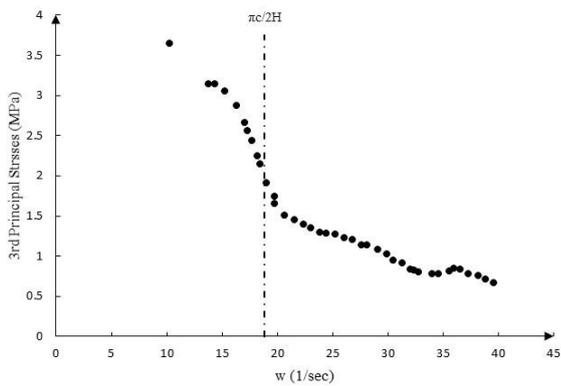

Fig. 9: Sensitivity of compressive principal stress in the toe vs. the input frequency of the earthquake

Accordingly, by increasing the input frequency in the force equation, the values of structural responses fluctuating in the hydrodynamic pressure decreases. This is very sensitive to the input frequency. For a better understanding of the effect of the input frequency in the sensitivity diagrams, a different frequency of structural responses is selected and shown in Table 2.

Table 2: Numerical values of maximum structural responses at different frequencies

Response	$w = 15 \ (\frac{1}{sec})$	$w = 20 \ (\frac{1}{sec})$	$w = 25 \ (\frac{1}{sec})$
Hydrodynamic pressure	1.05 MPa	0.92 MPa	0.49 MPa
Dam crest displacement	23 cm	13 cm	8.5 cm
Tensile principal stress	3.68 MPa	2.15 MPa	1.38 MPa
Compressive principal stress	3.05 MPa	1.50 MPa	1.27 MPa

5. Conclusion

In this research, the effect of earthquake frequency parameter on dynamic response of the concrete gravity dam is studied by LHS method using the finite element model. In the selected method, the Gaussian function has been used as a probabilistic function for probabilistic analysis. The frequency in the equation of acceleration of the motion of the system is considered as the input variable in the analysis. Considering all the conditions governing the problem and the interaction between the dam and the reservoir with ANSYS software, the concrete gravity dam was modeled two-dimensionally. The results show the interdependence between the earthquake frequency and natural frequency of reservoir. They indicate that critical conditions occur when the values of these frequencies get closer to each other. The obtained results illustrate that the highest pressure occurs in the near to *(2n-1)πc/2H* frequency, which is the natural frequency of the reservoir. Also it is obvious from results that in all frequency values, the hydrodynamic pressure exceeds 200 *KPa* and dam crest displacement exceeds 5 *cm*. The probability of 1.5 *MPa* tensile stress at the heel and 1.25Mpa compressive stress at the toe is about 50%.

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