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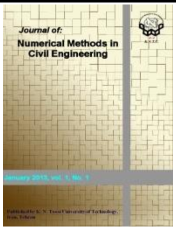

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Optimal number and location of parking facilities in the presence of autonomous vehicles

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Abstract:

Worldwide surveys have shown that autonomous vehicles will enter the transportation networks in the following decades. Therefore, investigating and analyzing the impacts of autonomous vehicles on traffic has been one of the most exciting issues. Autonomous vehicles affect facilities management, including parking location. For example, autonomous vehicles will change parking patterns. Conventional vehicle drivers must first find a spot to park their vehicle and then walk to their destination. In contrast, autonomous vehicle users can drop off right at their destination and do not experience walking or searching time for parking. Hence, modeling autonomous vehicles' effect on parking facilities' location is an important issue. This study seeks to present the optimal location of parking facilities in a mixed AV-HV traffic flow. We consider two structure layouts: (i) a corridor and (ii) a grid city. Also, we use the continuum approximation approach to model the problem and derive closed-form solutions. We prove that the demand (the infrastructure cost) increases (decreases) the required parking facilities. Numeric examples show that the share of autonomous vehicles decreases the number of parking facilities and the total cost.

1. Introduction

Finding parking is a major part of travel when travelers use personal vehicles. Travelers drive their vehicle from their origin to the nearest parking facility, then reach their destination by walking. The congestion and the cruising time (elapsed time to find a parking spot) increase because of insufficient parking spot numbers or inappropriate parking allocation. Shoup reports that more than 30% of the traffic jams are because of vehicles cruising for parking [1]. IBM indicates that drivers, on average, spend 20 minutes searching for a parking spot in a transportation network [2]. Also, the insufficient number of parking facilities causes illegal parking, leading to more accidents and less traffic speed [3]. In some countries, there are more parking spots than needed. For example, the area designated for parking in the US is about 6,500 square miles [4], but the inappropriate allocation of parking facilities causes traffic jams.

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We cannot build parking as much as to cover the transportation network because of its costs, emissions, and required land areas. Surveys determine that emissions from parking infrastructures cost the US between \$4B and \$20B annually [5]. Government has to pay approximately \$180K for one parking space in Hong Kong [4]. Moreover, we must designate significant lands to provide the required parking facilities. Litman reports that off-street parking covers 5-10% of the land in suburban areas, and in downtown areas, off-street parking covers 30-50% of the land area [6]. Previous studies indicate that building parking cannot individually decrease wasted time and may increase costs and emissions. Finding the optimal number and location of parking facilities is essential because it reduces costs paid by governing agencies, operators, and consumers [7].

In the last decade, autonomous vehicles (AVs) have attracted much attention and have gone from theoretical simulations to the emergence of real-world operations. Many car manufacturers such as Audi, Ford, GM, Toyota, Nissan, and Volvo are studying AVs in the testing phase [8]. Testing of AVs and AV pilot services is now existed in at least 36 states in the U.S. [9]. Generally, AV studies include various scopes such as vehicle testing, pilots, permits, and relevant state regulations [10]. Emerging AVs can change trip patterns of goods and travelers, vehicle crash rates, public transit usage, walking and cycling, and the need for automobile parking [10-11]. Hence, city officials need to invest in updating transportation infrastructures to adapt to AVs. For example, they may choose to implement sensors with bi-directional communication ability and re-evaluate on-street and off-street parking models [12].

Emerging AVs can change trip patterns significantly. For example, travelers who use conventional vehicles (CVs) have to drive from their origin to the nearest parking facility and then walk toward their destination. Whereas travelers that use AVs travel from their origin to their destination, then AVs drive from the destination to the nearest parking facility without a driver [8]. This pattern affects CV drivers, AV users, and decision-makers. Decision-makers do not have to build parking facilities in areas with high rent costs. AV users can save time, and CV drivers no longer need to compete for parking spots. Generally, AVs can change traffic flow and required parking space. In the presence of AVs, the time that travelers spend from their origin to their destination decreases; hence, passengers are willing to use AVs. Moreover, AVs can change the number of parking facilities and the average space per vehicle.

AVs can increase the capacity of parking spaces. For example, the average space assigned to each AV is decreased by 2 square meters in automated and intelligent parking systems [4]. Audi is trying to accomplish a parking pilot in Boston. Results predict that the required parking space can be saved approximately in AVs' presence [8]. Moreover, PARMAR highlights the advantages of AVs and their effects on urban environments, such as parking and land use [13]. Using AVs decreases the time travelers experience in their vehicles, so occupant-free vehicles can traverse more distance searching for parking away from their destination. The benefits of AVs motivate car manufacturers to focus on the effect of AVs on parking provision and providing intelligent and automated parking facilities. For example, Audi is investing in programs evaluating AVs' impact on land revitalization [4].

Several questions arise if AVs enter the transportation network. What effects do AVs have on the optimal location and the number of parking facilities? What is the relationship between the origin and demand distribution functions and the location of parking facilities? How does the network layout (corridor and grid) influence the problem modeling procedure? In this study, we analyze the problem and try to answer these questions. Moreover, the discrete modeling methodology and the continuum approximation (CA) approach are two different methods to model and solve transportation problems. In this paper, we distinguish between these methods and highlight the performance and benefits of the CA approach.

This study evaluates the parking facility location problem w.r.t. AVs' presence and analyzes the effect of some governing agencies' decisions on the optimal results. Most previous studies explore the corridor system layout [4] and ignore other system layouts (such as grid layout). We study both corridor and grid network layouts. At first, we present discrete equations and the optimization function. Then, we eliminate the complexity of the problem modeling procedure using the CA approach [14]. This approach considers some simplifying assumptions, but it provides meaningful insights and can investigate the impact of various variables and parameters on the social cost and the optimal number of parking facilities. Hence, the simplified model helps governing agencies make strategic decisions and analyze the effects of AVs. Also, governing agencies can pursue the best operational decisions.

We organize the rest of the paper as follows. Various scopes of studies are reviewed in Section 2, where we try to emphasize highly relevant studies. Sections 3 and 4 explore the corridor and the grid structure layouts, respectively. Section 5 confirms the presented equations and findings by numeric examples. Finally, section 6 summarizes the paper and provides some ideas for future works.

2. Literature Review

2.1 Facilities location using CA

In recent years, there has been an increasing tendency to the facility location problem. Previous studies considered the facility location problem w.r.t. the different assumptions, modeling procedures, and perspectives [15-17]. Facility location problems can be classified in different ways. One way is to classify the literature w.r.t. the modeling approach and present two classes: (i) studies that use a discrete approach and (ii) studies that use a continuous approach to model a facility location problem. The papers placed in the first may focus on finding the optimal locations of facilities [18-19]. Some others may consider allocation and routing strategies to minimize the total cost or maximize the network profit [20-22]. Although discrete models are essential and derive functional equations, but cannot export meaningful insights. Hence, continuous models are applied to derive closed-form solutions. In the following, we investigate the CA-based literature.

CA is a superseded approach to various problem modeling. This method was proposed for the first time by Newell [23-24], who replaced discrete parameters with continuous ones. CA is used in finding optimal long-term decisions. This method gives researchers some advantages over than discrete network modeling approach. First, it helps to derive closed-form solutions that give significant insights into the effect of decision variables on the objective function. Second, it eliminates the complexity of problem modeling by making assumptions about the network structure and the demand distribution pattern. For instance, Edrisi et al. assumed that origins and destinations are independently and uniformly distributed in a corridor [25]. The above explanations highlight the importance of the CA approach, so we explore the CA-based facility location literature in the following.

The CA-based facility location problems can be categorized into four general classes [26]. The first class is named classical facility location and deals with finding the number and location of facilities. Studies of the first class consider a set of deterministic and known assumptions. Some papers focus only on facility location, but others enter other operational considerations into the model. For instance, Wang et al. studied a corridor to solve an optimal location and pricing model [27]. Bouchery and Fransoo studied an allocation and location optimization problem to evaluate an inter-modal network's transportation cost and carbon emissions [28]. Ouyang et al. investigated a continuous network to optimize the location of facilities concerning traffic equilibrium [29]. They conducted numeric examples to compare the performance of discrete and approximate models. Byrne and Kalcsics considered a network (where a set of given facilities are present, and the demand is continuously and uniformly distributed) to find the optimal location of new facilities [30].

The second class of facility location problems examines the probability of facility disruption. In this situation, a facility may fail, and then its customers have to be served by other facilities. Cui et al. combined mixed-integer programming and the CA model to minimize total cost under normal and failure situations [31]. Lim et al. developed previous models by entering the probability parameter of disruption misestimation [32]. Bai et al. studied potential operational disruptions in bio-ethanol supply chains using discrete and continuous models [33]. The third class consists of papers that study facility location problems in a competitive environment. Dasci and Laporte assumed that demand distributes continuously to optimize the optimal facility density and compare different strategies such as leader and follower layout [34]. Wang et al. used the Stackelberg-Nash game to optimize the location of facilities in the biofuel and food industry [35].

The last class consists of papers that study the discretization of CA solutions to find the optimal location and service area assigned to each facility. Papers discretize CA solutions with the Voronoi diagram. Some studies applied the Voronoi diagram method to discretize continuous models and implement results for real cases [36-37]. Moreover, Ouyang and Daganzo presented a disk method to fix the location of facilities w.r.t. service area density derived from CA models [38]. Moreover, some studies considered a single source facility location to find the optimal assigned capacity to each facility and the optimal location of facilities [39]. They presented the discrete and continuous models, and the solutions were driven using an iterative metaheuristic approach and VNS-based metaheuristic technique. CA allows us to derivate closed-form models and provide meaningful insights for policy decision-makers, but integer-based models are limited in this capacity [40]. Moreover, CA release the complexities of dimensionality of integer programs. However, the application of CA in modeling parking location w.r.t. AVs is currently limited or non-existing.

2.2 Modeling of parking

Self-park capability can affect the performance and acceptance of AVs. For example, a survey declares that 43.5% of people believe that AVs' best advantage is their self-parking capability [41]. CV drivers try to find an on-street or off-street parking spot close to the destination. However, AVs carry their passengers to the destination and are then dispatched to farther and cheaper lots [42]. Hence, we explore the parking literature and AV parking models in the following.

Parking studies can be classified into different categories. He et al. studied the pricing aspect of parking [43]. They formulated a parking game model to solve the optimal parking space assignment model concerning prices. Liu and Geroliminis focused on the cruising-for-parking problem and presented a model to optimize the morning commute problem in a congested downtown network [44]. Nourinejad and Roorda used bilateral searching and meeting theory to evaluate parking enforcement policies in three different market regimes [3]. Other aspects such as parking competition, optimal parking control strategies, and commercial vehicle parking have been studied in detail [45-47].

The previous papers are not applicable to the AV parking problem because the parking pattern of AVs is different from CVs pattern. Nourinejad et al. indicated that car-park spaces assigned to CVs should have two rows of vehicles on each island, but those assigned to AVs can have multiple rows [4]. This type reduces infrastructure costs because the required parking space decreases. However, it can lead to blockage if the operator does not correctly relocate some vehicles. They optimized the number of vehicle relocations using a mixed-integer non-linear program. Nourinejad and Roorda studied the impact of AVs on cruising for parking in a mixed AV-CV traffic flow [48]. They considered a corridor with a uniform and linear parking supply function and showed that the travel costs of AVs and CVs decrease w.r.t. the AV penetration rate. Nourinejad and Amirgholy focused on a morning commute problem and introduced a new parking supply design scheme and temporal and spatial parking pricing strategies to optimize the system cost [49]. Also, some studies investigated the impact of shared AVs on parking behavior [50-52]. AVs can drop passengers at their destinations and go to cheaper parking spots. Exploring the previous studies shows little literature on this type [8,53-54]. Exploring the effect share of AVs on the optimal number and location of parking facilities is still a gap in the literature. Hence, we divide the problem into two structure layouts (corridor and grid) and analyze the effect of AVs on the optimal location, number of required parking, and the social cost involved.

2.3 Contribution of this paper

Although previous papers study the parking location, the presented models follow a network-based integer approach and are computationally exhaustive. Moreover, most of them do not consider the effect of AVs on the optimal number and location of parking facilities (to the best of our knowledge). In this paper, we enter the share of AVs parameter into the equations to investigate the effect of AVs. Previous papers that consider AVs concentrate on corridors and neglect grid structure layout.

Also, they focus on many-to-one demand patterns. We study corridor and grid structure networks and consider desired origin and destination distribution functions to generalize our research. CA is an alternative to facility location design, but its application in finding parking locations is limited in previous studies. This paper uses the CA approach to formulate the cost terms, consequently, the total cost term. This approach allows us to derive the service area for each parking facility which is rare in the parking location literature (to the best of our knowledge). We present heuristic algorithms for each system layout (corridor and grid) to compare the results of discrete solutions and CA findings. Hence, we can survey the precision and performance of the CA approach.

2.4 Problem definition

We assume that origin and destination are distributed independently. We consider two system layouts: (i) a corridor and (i) a grid network. We model the corridor in Section 3 and the grid network in Section 4. Table 1 summarizes the nomenclatures of the paper. In this paper, we consider three assumptions. We assume that demand is inelastic and insensitive to the cost of travel, passengers choose the nearest parking to park their vehicles, and we do not consider the capacity constraint. These assumptions are consistent with the CA literature.

Table 1: Nomenclature.

Parameters			
γ_d^C	Marginal cost of distance passed by CVs ($\$/km$)	γ_o^A	Marginal cost of distance passed by AVs between the origin and the destination ($/km)
γ_w^C	Marginal cost of distance passed by CVs ($\$/km$)	γ_f^A	Marginal cost of distance passed by AVs between the destination and the parking facility ($/km)
$O(X)$	Origin distribution function ($pass/km^2$)	$D(X)$	Destination distribution function ($pass/km^2$)
L	Length of corridor and city dimension (km)	$\overline{D}$	Total demand ($pass/hr$)
β	Share of AVs of total demand	P_i	Parking facility i
Variables			
C_d^C	Total distance passed by CVs (km)	C_o^A	Total distance passed by AVs between the origin and the destination (km)
C_w^C	Total distance walked by passengers (km)	C_f^A	Total distance passed by AVs between the destination and the parking facility (km)
F_{in}	Infrastructure and operating cost of parking facilities ($\$/hr$)	S_i	Region served by parking facility i (km^2)
TC	Total cost of the system ($\$/hr$)	N	Number of parking facilities

3. Corridor

3.1 Modeling

We assume that origins are distributed w.r.t. the origin function $O(x)$ and destinations are distributed w.r.t. the destination function ($D(x)$, where x is measured from the leftmost corner of the corridor. We assume that the origins and destinations are independent. A proportion β of the travelers use AVs, and the remaining $1-\beta$ drive CVs (we define β as the proportion of AVs to the total demand). There are N parking facilities placed at locations $X \in$

$\{x_1, x_2, \ldots, x_N\}$. We assume that travelers choose the nearest parking to their destinations. For better exposition, we present the mid-point of two parking facilities i and $i+1$ as:

$$x_i' = \frac{x_i + x_{i+1}}{2} \qquad \forall i = 1,2,\ldots,N-1, \quad x_0' = 0, \quad x_N' = L. \tag{1}$$

The influence area of each parking facility is the area that its demand is attracted by that parking facility. Using (1), the influence area of parking i is $S_i = [x_{i-1}', x_i')$.

The travel of CVs consists of two parts: (i) driving between origin and a parking facility that is the nearest to the destination, and (ii) walking from the parking facility to the destination (these two parts of travel are demonstrated in Figure 1a).

The mean distance that passengers drive to access parking i is:

$$C_{d,i}^C = \frac{\int_{x_{i-1}'}^{x_i'} |x - x_i| O(x)dx}{\bar{D}} = G(x_i), \tag{2}$$

which depends on the origin-distribution functions and the location of parking facilities. Hence, the total distance traversed by CVs can be written as:

$$C_d^C = (1-\beta)\sum_{i=1}^{N}(G(x_i)\int_{x_{i-1}'}^{x_i'} D(x)dx). \tag{3}$$

Passengers that use CVs walk from parking to their destination. Hence, the total distance walked from the nearest parking to their destination is:

$$C_w^C = (1-\beta)\sum_{i=1}^{N}(\int_{x_{i-1}'}^{x_i'} |x - x_i| D(x)dx). \tag{4}$$

AV passengers' path consists of two parts: (i) they drive from their origins to their destinations, and (ii) AVs drive independently from the destination to the nearest parking (Figure 1b). We note that the distance from the origin to the destination that passed by a desired AV is not depended to the number and the location of parking facilities. Hence, we calculate the total distance passed by occupant AVs as:

$$C_o^A = \frac{\beta \int_0^L |x - x_i| D(x) O(x)dx}{\bar{D}} = \beta \bar{L}. \tag{5}$$

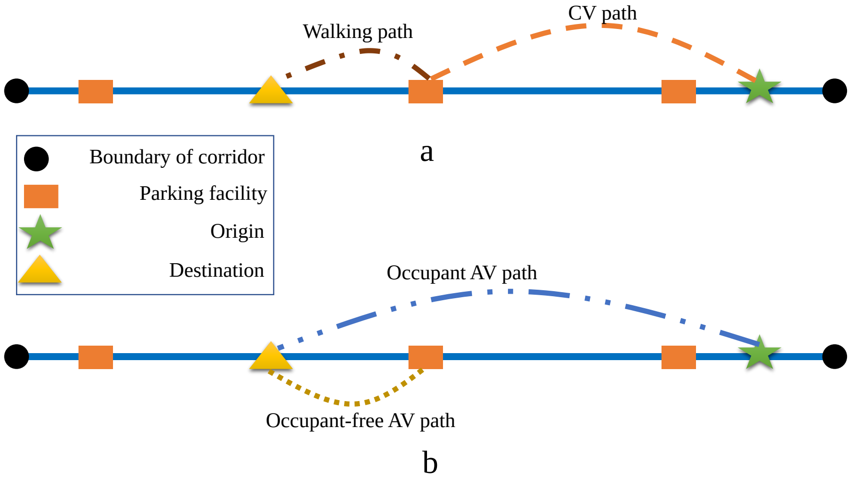


Fig. 1: a. CV trip pattern, and b. AV trip pattern in a corridor.

Note that the integral of the origin function or the destination function over the corridor is equal to the total demand. Hence, we input 1/D in (5) to avoid double demand considering. Occupant-free AVs drive from their passenger's destination to the nearest parking. The total distance passed by occupant-free AVs is:

$$C_f^A = \beta\sum_{i=1}^{N}(\int_{x_{i-1}'}^{x_i'} |x - x_i| D(x)dx). \tag{6}$$

In addition to transportation costs, we should find the parking facilities infrastructure and operating costs. Therefore, we present the fixed and operating costs of parking facilities as:

$$F_{in} = \sum_{i=1}^{N}(f(x_i) + a(x_i)\int_{x_{i-1}'}^{x_i'} D(x)dx + b(x_i)(\int_{x_{i-1}'}^{x_i'} D(x)dx)^{\alpha}). \tag{7}$$

Equation (7) is a relevant function in previous studies [55]. The first term indicates that the facility cost is related only to the location of the parking facility, and the second and third terms depend on the location and the attracted demand. The next step is to present the total cost function. We calculate the total cost as:

$$TC = F_{in} + \gamma_f^A C_f^A + \gamma_o^A C_o^A + \gamma_w^C C_w^C + \gamma_d^C C_d^C. \tag{8}$$

Note that γ_f^A, γ_o^A, γ_w^C, and γ_d^C are the marginal cost of distance passed by occupant-free AVs, occupant AVs, CVs, and passengers, respectively. We aim to find the location and the number of parking facilities as decision variables. We present the below heuristic algorithm to find the near-optimal solutions of (8) as:

- Step 1. We estimate the number of required parking facilities (N).
- Step 2. We generate a primary solution by distributing N facilities among the corridor and determining the service area of each facility using $S_i = [x'_{i-1}, x'_i)$.
- Step 3. We use meta-heuristic algorithms such as genetic or particle swarm optimization to find the optimal location of facilities by minimizing the total cost.
- Step 4. We repeat the process (Steps 1-3) for a set of number of locations.
- Step 5. We find the optimal location and the optimal number of parking facilities with respect to the total cost.

The presented algorithm is difficult to solve because we have no idea about the number of required facilities. Hence, we have to try different values for the number of parking facilities. Moreover, we cannot derive closed-form solutions. For example, suppose we want to investigate the impact of the share of AVs. In that case, we have to implement the presented algorithm for different values of β, leading to a time-consuming procedure. So, we use the CA approach to eliminate the complexities of the problem and present significant insights in Subsection 3.2.

3.2. CA approach-based modeling of corridor

The model presented in the previous subsection is non-convex and non-linear; hence, it is difficult to solve. Using CA, we rewrite the previous equations. The total distance passed by CVs can be rewritten as:

$$\begin{aligned} C_d^c &= (1-\beta)\sum_{i=1}^{N} G(x_i)S_iD(x_i) \\ &= (1-\beta)\sum_{i=1}^{N} G(x_i)S_iD(x_i)\int_{S_i}\frac{1}{S_i}dx \\ &= (1-\beta)\sum_{i}\int_{S_i}\frac{G(x_i)S_iD(x_i)}{S_i}dx \\ &= (1-\beta)\int_0^L \frac{G(x)S_s(x)D(x)}{S_s(x)}dx \\ &= (1-\beta)\int_0^L G(x)D(x)dx, \end{aligned} \tag{9}$$

where $S_s(x) = S_i,\ \forall x \in S_i$ and S_s is a step function. The above approach is relevant in the CA methodology and has been used in previous studies [26]. We suppose that the parameters vary slowly. Although, as pointed out by Daganzo [14], the CA approach gives acceptable results if large differences occur in close neighborhoods. Similar to Ansari et al. [26], we rewrite the total distance walked by passengers from the parking facility to the destination as:

$$C_w^C = (1-\beta)\int_0^L \frac{S_s(x)D(x)}{4}dx = (1-\beta)\int_0^L \frac{S(x)D(x)}{4}dx. \tag{10}$$

It has complexity yet to solve the objective function if we use $S_s(x)$. Hence, we replace the step function with the continuous function $S(x)$. Similarly, we rewrite the total distance passed by occupant-free AVs as:

$$C_f^A = \beta\int_0^L \frac{S(x)D(x)}{4}dx. \tag{11}$$

Using the same approach, (7) is replaced by:

$$F_{in} = \int_0^L \frac{f(x) + a(x)S(x)D(x) + b(x)\big(S(x)D(x)\big)^2}{S(x)}dx. \tag{12}$$

Placing (5) and (9)-(12) in (8), we get:

$$\begin{aligned} TC = \gamma_o^A\beta\bar{L} + \int_0^L \frac{f(x) + a(x)S(x)D(x) + b(x)\big(S(x)D(x)\big)^2}{S(x)}dx + (\gamma_f^A\beta + \gamma_w^C(1-\beta))\int_0^L \frac{S(x)D(x)}{4}dx \\ + \gamma_d^C(1-\beta)\int_0^L G(x)D(x)dx. \end{aligned} \tag{13}$$

The first term in (13) represents the total distance passed by occupant AVs and is independent of the number and the location of parking facilities. Hence, we remove it from the optimization process. Nevertheless, other terms are affected by the number and the location of facilities. We know that the integral minimizing is equal to the integrand minimizing at all points. So, we minimize the following equation instead of (13):

$$TC' = \frac{f(x) + a(x)S(x)D(x) + b(x)\big(S(x)D(x)\big)^2}{S(x)} + \frac{(\gamma_f^A \beta + \gamma_w^C (1-\beta))S(x)D(x)}{4} + \gamma_d^C (1-\beta)G(x)D(x). \tag{14}$$

Multiplying the first derivative of (14) by S^2, we get

$$S^2 \frac{\partial TC'}{\partial S} = \begin{cases} -f - (1-\alpha)bd^\alpha S^\alpha + \dfrac{\left(\gamma_f^A \beta + \gamma_w^C (1-\beta)\right)DS^2}{4} = 0 \quad for\ 0 < \alpha < 1 \\ -f + \dfrac{\left(\gamma_f^A \beta + \gamma_w^C (1-\beta)\right)DS^2}{4} = 0 \quad for\ \alpha = 1 \end{cases}. \tag{15}$$

We omit (x) in all variables to improve clarity. According to (15), we find that the optimal continuous service region function is related to the parameter α. For example, for $\alpha = 1$, S^* can be written as:

$$S^* = \sqrt{\frac{4f}{\left(\gamma_f^A \beta + \gamma_w^C (1-\beta)\right)D}}. \tag{16}$$

Proposition 1. *The infrastructure cost increases and the demand decreases the optimal continuous service area function.*

According to Proposition 1, the infrastructure cost increases the optimal continuous service area function. If the infrastructure cost increases, building so many parking facilities is not beneficial, and we have to decrease the number of parking facilities. Therefore, the service region designated for each facility increases. If the demand increases, the demand designated to each parking facility increases; consequently, (7) increases. Hence, we need to increase the number of facilities (consequently decreasing the facilities' service region) to reduce the number of passengers dedicated to each parking facility.
Similarly, we calculate the optimal number of parking facilities as:

$$N^* = \int_0^L \frac{1}{S^*} dx. \tag{17}$$

Proposition 2. *The share of AVs affects the optimal number of parking facilities w.r.t. γ_f^A and γ_w^C.*

We find that the share of AVs affects the optimal continuous service region function, consequently, the optimal number of parking facilities. It is a logical assumption that the marginal cost of distance walked by passengers is higher than the marginal cost of distance passed by AVs between the destination and the parking facility. Hence, we can conclude that the share of AVs decreases the optimal number of parking facilities. Using $S_i \int_{S_i} D(x)dx$, we can calculate the assigned demand for the parking facility placed at x_i if we follow the discrete approach. Following the CA approach, we calculate the optimal continuous assigned demand function as:

$$P^* = S^* D = \sqrt{\frac{4fD}{\left(\gamma_f^A \beta + \gamma_w^C (1-\beta)\right)}}. \tag{18}$$

Proposition 3. *The infrastructure cost and the total demand increase the optimal continuous assigned demand function.*

The infrastructure cost increases (decreases) the optimal service region (the optimal number of parking facilities). Also, the demand decreases the optimal service region. Consequently, it causes an increase in the optimal number of parking facilities. In this section, we formulate the one-dimensional space. The following section models a city with a grid structure (two-dimensional space).

4. Grid Network

4.1 Modeling

We have two distance metrics in a two-dimensional space: (i) Euclidean and (ii) Manhattan. We consider a city with a grid structure ($L * L$) and calculate the distance between two random points according to the Manhattan metric. As explained in the previous section, the trip pattern is different for CVs and AVs. Figure 2a shows the CVs trip pattern that consists of C_d^C and C_w^C. Figure 2b shows the AVs trip pattern that consists of C_o^A and C_f^A. Similar to the previous section, we assume that there are N parking facilities and $O(x,y)$ and $D(x,y)$ are the origin and destination distribution functions, respectively. If (x_i, y_i) is the location coordinate of the parking facility i, we can write the average distance passed by CVs from the origin to parking facility i as:

$$C_{d,i}^C = \frac{\int_0^L \big||x - x_i|\big| O_x(x)dx}{\overline{D}} + \frac{\int_0^L \big||y - y_i|\big| O_y(y)dy}{\overline{D}} = G(x_i, y_i). \tag{19}$$

Note that $O_x(x)$ and $O_y(y)$ are the origin distribution functions along x and y axes, respectively. Equation (19) is dependent only on the location of the parking facility and the origin function. For example, if origins are distributed uniformly over the city, we can rewrite (19) as:

$$C_{d,i}^{C} = \frac{x_i^2 + (L - x_i)^2 + y_i^2 + (L - y_i)^2}{2L}. \qquad (20)$$

The total distance passed by CVs can be written as:

$$C_d^C = (1 - \beta)\sum_{i=1}^{N} (G(x_i, y_i) \int_{S_i} D(X)dX). \qquad (21)$$

Note that S_i is the service area of parking facility i and is defined as:

$$S_i = \{X| \ \|X - X_i\| < \left\|X - X_j\right\|\} \qquad for\ i \neq j,\ \ j \in N. \qquad (22)$$

The next step is to define the total distance walked by passengers from parking facilities to destinations as (23).

$$C_w^C = (1 - \beta)\sum_{i=1}^{N} \int_{S_i} \|X - X_i\| D(X)dX. \qquad (23)$$

The total distance passed by AVs from origins to destinations is calculated as (24).

$$C_o^A = \frac{\beta \int_S \|x - x_i\| O_x(X) D_x(X)dX + \int_S \|y - y_i\| O_y(Y) D_y(Y)dY}{\bar{D}} = \beta\bar{L}. \qquad (24)$$

We recall that (24) seems complicated, but we can derive it easily if we know the origin and destination distribution functions. For example, if the origins and destinations distribution functions are uniformly distributed among the city, (24) is derived as:

$$C_o^A = 2\beta\bar{D}L/3. \qquad (25)$$

Similarly, the total distance passed by occupant-free AVs is:

$$C_f^A = \beta \sum_{i=1}^{N} \int_{S_i} \|X - X_i\| D(X)dX. \qquad (26)$$

We can find the infrastructure and the total costs by updating (7) and (8) for two-dimensional space. However, finding the optimal number and the location of parking facilities is difficult. The biggest challenge is how to find the service areas of facilities. Some studies use the Voronoi diagram to calculate the service region and, consequently, the service area of facilities [37]. We replace the city $(L * L)$ with a mesh grid $(m * m)$. Each node in the mesh grid is equivalent to a point in the city. To solve the model, we present the following heuristic algorithm:

- Step 1. We divide each city dimension into m segments, and we have a mesh grid $(m * m)$ that covers the city $(L * L)$.

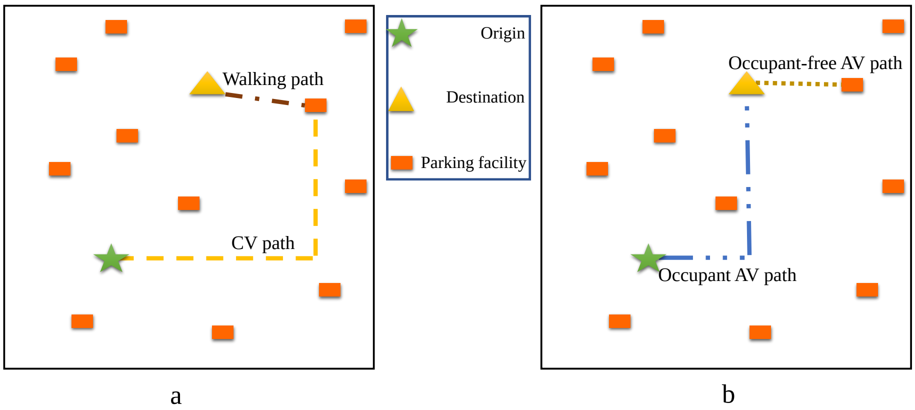


Fig. 1: a. CV trip pattern, and b. AV trip pattern in a grid network.

- Step 2. We estimate the number of required parking facilities (N) and generate a primary solution by distributing N facilities throughout the city.

- Step 3. We assign each node in the mesh grid to the nearest parking facility. Hence, we can find each service area.

- Step 4. We use meta-heuristic algorithms such as genetic or particle swarm optimization to find the optimal location of facilities by minimizing the total cost.
- Step 5. We repeat the process (Steps 2-4) for a set of number of locations.
- Step 6. We find the optimal number and the location of parking facilities.

The presented algorithm is complicated and challenging to solve. Because we have no idea about the number of required facilities, we have to try different values for the number of parking facilities. Similar to the previous section, we reformulate the problem using CA in the following subsection.

4.2. CA approach-based modeling of grid network

In this subsection, we eliminate the complexity of the problem using CA. We reformulate (21) as:

$$C_d^C = (1-\beta)\int_S G(x,y)D(x,y)dxdy. \quad (27)$$

Similarly, (23) is rewritten as:

$$C_w^C = (1-\beta)\int_S KD(x,y)\sqrt{S(x,y)}dxdy. \quad (28)$$

Note that K is a constant that depends on the distance metric and the shape of the service region. Previous studies suggest different values for K. In this paper, K is 0.454 [26]. Similarly, the total distance passed by occupant-free AVs is:

$$C_f^A = \beta\int_S KD(x,y)\sqrt{S(x,y)}dxdy. \quad (29)$$

The infrastructure and total costs can be reformulated similarly to Subsection 3.2. Using the same approach, we find the optimal continuous service area function for a two-dimensional space as (30). Note that α affects the continuous service region function as shown in (30). For example, for $\alpha = 1$ and $\alpha = 3/4$, S^* can be written as (31).

$$S^2\frac{\partial TC'}{\partial S} = \begin{cases} -f-(1-\alpha)bd^\alpha S^\alpha + \frac{\left(\gamma_f^A\beta+\gamma_w^C(1-\beta)\right)DK}{2}S^{3/2} = 0 \quad for\ 0<\alpha<1 \\ -f+\frac{\left(\gamma_f^A\beta+\gamma_w^C(1-\beta)\right)DK}{2}S^{3/2} = 0 \quad for\ \alpha = 1 \end{cases}. \quad (30)$$

$$S^* = \begin{cases} \left(\frac{\upsilon}{\phi}\right)^{\frac{4}{3}}(1+\sqrt{1+\frac{2f\phi}{\upsilon}})^{4/3},\ \upsilon = \frac{bd^{3/4}}{4},\ \phi = \left(\gamma_f^A\beta+\gamma_w^C(1-\beta)\right)DK,\ for\ \alpha = 3/4 \\ \left(\frac{2f}{\left(\gamma_f^A\beta+\gamma_w^C(1-\beta)\right)DK}\right)^{2/3},\quad for\ \alpha = 1 \end{cases}. \quad (31)$$

Proposition 4. *The origin distribution function does not affect the optimal continuous service region function.*

According to (21) and (27), $G(x,y)$ represents the origin distribution function, but the CA approach removes $G(x,y)$ from (30). Hence, the origin distribution function cannot affect the continuous service area function.

5. Numeric Examples

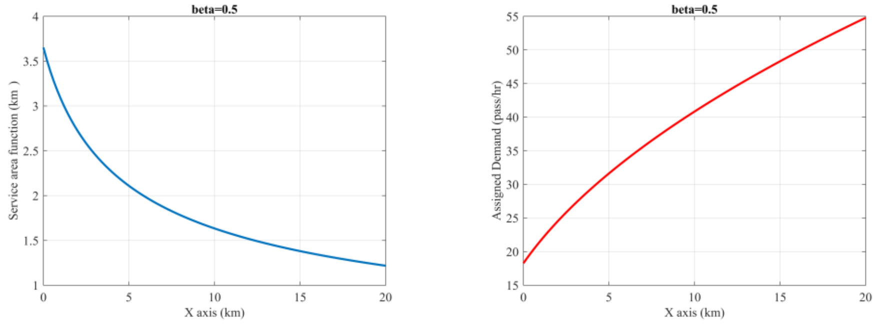
5.1 Corridor

This section explores the presented equations and findings by numeric examples. Input parameters are given in Table 2. The length of the corridor and the total demand are 20 km and 500 $pass/hr$, respectively. Input data show that the walking distance passed by passengers is more important than the distance passed by CVs and AVs, which is logical. Figure 3a shows that the service area function decreases along the axis ($\beta = 0.5$). The destination distribution function increases along the axis. Hence, the service region of parking facilities should decrease. Figure 3b shows that the assigned demand for parking facilities increases along the corridor. For example, if a parking facility is located at $x = 20\ m$, its capacity should be at least 55 $pass/hr$ to serve the assigned demand. However, while the service area function decreases along the x-axis, the assigned demand function increases. The result is because of the destination distribution function that increases along the axis. Hence, $S(x)D(x)$ increases.

Table 2: Input parameters.

β	[0,1]	γ_d^C	2 ($\$/km$)
L	20 km	γ_w^C	5 ($\$/km$)
$D(x)$	$100/L + 800x/L^2(pass/hr)$	γ_o^A	2 ($\$/km$)
$f(x)$	100 ($\$/hr$)	γ_f^A	1 ($\$/km$)

In Figure 4a, the results show that the share of AVs increases the service area function. The input data show that CVs' parameters are greater than AVs' parameters and that β decreases the effects of CVs' parameters; hence, the results are acceptable. As the service area increases, the region served by a random parking facility increases, so we need fewer parking facilities to cover the corridor. Figure 4b confirms the results. We note that the results derived from figures can be changed if we vary the input parameters.



a b

Fig. 3: a. Service area function, and b. Assigned demand function in the corridor layout.

The next step is to evaluate the effect of β on the total cost. Fig. 5a shows that β decreases the total cost.

Hence, we can conclude that increase in the share of AVs is a good option for society. We compare the CA and the discrete approaches for β in Figure 5b. Figure 5b shows that the service areas assigned to parking facilities by the CA method are approximately the same as those assigned by the discrete approach. We observe an average 0.2% difference between the calculated service area by the CA approach and the calculated service area by the discrete approach, which is negligible. This result highlights the performance of the CA approach because we can derive the optimal results in a short time and with acceptable accuracy.

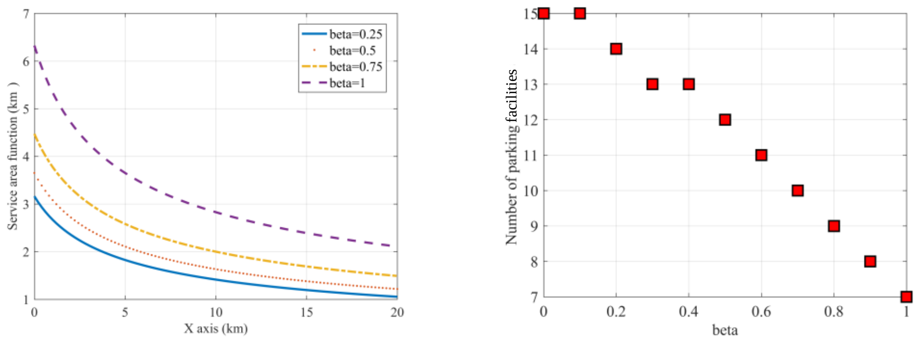


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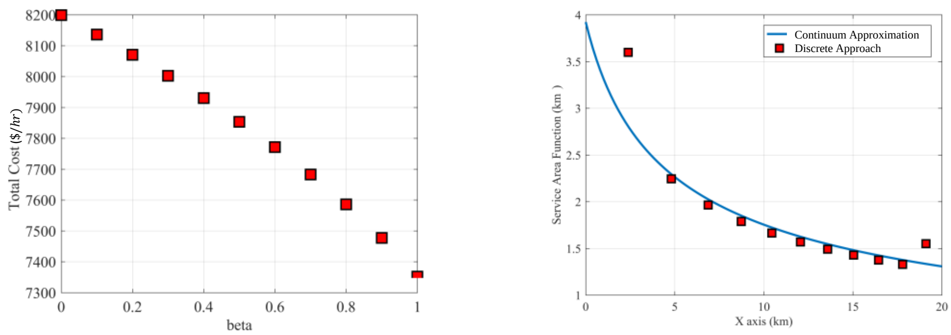
Fig.4: Effect of β on a. Area function, and b. Number of parking facilities in the corridor layout.



a b

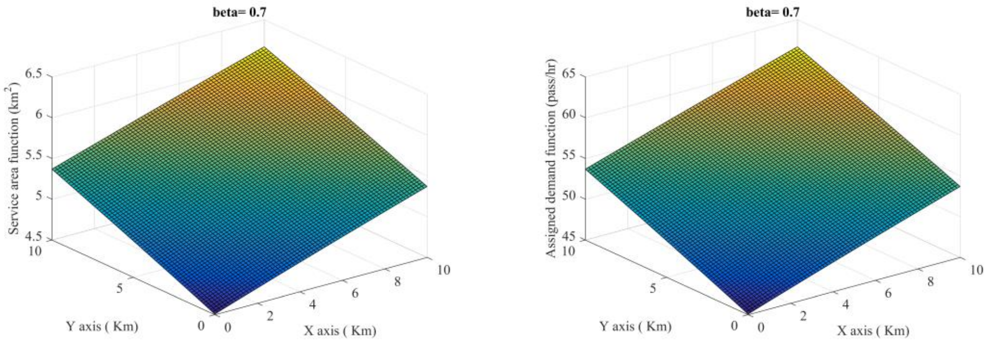
Fig. 5: a. Effect of β on the total cost, and b. Comparison of the results of CA and discrete approaches in the corridor layout.

5.2 Grid network

In this subsection, we study a grid city ($10 * 10\ km$). We use input data but define the destination distribution function as D/L^2 and the infrastructure and operating cost as $50 + 30(x+y)/(2L)$. Figure 6a shows that the service area function increases along the axes. Equation (31) indicates that the service area behaves like the infrastructure and operating cost. Hence, the results of Figure 6a are reasonable. Also, the assigned demand function behaves like the service area and the infrastructure and operating cost (as shown in Figure 6b).

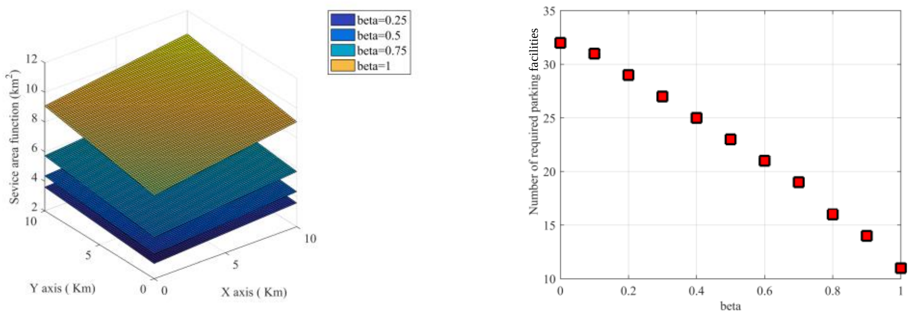
We now determine the impact of β on the service area and the number of parking facilities. Figure 7a shows that β increases the service area function. This result indicates that β decreases the number of required parking facilities (as shown in Figure 7b).

The numeric example proves that AVs decrease the total cost (shown in Figure 8a). For $\beta = 1$, we solve the problem using the discrete approach. Figure 8b shows the optimal location of parking facilities and their service regions. We find an averagely 6% difference between the calculated service area by the CA approach and the calculated service area by the discrete approach, which is negligible. For $\beta = 1$ and $N = 11$, the total cost of the discrete approach is 15540 ($\$/hr$), and the total cost of the CA approach is 15478 ($\$/hr$). Therefore, there is an average 0.4% difference between the calculated total cost derived from the CA approach and that derived from the discrete approach. This result highlights the power and the performance of the CA approach.



a b

Fig. 6: a. Service area function, and b. Assigned demand function in the grid city layout.



a b

Fig. 7: Effect of β on a. Service area function, and b. Assigned demand function in the grid city layout.

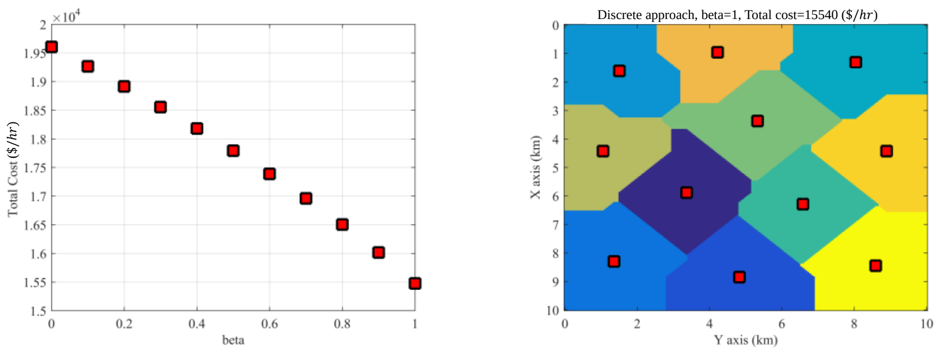


Fig. 8: Effect of β on total cost, and b. Optimal location of parking facilities in the grid city layout.

6. Conclusion

The present study considers the presence of AVs and AVs simultaneously and finds the optimal location and the optimal number of parking facilities. We consider two system layouts: (i) a corridor and (ii) a grid city. We calculate the distance between points according to the Manhattan metric in the grid city. To address the problem, we present the general form of the problem and then use the CA approach to investigate the problem.

To generalize the defined problem, we define origin and destination distribution functions independently. At first, equations are formulated, and a heuristic algorithm is presented for the corridor and the grid city. Then, we replace the cost terms using CA to derive closed-form solutions.

The numeric examples show that the origin distribution function does not affect the optimal service area. However, the destination distribution and the infrastructure and operating cost functions affect the optimal service area. The AV ratio decreases the total cost and the required parking facilities. Moreover, we find the problem's solution using the presented heuristic algorithm and show that the CA approach derives the total cost and the service area of facilities as close as possible to the results of the discrete approach. This result highlights the power and the performance of CA.

We present some recommendations for future works. This paper investigated a corridor and a grid city. Future research is recommended to study the parking facilities location problem in a ring-radial city. This study neglects the cruising for parking and on-street parking. Other studies can enter these patterns into the equations to achieve a more realistic analysis.

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