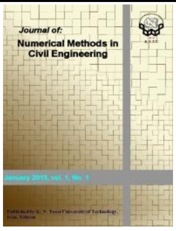

Numerical Methods in Civil Engineering

Buckling optimization of elastic IPE and circular sections columns considering boundry condition effects

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ARTICLE INFO

Article history:
Received:
November 2016
Revised:
April 2017
Accepted:
May 2017

Keywords:
Optimum design
Buckling
Resistance
Eigenvalue
Sensitivity number
Elastic column

Abstract:

The theory of optimization has improved remarkably during the last four decades. The main part of optimizing investigations has been focused on enhancing buckling resistance which does not violate the economic feasibility of final design. The finite element analysis which is called ESO (Evolutionary Structural Optimization) is presented for optimum (or most ideal) design of columns to increase the buckling resistance of structures. To attain the significant design variables, this method can be employed to choose an appropriate, affective and economical way. During an iterative process, the above approach ensures the attainment of global maximum critical load under the imposed equality volume constraint, type of boundary conditions and type of cross sections. Precise results and numerical examples have been shown and useful diagrams have been developed for the cases of simple, clamped and clamped free supported by different types of cross-sectional areas. The model has succeeded in arriving at the global optimal column designs possessing the absolute maximum buckling load without violating the economic feasibility requirement. As a matter of fact, the cross sectional area of column changes whereas, the total volume of column remains constant. As a result, the buckling forces increase. According to this study critical buckling load of columns decreases by changing the boundary condition from clamped to clamped free and then simply supported.

1. Introduction

Optimizing the buckling constraints problems is complicated since two factors, namely, analysis and eigenvalue solution, have to be considered at each step.

In this research, many optimization procedures were carried out on steel frames [1-2]. By focusing on a single column with different boundary conditions and trying to enhance the buckling resistance of the column under varying situations.

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The main aim of this investigation is to maximize the buckling load to 3 percent higher than the uniform column, by redistributing the isotropic column cross sectional area.

According to research, other scientists have investigated optimization of the buckling load by using energy approach for continuous structural models or discretized models by finite element method [3]. Spillers and valley (1990) developed Keller's classic solution for optimal design of columns [4]. The application of ESO (Evolutionary Structural Optimization) method on stiffness optimization and frequency optimization of plates was presented by Chu et al. (1996) and Manickarajah et al.(1995) [5-6]. Xie and Steven (1997) applied the ESO method to optimum design of buckling resistance of frames; he applied this approach with an iterative optimization on planer frames in order to avoid buckling under the equality of volume constraint [7].

Ruocco et al. (2016) studied the internal rotational spring stiffnesses and developed Hencky bar-chain model for buckling analysis of non-uniform column [8]. Godat et al. (2012) investigated the stability of local buckling behavior of tubular polygon columns under concentric compression through experimental tests consisting of six stub columns with three different cross-sections [9]. Li et al. (2011) investigated the stability of composite columns and parameter optimization against buckling [10].

The local modification of every element is handled by gradually shifting the material from the strongest to the weakest part of the structure. A comprehensive discussion on the ESO method has been presented in a recent book by Xie and Steven (1996) [11].

Scholars have pursued a precise approach of distinctly expressing the optimization problem and defining sensitivity numbers based on sensitivity analysis of the objective function in most of the recent investigations on BESO. For example, one can refer to Huang and Xie (2007, 2009, 2010a, b) [12-15], Ghabraie (2009) [16], Ghabraie et al. (2010) [17], and Nguyen et al. (2014) [18]. To the best knowledge of the author, despite the visible similarity between ESO and BESO, no convincible mathematical solution has been suggested for ESO.g. and the solution by Tanskanen (2002) is not justifiable [19]. The nobility of this research is concentrating on the fact that it considers the effect of various boundary conditions and the number of elements simultaneously with regards to column optimization process.

2. The Eso Method For Buckling Optimization

ESO method is an iterative method for redistribution of column cross-sectional area, through the use of two sensitivity numbers, which are calculated for each element of a column by using finite element model.

The sensitivity numbers are used to assess the effects of small changes in thickness of some elements on the critical value of the buckling load factor. During the optimization process the main task is to raise the critical buckling eigenvalue. In applying these changes, the structural volume is constrained to remain constant causing the higher eigenvalues to decrease simultaneously.

To increase the buckling resistance of a column the cross sectional area is increased in the elements with the greatest sensitivity to a thickness increase and reduced in those with the greatest sensitivity to a thickness decrease. In applying these changes, the segment volume remains constant. This iterative procedure shifts the material gradually from the strongest part to the weakest part.

3. FEA Analysis

In order to increase the buckling resistance of a column, it is necessary to calculate measures of the sensitivity of critical load factor λ to an increase or decrease in the variable dimension of an arbitrary finite element by redistributing the cross sectional area,. The measures are obtained by quantities that are defined as sensitivity numbers. The sensitivity numbers are calculated from Eq(1):

$$\left([K]-\lambda_j\left[K_g\right]\right)\{u_j\}=0 \tag{1}$$

The symbols [k], $\left[K_g\right]$, λ_j and $\{u_j\}$ denote the global stiffness matrix, the global geometric stiffness matrix, the jth eigenvector, respectively.

The value of critical buckling load is the smallest positive eigenvalue λ_j for compression-loaded columns. By multiplying Eq. (1) By the transpose of eigenvector $\{u_j\}$; i.e.

$$\lambda_j=\frac{\{u_j\}^T[K]\{u_j\}}{\{u_j\}^T\left[K_g\right]\{u_j\}} \tag{2}$$

$$\frac{\partial\lambda_j}{\partial x}=\frac{\{u_j\}^T\left(\frac{\partial[K]}{\partial x}\left(\{u_j\}^T\left[K_g\right]\{u_j\}\right)-\frac{\partial\left[K_g\right]}{\partial x}\left(\{u_j\}^T[K]\{u_j\}\right)\right)\{u_j\}}{\left(\{u_j\}^T\left[K_g\right]\{u_j\}\right)^2} \tag{3}$$

$$\frac{\partial\lambda_j}{\partial x}=\frac{\{u_j\}^T\quad\frac{\partial[K]}{\partial x}\quad\lambda_j\frac{\partial\left[K_g\right]}{\partial x}\quad\{u_j\}}{\left(\{u_j\}^T\left[K_g\right]\{u_j\}\right)^2} \tag{4}$$

By normalizing the eigenvectors with respect to global geometric stiffness matrix such that Eqs. (4) and (5) reduces:

$$\{u_j\}^T\left[K_g\right]\{u_j\}=1 \tag{5}$$

The sensitivity numbers are defined by considering a small change in the ith segment dimension.

$$\Delta\lambda_j=\{u_j\}^T\left([\Delta K]\quad\lambda_j\left[\Delta K_g\right]\right)\{u_j\} \tag{6}$$

The change in the global stiffness matrix [Δk] in Eq.(6) is equal to the change in the matrix stiffness of ith element.

If the modification to each element is kept sufficiently small, $\left[\Delta K_g\right]$ can be neglected. In this case, the change in the jth element eigenvalue caused by modificating ith element stiffness, is given by Manickarajah et al.(1998).

$$\Delta\lambda_{ij} = \{u_j\}^T([\Delta K]_i)\{u_j\} \tag{7}$$

The sensitivity numbers are obtained directly from Eq (4). For an increase in dimension of Δt in ith element:

$$[\Delta K_i] = [\Delta K_i]^+ = [K_i(t+\Delta t)] - [K_i(t)] \tag{8}$$

And also, for a reduction in dimension of Δt in ith element:

$$[\Delta K_i] = [\Delta K_i]^- = [K_i(t-\Delta t)] - [K_i(t)] \tag{9}$$

Hence, to estimate the effect of dimension changes on the critical load factor, the following two sensitivity numbers need to be calculated for each element:
The term for dimension increase:

$$\alpha_i^+ = \{u_1\}^T[\Delta K_i]^+\{u_1\} \tag{10}$$

And the other for dimension reduction:

$$\alpha_i^- = \{u_1\}^T[\Delta K_i]^-\{u_1\} \tag{11}$$

4. Optimization Procedure

An iterative/repetitive procedure is used for resizing the cross-section dimension, so that the material is gradually shifted from the strongest part of the structure to the weakest part. In this method, two parameters that are included, resizing ratio of element dimensions and steps of this change, are defined before starting the process for increasing the accuracy and convergence of algorithm, while chosen parameters are usually kept constant, throughout the optimization process.

$$\begin{aligned}&\max(\min(|\lambda_i|, i = 1,\ldots\ldots\ldots\ldots, m))\\&subject\{volume = cte\}\end{aligned} \tag{12}$$

The iterative procedure is given as follows:

Step1. Determining the geometric and mechanical properties and discretizing the column by using a fine mesh of finite elements. The mesh should be adequate for representing the buckling stress distribution and the buckling mode.

Step2. Solve the buckling eigenvalue problem and corresponding eigenvector.

Step3. Calculate the sensitivity numbers α_i^+ and α_i^- for each element.

Step4. Increase the dimension of element with the greatest value of α_i^+ and decrease the dimension of the same number of elements with the greatest value of α_i^-.

Step5. Repeat steps 2-4 until the difference between critical buckling loads in two successive stages is kept constant.

In general, the accuracy of the optimum solution is improved with a smaller resizing ratio of cross-section dimension and a smaller step size, but at the expense of higher computational costs. It can be pointed out that different values of resizing ratio and step size can be used at different stages of the optimization process but, in order to achieve the accurate design, it would be better to make these parameters smaller at final stages or keep them constant.

5. Numerical Examples

In this section three examples of column with different type of cross-section and variety of boundary conditions are presented. All of these cases have 10 elements. 10% of the elements are subjected to resizing by 0.5cm step size at each iteration. In fact, before optimisation, the cross section of all elements was considered uniform and it changes during optimization procedure.
The optimum designs for simply supported column with rectangular cross-section (4×5cm) and length of 400cm are shown in Figures 1 and 2.

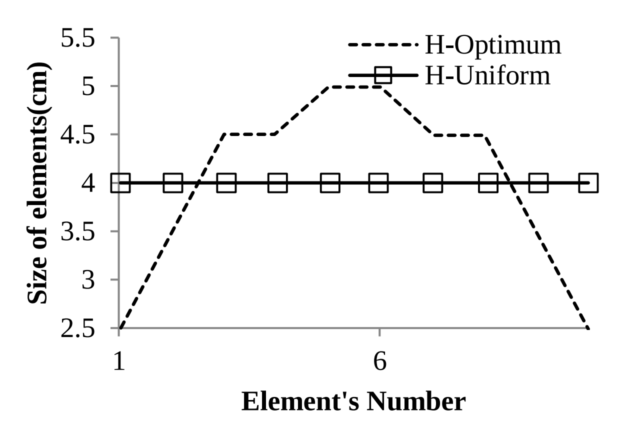

Fig.1: Optimum shapes of simply supported column.

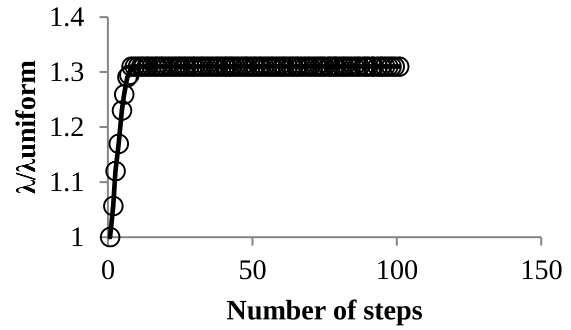

Fig.2: Optimization histories of simply supported column.

According to figures 1 and 2, 100 iteration is needed for achieving to desired shape. In optimal geometry, the width of cross section in middle elements increases to maximum 5 cm while it decreases to 2.5 cm in two end elements.

The next case is for clamped column with circular cross-section (r =2.5cm) and length of 400cm which is shown in Figures 3 and 4. Here again radius of cross section changes for different element during 40 optimization cycles. These changes vary from 2 cm to maximum 3 cm in different elements. Finally, the clamped free column with IPE cross section (h=160 mm) and length of 400cm is shown in Figures 5 and 6.

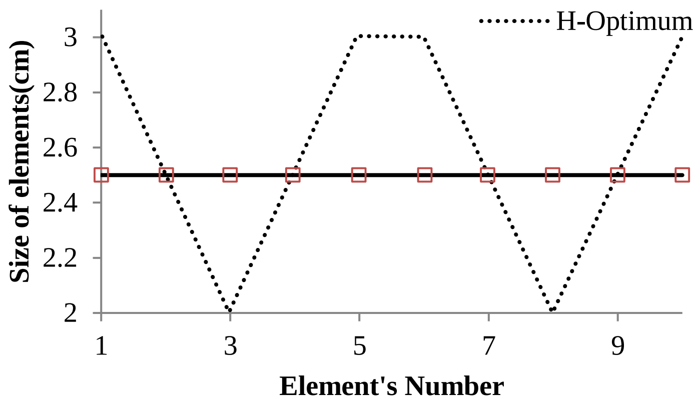

Fig.3: Optimum shapes of clamped column.

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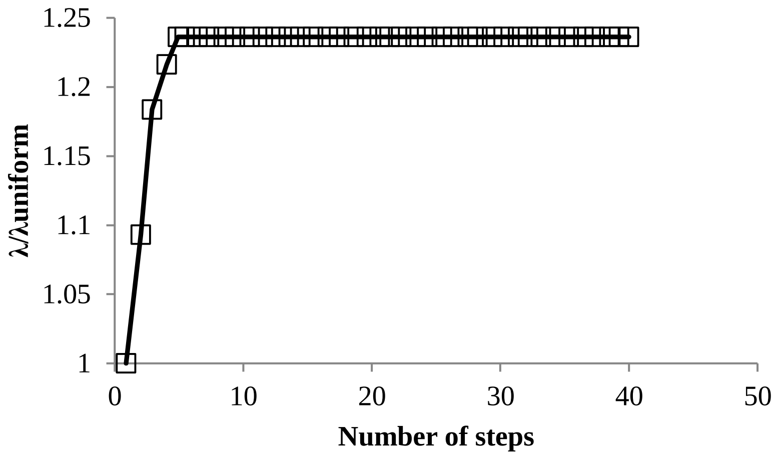

Fig.4: Optimization histories of clamped column.

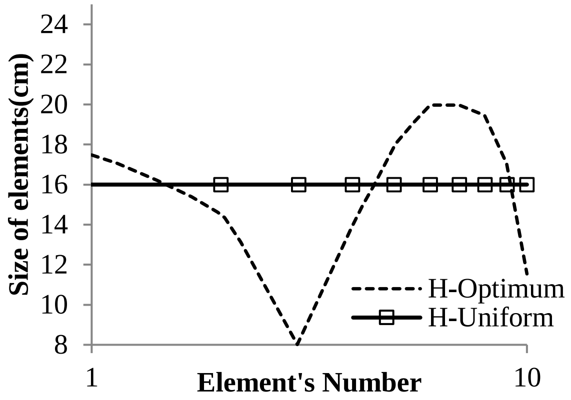

Fig.5: Optimum shapes of clamped free column.

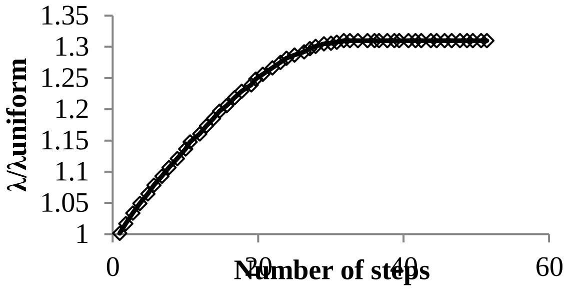

Fig.6: Optimization histories of clamped column.

Table.1: P_{cr} for the circular cross-section and simply supported column (R=2.5 cm, A=20 cm2)

Pcr	Number of iterations	$\frac{\lambda}{\lambda_{unf}}$,N=40	P_{cr} (kg/cm^2)	Number of iteration	$\frac{\lambda}{\lambda_{unf}}$,N=10	Step size	Resizing ratio
5.6639e3	23	1.4252	5.3640e3	11	1.3497	0.2	10%
5.5418e3	10	1.3944	4.9593e3	4	1.2479	0.5	10%
5.6359e3	9	1.4233	5.2969e3	7	1.3368	0.2	50%
5.4412e3	6	1.3691	4.7676e3	5	1.2036	0.5	50%

Table.2: P_{cr} for IPE cross-section and simply supported column (A=20 cm2 , IPE160)

Pcr	Number of iterations	$\frac{\lambda}{\lambda_{unf}}$,N=40	P_{cr} (kg/cm^2)	Number of iteration	$\frac{\lambda}{\lambda_{unf}}$,N=10	Step size	Resizing ratio
5.0226e3	170	1.3916	4.7904e5	77	1.3213	0.2	10%
5.0483e3	73	1.3912	4.7831e5	31	1.3193	0.5	10%
5.0469e3	55	1.3928	4.7939e5	34	1.3222	0.2	50%
5.0433e3	23	1.3911	4.7815e5	17	1.3210	0.5	50%

Table.3: P_{cr} for the circular cross-section and clamped supported column (R=2.5 cm, A=20 cm2)

Pcr	Number of iterations	$\frac{\lambda}{\lambda_{unf}}$,N=40	P_{cr} (kg/cm^2)	Number of iteration	$\frac{\lambda}{\lambda_{unf}}$,N=10	Step size	Resizing ratio
2.0882e4	21	1.3253	2.0664e4	11	1.3251	0.2	10%
2.0128e4	23	1.2662	1.9349e4	4	1.2360	0.5	10%
2.0719e4	6	1.3155	1.9507e4	5	1.2487	0.2	50%
1.7871e4	15	1.2339	1.7954e4	7	1.1292	0.5	50%

Table.4: P_{cr} for IPE cross-section and clamped supported column (A=20 cm2 , IPE160)

Pcr	Number of iterations	$\frac{\lambda}{\lambda_{unf}}$,N=40	P_{cr} (kg/cm^2)	Number of iteration	$\frac{\lambda}{\lambda_{unf}}$,N=10	Step size	Resizing ratio
1.9246e6	123	1.3270	1.8823e6	66	1.2973	0.2	10%
1.9215e6	46	1.3250	1.8811e6	24	1.2937	0.5	10%
1.9765e6	44	1.3629	1.8813e6	32	1.2970	0.2	50%
1.9567e6	19	1.3492	1.8742e6	13	1.2921	0.5	50%

Table.5: P_{cr} for the circular cross-section and clamped free supported column (R=2.5 cm, A=20 cm2)

Pcr	Number of iterations	$\frac{\lambda}{\lambda_{unf}}$,N=40	P_{cr} (kg/cm^2)	Number of iteration	$\frac{\lambda}{\lambda_{unf}}$,N=10	Step size	Resizing ratio
1.1740e4	27	1.4440	1.0559e4	11	1.2987	0.2	10%
1.0683e4	9	1.3140	9.8802e3	4	1.2270	0.5	10%
1.1479e4	11	1.4119	1.0605e4	9	1.3043	0.2	50%
1.0137e4	6	1.2840	1.0194e4	2	1.2583	0.5	50%

Table.6: P_{cr} for IPE cross-section and clamped free supported column (A=20 cm2 , IPE160)

Pcr	Number of iterations	$\frac{\lambda}{\lambda_{unf}}$,N=40	P_{cr} (kg/cm^2)	Number of iteration	$\frac{\lambda}{\lambda_{unf}}$,N=10	Step size	Resizing ratio
1.0425e6	181	1.4055	9.7209e5	82	1.3106	0.2	10%
1.0408e6	71	1.4027	9.7103e5	33	1.3091	0.5	10%
1.0427e6	61	1.4059	9.7193e5	41	1.3104	0.2	50%
1.0397e6	36	1.4025	9.7065e5	17	1.3086	0.5	50%

6. Results

In this section all of the charts are provided for columns with circular and IPE cross-section under three different boundary conditions which are simply, clamped and clamped free supported. In all of the cases the length of column 400cm and Young's modulus E=2.1e6 $\frac{\text{kg}}{\text{cm}^2}$ are assumed (Tables 1-6).

According to presented charts, the following results are obtained:

- By increasing the number of elements, the critical buckling load increases.
- By decreasing the resizing ratio (the percentage of resized elements of total number), the critical buckling load increases.
- By reduction of the step size, the optimum critical load increases.

According to the Figures 7 and 8, the critical buckling load of the circular cross-section column can be decreased by changing the boundary condition from clamped to clamped free and then simply supported. On the other hand, it can be increased by applying same boundary conditions respectively as mentioned above. Therefore, this method presents the greatest value of $\frac{\lambda}{\lambda_{unf}}$ for simply supported column.

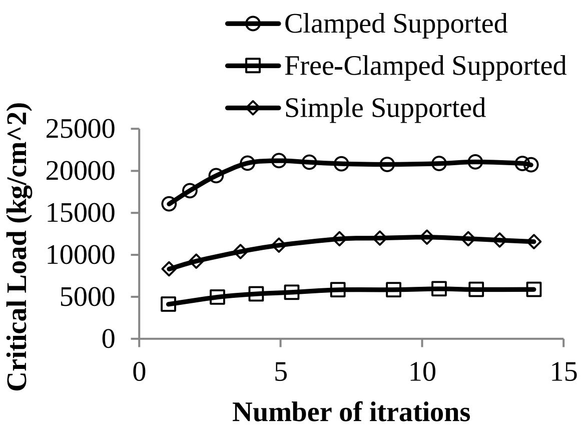

Fig.7: Iteration histories of critical buckling load for circular cross-section column with different boundary condition.

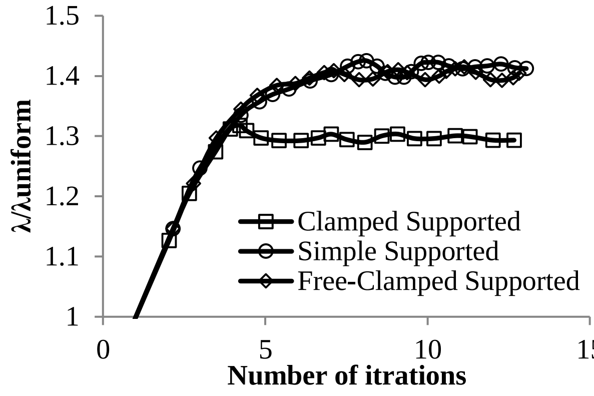

Fig.8: Iteration histories of circular cross-section column with different boundary condition.

According to Figures 9 and 10, the critical buckling load of IPE section column decreases by changing the boundary condition from clamped to clamped free and then simply supported. This method presents the greatest value of $\frac{\lambda}{\lambda_{unf}}$ for clamped free column.

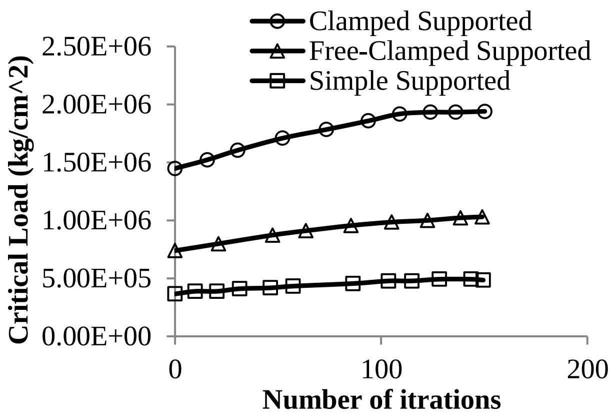

Fig.9: Iteration histories of critical buckling load for IPE cross-section column with different boundary condition.

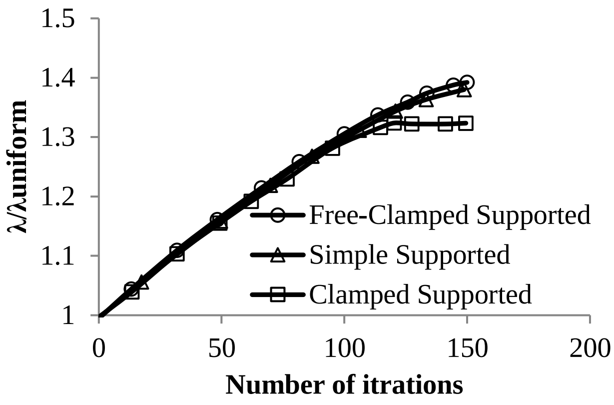

Fig.10: Iteration histories of for IPE cross-section column with different boundary condition.

7. Conclusion

The paper's aim is about applying ESO method as a way to optimize buckling forces in elastic column. In this procedure, the cross sectional area of column changes while the total volume of column remains constant. This change transforms the buckling modes, which in turn increase the buckling forces that depends directly on the number of elements in the column. According to this study critical buckling load of columns decrease by changing the boundary condition from clamped to clamped free and then simply supported. The critical buckling load of the circular cross-section column decreases by changing the boundary condition from clamped to clamped free and then simply supported, but increases by applying respectively the same boundary conditions as mentioned above. Therefore, this method presents the greatest value of $\frac{\lambda}{\lambda_{unf}}$ for simply supported column. The results obtained in this study are verified by series of theoretical relationships which serve at some level, to the verification of the results.

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[DOI: 10.29252/nmce.1.4.38]