Evaluation of the effect of reservoir length on seismic behavior of concrete gravity
dams using Monte Carlo method


ARTICLE INFO
Article history:
Received: June 2020.
Revised: July 2020.
Accepted: August 2020.

Keywords:
Concrete gravity dams; Truncated boundary; Monte Carlo; Probabilistic method; Reservoir length

Abstract:
In present study, the effect of reservoir length on seismic performance of concrete gravity dam
has been investigated. Monte Carlo probabilistic analysis has been used to achieve a
sensitivity of the responses to variation of truncated reservoir length in finite element model.
The ANSYS software based on finite element method is applied for modeling and analysis. The
Pine Flat dam in California, under components of El Centro, San Fernando and North Ridge
earthquake, is modeled as a case study to evaluate the effect of reservoir length on seismic
behavior and optimization. The foundation flexibility has been considered in modeling and
Sommerfeld boundary condition has been used for reservoir truncated boundary condition. In
Monte Carlo probabilistic analysis, the reservoir length has been considered as input variable
and maximum dam crest displacement, maximum hydrodynamic pressure in reservoir and
maximum tensile principal stress in heel and compressive principal stress in toe of dam have
been selected as output parameters. The Latin Hypercube sampling method has been applied
with unique distribution function for input variable. Obtained results show the sensitivity of
output responses to variation of reservoir length. Considering sensitivity results, it is possible
to select the optimum length of reservoir for finite element model.

1. Introduction
Optimization means to achieve the best result in an
operation, with certain restrictions and limitations in place.
In optimizing a system, the authority of changing the
structure is implicitly a default. In general, changing
potential is expressed based on a range of changes in a
number of parameters. Such parameters are usually called
designing variables in optimization term.
In most of the structure designing problems, the continuity
of design variables are commonly neglected in solving
optimization problem. When the optimized variable is
obtained, then the value of design variable is turned into
nearest available discrete value. The reason of this action is
that, solving an optimization problem with discrete design
variables is more difficult compared to a similar
respective problem with continuous design variables.

Concrete dams are considered as the most important and
costly projects among infrastructures. The effects of
hydrodynamic pressure during an earthquake must also be
estimated in addition to the hydrostatic pressures in order
to accurately calculate the reservoir water pressure to the
body of concrete dams. The problem has been posed as
dynamic interaction of the dam and reservoir during
earthquakes and is considered as one of the major factors of
designing new dams and safety assessment of existing
dams in seismic areas.

In an earthquake, the dam that is connected to the ground
vibrates, while water behind the dam is not directly
affected by the seismic motion of ground due to the low
shear forces between the bed of the reservoir and reservoir
water and only hydrodynamic pressure induced by dam
vibrations spread upward the reservoir. Today, the finite
element method is widely used in modeling concrete dams
due to its capability in defining mathematical models with
complex geometry and different materials. Hence, utilizing
it to solve reservoir equations can lead to an appropriate
ordinance between solving methods of dam and reservoir

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equations and significantly reduce the complexity of the problem. However, modeling large and unlimited reservoirs with this method has some drawbacks. Since taking into account the whole reservoir in the model is very expensive and unpractical, the model must be disconnected in an appropriate distance from the body of dam in a way that it has no effect on seismic responses of the dam.

Zienkiewicz and Bettis (1978) proposed a model for unlimited reservoir conditions to solve the pressure wave equation in the reservoir [1]. They showed that the Sommerfeld boundary condition is quite suitable for long reservoir modeling and can be used to discrete the finite element elements of the fluid range. The truncated upstream boundary must be placed at a far distance from the dam in finite element model. Zienkiewicz concluded that if the far end boundary is selected as almost twice the depth of the reservoir away from the dam, the condition would lead to relative precise results in most of the frequency ranges of loading and the obtained results can be acceptable.

Saini et al. (1978) [2] and Chopra and Chakrabarti (1981) [3] studied the dam-reservoir interaction problem in the frequency domain using the finite element model. Finite element analyses in time-domain were performed by Sharan (1985 and 1986) [4-6] and Tsai et al. (1992) [7]. For the reservoir with irregular geometry, numerical methods such as finite element method must be used, because an analytical solution cannot obtain the results for the arbitrary boundary and geometry of the system. Jablonski and Humar (1990) [8] applied the boundary element method in frequency domain for seismic analysis of concrete dams.

The Monte Carlo method is a simulation method, one of the common goals of which is to estimate the specific parameters and probability distributions of random variables. One of the most commonly used methods for solving complex problems is probability analysis [9]. The Monte Carlo method is divided into two methods: Direct Sampling, and Latin Hypercube Sampling. The LHS method is a more advanced and appropriate form of Monte Carlo simulation. It performs 20 to 40 percent fewer simulation loops to obtain the results similar to Direct Sampling. In this research, the Latin Hypercube Sampling method is used for Monte Carlo analysis.

In the field of probabilistic and sensitivity analysis, Alembagheri and Seyedkazemi (2014) [10] conducted a probabilistic study in which the seismic behavior of the concrete gravity dam is addressed with regard to concrete tensile behavior parameter as the sensitivity parameter. The results of their research showed that accurate examination of tensile behavior and final failure of concrete in concrete gravity dams requires a proper definition of nonlinear models of materials. Using Monte Carlo probabilistic method, Pasbani Khiavi (2015) [11] investigated the reservoir bed characteristics effect on reducing the pressure induced in the reservoir. Results confirmed a high dependence of responses to the reservoir bottom absorption. Additionally, Pasbani Khiavi (2017) [12] investigated the influence of the concrete stiffness on the seismic responses of concrete gravity dams by the Monte Carlo simulation. According to the results, the optimized value of the concrete Young Modulus to access the confident response of the structure was achieved, which was economically important.

Pasbani Khiavi et al. (2020) used the Monte Carlo simulation in seismic optimization of concrete gravity dams using isolation layer. In their research, the optimum geometry of isolation layer was obtained using Monte Carlo with LHS method [13].

2. LHS method

Latin Hypercube Sampling (LHS) may be viewed as a stratified sampling scheme designed to ensure that the upper or lower ends of the distributions used in the analysis are well represented. Latin Hypercube Sampling is generally recommended over Direct Sampling method when the model is complex or when time and resource constraints are an issue [14]. The nature of LHS does not determine the appropriate sample size to achieve a certain confidence level. There is no specified value for sample size N to achieve a certain confidence level in LHS [15].

By sampling N times from the parameter distributions, this procedure creates a population of N possible instances of the structure, each of which needs to be analyzed. The use of relatively high N that is substantially larger than the number of parameters will always result in reasonably accurate estimates for practical purposes. The optimal N to use is a function of the number of random variables and their influence on the response [16].

The basis for all LHS simulation steps is generating random numbers being uniformly distributed between 0 and 1. When there is an understanding of U which is related to the uniform distribution of random number U between 0 and 1, it is possible to produce x related to the uniform distribution of random number x between both values a and b (a ≤ x ≤ b).

The process part with events can be simple or very complex and may contain many loops and algorithms and even multiple random generators. Besides, it is possible to extract quantitative data from any point of the algorithm and analyze them as output variables. Monte Carlo simulation methods can be used in all fields of science and engineering to predict the real and virtual behavior of systems and define different scenarios.
Most of the engineers who utilize general sophisticated software packages for structural analysis often have no access to general sophisticated software packages, and don’t have sufficient knowledge yet about the details of structure analysis algorithms that have been used in these soft wares. Therefore, the main challenge of structural optimization researchers is to regulate approaches and methods proper to these types of soft wares. Another main challenge is high computing costs of analyzing most of today's complex structures such as concrete dams. One of the modern approaches of structures’ optimization is utilizing Monte Carlo probabilistic analysis. Therefor in present research, Monte Carlo probabilistic analysis of ANSYS Software has been used to optimize the disconnected distant border of dam.

3. The governing equations

In this part, the solid and fluid domain equations are presented while the water inside the dam is assumed to be inviscid, incompressible, and with small displacements. Also, the dam is considered solid elastic with a linear behavior of materials [3, 17].

3.1. Modeling of the dam

The governing equation of the dam behavior is the equation of motion. However, for the consideration and comprehensive definition of the interaction between the fluid and the structure, the load applied due to the hydrodynamic pressure of the fluid at the interface structure and fluid, must be added to the equations of structure:

\[ M \ddot{u} + C \dot{u} + K u = M \ddot{u}_g + F_{Pr} \]  
(1)

In Eq. (1), M, C and K represent mass, damping, and stiffness matrices, respectively. \( \ddot{u} \) shows the relative movement vector, \( \ddot{u}_g \) refers to the ground acceleration vector, \( F_{Pr} \) is the hydrodynamic force vector at the dam and reservoir interface [1].

3.2. Modeling the reservoir

The equation of dynamic behavior of the reservoir should be considered with Navier-Stokes, momentum, and fluid continuity equations with problems related to the acoustic interaction between the structure and fluid. Given that the water inside the reservoir is inviscid, incompressible with small displacement, the equations of continuity and momentum are summed up to the wave equation. Also, the pressure applied to the dam by the reservoir at the contact point is considered to form the interaction matrix [17].

\[ \frac{1}{c^2} \frac{\partial^2 P}{\partial t^2} - \nabla^2 P = 0 \]  
(2)

Where \( c = \sqrt{\frac{k}{\rho_0}} \) is the velocity of the acoustic waves in the fluid and \( k \) is the bulk modulus of water. Equation (2) is the basis of acoustic issues and is known as the Helmholtz Equation, which is derived from hydrodynamic pressure.

4. Dam and reservoir interaction

In the past, the dynamic effects of reservoir in terms of dam-reservoir coupled system were approximately calculated using added mass method. It is clear that response approximation by added mass method is not appropriate for problems such as evaluating the distribution of cracks in dam body and it is necessary to look for more precise solutions. The dam-reservoir system can be considered as a coupled system, because of two different physical domains in the form of coupled system. In such issues, the responses of structure and fluid are considered simultaneously due to the interaction between them. The interaction effect of dam-reservoir is considered using coupled equations including two second-order differential relations as following:

\[ [M][\ddot{u}] + [C][\dot{u}] + [K][u] = [f_1] - [M][\ddot{u}_g] + [Q][P] = [F_1] + [Q][P] \]  
(3)

\[ [G][\ddot{P}] + [C'] [\dot{P}] + [K'][P] = [F_2] - \rho [Q^T][\ddot{u}] \]  
(4)

\[ [M], [C] \text{ and } [K] \text{ are matrixes of mass, damping and stiffness of structure, respectively. In addition, } [G], [C'], \text{ and } [K'] \text{ are equivalent matrixes of mass, damping and stiffness of reservoir, respectively; } [Q] \text{ is coupled matrix and } [f_1] \text{ is the vector of sum of body forces imposed on the body of dam. Vector } [F_2] \text{ is the sum of forces created due to the base acceleration } [\ddot{u}_g] \text{ in dam-reservoir interface and total acceleration } [\ddot{u}] \text{ in dam-foundation interface. } [P] \text{ is hydrodynamic pressure imposed on the reservoir and dam interface and } \rho \text{ is the density of water.} \]

The coupled matrix of reservoir pressure and the interaction forces between the dam and reservoir is as follows:

\[ [Q][P] = [f] \]  
(5)

Where, \( [f] \) is the vector of force imposed on the dam caused by hydrodynamic pressure.

5. Boundary condition

Sommerfeld boundary condition has been used for reservoir truncated boundary condition defined as follows:

\[ \frac{\partial P(x,y,t)}{\partial x} = \frac{1}{c^2} P(x,y,t) \]  
(6)

In which, \( P(x, y, t) \) is hydrodynamic pressure. \( C \) is the velocity of wave pressure in water and \( x \) and \( y \) are the coordinate axes. The boundary conditions in dam-reservoir
interface and reservoir-foundation are dominated using below equation:

$$\frac{\partial P(x,y,t)}{\partial n} = -\rho a_n(x,y,t)$$  \hspace{1cm} (7)

Where, \(a_n(x,y,t)\) is dam normal acceleration in the contact area. By neglecting free surface wave, the boundary condition of reservoir surface is considered as follows:

$$P(x,h,t) = 0$$  \hspace{1cm} (8)

In which, \(h\) is the height of reservoir.

6. Case study

Pine Flat dam with a height of 122 m was simulated in a two-dimensional form as a case study of present research. The geometry of Pine Flat dam and dam-reservoir-foundation system created using the ANSYS software have been represented in figures (1) and (2), in which \(H\) is the height of reservoir. The materials of concrete dam and foundation are considered with homogeneous, linear and isotropic behavior and the reservoir water is considered as compressible, non-viscous and non-rotating with small displacement fluid. All dimensions are based on SI system. The characteristic of material parameters of system have been shown in table (1). For the reservoir, density and bulk modulus corresponding to water are considered as 1000 \(kg/m^3\) and 2.1 GPa.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbol</th>
<th>Dimensions</th>
<th>Dam</th>
<th>Foundation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>(\nu)</td>
<td>(kg/m^3)</td>
<td>2483</td>
<td>0</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>(\rho)</td>
<td>-</td>
<td>0.2</td>
<td>0.33</td>
</tr>
<tr>
<td>Elasticity modulus</td>
<td>(E)</td>
<td>GPa</td>
<td>22.4</td>
<td>22.4</td>
</tr>
</tbody>
</table>

Fig. 1: Geometry of Pine Flat concrete gravity dam

7. Model analysis and results evaluation

ANSYS software based on finite element method was used for seismic analysis of model. The software has the capability of seismic analysis by considering irregular geometry and interaction effects. For this purpose, proper elements representing compressible behavior of fluid have been considered. According to the conditions dominated on concrete gravity dam and geometry of reservoir, the model of system was considered in a two-dimensional form with plane-stress behavior and its interaction with foundations has also been considered in the model. SOLID182eight-node elements were used for discretization of the solid part and FLUID29 four-node elements were used for the fluid part including adjacent and non-adjacent fluid of structure. The unknown parameters including displacement and stresses of the dam body and induced hydrodynamic pressure in the reservoir were extracted by analyzing the model. For probabilistic analysis, the records of El Centro, San Fernando and North Ridge earthquakes were extracted from Peer website according to Table (2). The records of these earthquakes have been scaled compared to the 0.3g.

The Newmark method was used for numerical integration, in which its parameters were selected as \(\beta=0.25\) and \(\gamma=0.5\). Rayleigh method was used to apply damping effects and Sommerfeld boundary condition was used for truncated far boundary. Riley damping coefficients using the first and second frequency of system equal to \(\alpha=0.5202\) and \(\beta=0.0046\) were extracted. Latin Hypercube Sampling (LHS) method has been used in Monte Carlo probabilistic analysis and Uniform method has also been used for probability distributions of the input variables.

Table 2: The characteristic of selected earthquake records

<table>
<thead>
<tr>
<th>Distance to fault (Km)</th>
<th>Station name</th>
<th>earthquake</th>
</tr>
</thead>
<tbody>
<tr>
<td>20.72</td>
<td>Castaic - Old Ridge Route</td>
<td>Northridge-01</td>
</tr>
<tr>
<td>6.09</td>
<td>El Centro Array #9</td>
<td>Imperial Valley-02</td>
</tr>
<tr>
<td>1.81</td>
<td>Pacoima Dam (upper left abut)</td>
<td>San Fernando</td>
</tr>
</tbody>
</table>
One of the most important modeling steps in finite element softwares is the discretization of the system. Discretization in the model must be converged, so finer discretization does not affect the results considerably. Hence, through Monte Carlo probability analysis and with APDL programming in ANSYS software, optimum discretization of the model was achieved with regard to investigation of the effect of discretization size on seismic responses of concrete gravity dam. Therefore, through employing Monte Carlo probability analysis, it is possible to specify the optimum meshing for every earthquake.

In the Monte Carlo probabilistic analysis, Histogram diagrams of output variables must be converged to the number of simulations. In addition, the obtained results from Monte Carlo probabilistic analysis have proper convergence. For this purpose, the adjustments related to the simulating Monte Carlo analysis in ANSYS Software have been considered according to Table (3).

<table>
<thead>
<tr>
<th>The number of repetitions</th>
<th>The number of simulation loops</th>
<th>Distribution Type</th>
<th>Random variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>40</td>
<td>Uniform</td>
<td>Reservoir length</td>
</tr>
</tbody>
</table>

To select the optimum value of reservoir length by Monte Carlo probabilistic analysis, $\bar{\lambda}$ parameter is considered as input variable which indicates the ratio of reservoirs length to its height. In this parameter, the dam height is a constant value and only the length of reservoir is changed.

After conducting the analysis, the sensitivity of seismic responses of horizontal displacement of the dam crest, the maximum and minimum principal stresses in the toe and heel of the dam and the maximum hydrodynamic pressure in the reservoir related to $\bar{\lambda}$ parameter were extracted using ANSYS Software. The sensitivity of seismic responses to $\bar{\lambda}$ parameter affected by EL Centro Earthquake have been represented in figures (3) to (7).

Fig. 3: Sensitivity of maximum hydrodynamic pressure in the dam heel vs. the $\bar{\lambda}$ parameter during EL Centro earthquake

Fig. 4: Sensitivity of horizontal displacement of dam crest vs. the $\bar{\lambda}$ parameter during EL Centro earthquake

Fig. 5: Sensitivity of compressive principal stress in the toe vs. the $\bar{\lambda}$ parameter during EL Centro earthquake

Fig. 6: Sensitivity of compressive principal stress in the toe vs. the $\bar{\lambda}$ parameter during EL Centro earthquake

Figures (7) to (10) represent the sensitivity of seismic responses to $\bar{\lambda}$ parameter during San Fernando Earthquake.

Fig. 7: Sensitivity of maximum hydrodynamic pressure in the heel vs. the $\bar{\lambda}$ parameter during San Fernando earthquake
Figures (11) to (14) represent the sensitivity of seismic responses to reservoir length parameter during North Ridge earthquake.

Obtained results show that the reservoir length, which is 2 times greater than reservoir height, has no effect on responses. So, according to the obtained sensitivity curves it can be concluded that the selecting of reservoir height equals twice reservoir height presents acceptable responses in finite element model with Sommerfeld truncated boundary condition.

8. Conclusion

In the present study, a parameter sensitivity analysis of a dam-reservoir system was performed using Monte Carlo simulation with Latin hypercube sampling. A probabilistic analysis was used to identify the reservoir length as a particular parameter that had a significant effect on the responses. The Pine Flat dam was considered as case study to illustrate the effect of reservoir length on selected output.
parameters. Obtained results show how the reservoir length can affect seismic performance of finite element model. Considering the sensitivity curves of responses to the variation of reservoir length, it is possible to select the optimum length of truncated boundary for reservoir in finite element model. Also, it is obvious from obtained results, increasing reservoir length reduces the seismic responses of dam because of hydrodynamic pressure damping spread to the upstream of reservoir. In addition, increasing of reservoir length in the range of approximately three times the dam height has an effect on seismic responses of dam and by increasing the length more, no significant change is created in seismic responses.

It can be concluded that in seismic analysis of concrete gravity dam, the truncated boundary can be considered about three times the dam height in finite element model by applying Sommerfeld boundary condition for far end boundary.

References