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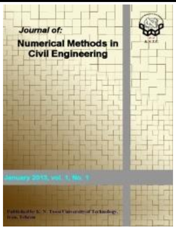

Numerical Methods in Civil Engineering

Approximation of undrained bearing capacity of strip foundations on heterogeneous marine clay

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Abstract:

The problem of bearing capacity of foundation under rigorous circumstances has always been of a great importance in geotechnical engineering. Various simple and also more sophisticated methods have been developed to improve the bearing capacity calculations. In this paper, the performance of three different approximate methods including the limit equilibrium analysis, upper and lower bound of the limit analysis, and the characteristic method of the slip line solution is evaluated and compared in dealing with marine soil deposits with the heterogeneous soil strength profile under the plane strain condition. The undrained bearing capacity of strip footing rested on a stratum with a linear variation of undrained shear strength was estimated using various methods. It was found that for all methods adopted in the bearing capacity estimation, the undrained bearing capacity increases with the strength density. As a result, the failure mechanism becomes shallower and narrower as the undrained shear strength increases with depth. Comparing the results obtained by the applied methods, the maximum bearing capacity is rendered by the limit equilibrium (assuming Terzaghi failure mechanism), resulting as an unsafe solution. On the contrary, the minimum bearing capacity is yielded by the stress characteristic method, as the most conservative and applicable solution.

1. Introduction

The calculation procedure of bearing capacity problem is recognized as for association of three different contributors named as the “cohesion” (c), surcharge” (q), and the “unit weight of soil” (γ), in the classical equation of bearing capacity [1]. However, validation of this equation has been a controversial issue for years such that a great number of studies were carried out ranging from fundamental questioning of the assumptions made in derivation of the equation to investigations on affecting factors such as footing shape and its depth [2-5]. In addition, the conventional limit equilibrium solutions reported by Terzaghi [1], Hansen [2] and Meyerhof [3,4] in derivation of the classical bearing capacity equations, different approximate analytical methods were also developed and used to attain more precise solutions.

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The upper and lower bound theorem of limit analysis [6-19] and characteristic method [20-30] are the most well-known approximate methods among many.

Theories and methods of bearing capacity calculation were initially developed for simple soil profiles and geometrical conditions which are more appropriate for homogenous media. However, a large number of studies have recently been focused on the real soil conditions, considering the effects of heterogeneity and anisotropy. The effect of soil heterogeneity is clearly manifested in the bearing capacity, as well as the failure mechanism. Hence, more accurate failure mechanisms and more rigorous methods of analyses are required in order to capture the effects of the heterogeneity. In this study, the performance of different approximate methods known as (1) the limit equilibrium with general shear failure mechanism, (2) the conventional upper-bound method comprising of the two different failure mechanisms i.e., Prandtl and the revised failure mechanism, optimized by the “Genetic Algorithm”, (3) the lower-bound with stress fan, (4) the upper and lower bounds of limit analysis in conjunction with the finite element method, and (5) the characteristic method in combination with the finite difference method, are evaluated under the plane strain condition. A detailed comparison that has been made in this study, makes the engineers able to use either the average bearing capacity coefficient or the bounds of solution, proposed by the various methods of analysis.

2. Variation of the Undrained Shear Strength

In the natural soil deposits, especially for the normally consolidated and lightly over-consolidated clays, the void ratio decreases with depth. Consequently, the stiffness and soil strength properties mutually would increase with depth [31]. Furthermore, the natural geological processes such as sedimentation, progressively change the deposited soil properties. This phenomenon was firstly analysed and reported by Gibson and Morgenstern in 1962 [32]. Gibson Soil was defined as a linear elastic material in which its stiffness and consequently, its shear strength parameters can increase linearly with depth. The results presented by different in-situ tests show a linear variation in the undrained cohesion of soil with depth, according to the increase of effective overburden stress with depth. According to this trend, many studies have been carried out with regard to clay inhomogeneity [19,33-38].

Davis and Booker [39] observed three different types of non-homogenous clay profiles: (1) Recent marine deposits without surface strength but with linear variation of shear strength with depth, (2) Normally consolidated deposit which are aged or weathered and have some surface strength (c_0) (as shown by Figure 1), and (3) the normally consolidated deposit with constant shear strength up to the transformation depth and thereafter with linear variation of the undrained shear strength parameter. A range of values as of 0.6 – 3.0 kPa/m has been reported for the rate of strength variation (λ) based on the experimental study of Davis and Booker [39]. Despite the wide variety of soil strength profiles, most of the existing solutions for the bearing capacity of footings are reported for the homogeneous clay deposits.

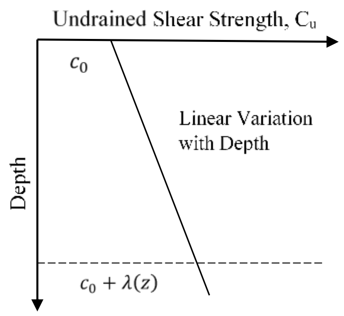

Fig. 1: Linear variation of undrained shear strength with depth.

3. Methodology and Analyses

3.1 The Limit Equilibrium Method

The limit equilibrium method can be considered as one of the most traditional and commonly used methods to approximate the solutions of the bearing capacity problems. This method is based on an iterative procedure for determination of the bearing capacity with regard to an arbitrary collapse mechanism, which may consist of any combination of straight lines or curves, arranged to give a failure mechanism. It would be imperative to calculate the equilibrium of the components by equating the resultant forces and moments acting on the failure surface to determine the bearing capacity of the foundation. It is also worth noting that any compatible mechanism should be examined in order to find the critical mechanism for which the loading is in a limit equilibrium state. Obviously, the accuracy of the collapse load as determined by the limit equilibrium approach is strongly dependent on the geometry of the assumed failure surface [40,41].

In this study, the bearing capacity calculations associated with the limit equilibrium analysis are established based on the assumption of a general shear failure mechanism, as shown in Figure 2. Such mechanism is certainly compatible with the relative movement between the adjacent blocks that does not have to occur within separation and/or volume changes. This failure mechanism was initially used by Terzaghi [1] with the wedge angle (θ) equal to the soil internal friction angle, as $\theta = \varphi$.

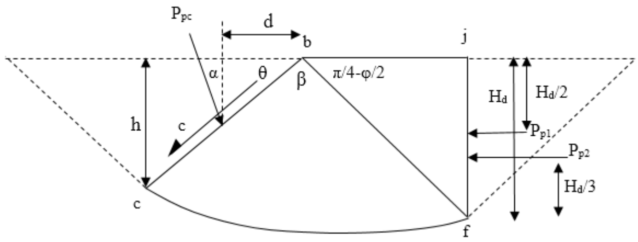

Fig. 2: General shear failure mechanism of the limit equilibrium method.

3.2 The Limit Analysis Method

Exact solutions to the soil mechanics problems such as the bearing capacity of the foundations can be obtained when the equilibrium equations of stresses, compatibility conditions and stress-strain relations are considered, simultaneously. However, calculation of the actual failure loads is not a straightforward task to do, demanding it to develop methods through which the limit loads can be obtained in a simpler way; i.e. without considering all of the conditions and constraints of a complete solution. The limit analysis is one of the most prevalent methods that simplifies the solution of stability problems by ignoring some of the conditions of the equilibrium and compatibility.

The limit analysis utilizes the bounds theorem of plasticity to calculate the lower and upper bounds of the actual load. The upper and lower bound theorems are applicable for materials with the perfectly rigid plastic behaviour and associated flow rule. By satisfying the equilibrium equations and stress-strain conditions simultaneously, the lower-bound of the limit-load can be found, whereas in the upper-bound limit analysis, the amount work done by the external loads is equated to that of the internal stresses in terms of the energy dissipation and the upper-bound of the limit-load that can be found accordingly. Obviously, the true plastic collapse load must fall between these two bounds, resulting by the exact limit-load. The limit load can be found with exact accuracy, provided that the upper and lower bounds are the same.

3.2.1 The Upper-Bound Limit Analysis

The calculations required for the upper-bound analysis are performed by equating the dissipated energy through the shear stresses along the slip lines of the failure mechanism to the external work done by a set of applied forces for any arbitrary mechanism of the plastic deformation. Such mechanism should be a kinematically admissible displacement field that satisfies any displacement constraints associated with the boundary value problem, compatibility, and the flow rule. Generally, the aim of the upper-bound limit analysis is to calculate the least upper-bound solution since the natural processes are accomplished in such a way that the spent energy has to be at the minimum level. Failure mechanisms can be constructed by the individual slip surfaces including the circular slip surfaces or a combination of different types of surfaces. Since the accuracy of the plastic collapse load as determined by the upper-bound limit analysis does strongly depend on the failure mechanism, different approaches of Prandtl mechanism, revised failure mechanism and the upper-bound limit analysis merged with the finite element method were employed in this study in order to generate the approximate solutions of the bearing capacity factors for a non-homogenous soil.

Figures 3 and 4 illustrate the Prandtl mechanism [42] with the corresponding displacement field and the revised failure mechanism which has the capability of being put into the practice in the calculation of the bearing capacity factors, considering the soil anisotropy and non-homogeneity in both drained and undrained conditions. Geometry of the problem is defined by unknown angles.

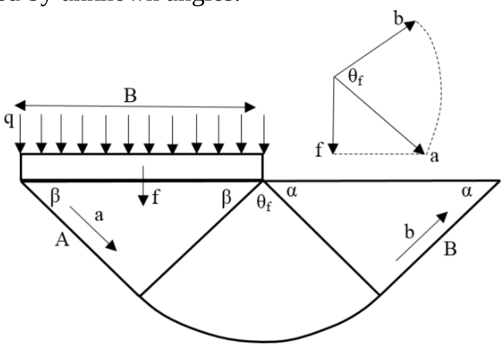

Fig. 3: Prandtl failure mechanism and the corresponding displacement field [42].

The other scheme of the upper-bound limit analysis is the finite element upper-bound approach. The finite element limit analysis is the general form of an upper-bound solution in which no specific failure mechanism is assumed. This leads to an enhancement of the accuracy of the results compared to the conventional upper-bound solution, which merely employs some explicit slip surfaces. The constraint equations and the objective function can be defined by applying the compatibility of strains, boundary conditions, and displacement discontinuities. The constraint equations and objective function are defined as a linear function of unknown nodal displacements. Moreover, the Mohr-Coulomb's failure criterion was linearized by an exterior polygon with *P* sides as depicted in Figure 5. By assembling all equalities and constraints inequalities and the objective function, a discrete formulation of the upper-bound theory leads to a constrained optimization problem. Since all of the constraint equations and objective functions are linear, the problem is then known as the linear programming in the mathematical terminology. The formulation used in this study follows that of Sloan, firstly proposed in 1989 [9].

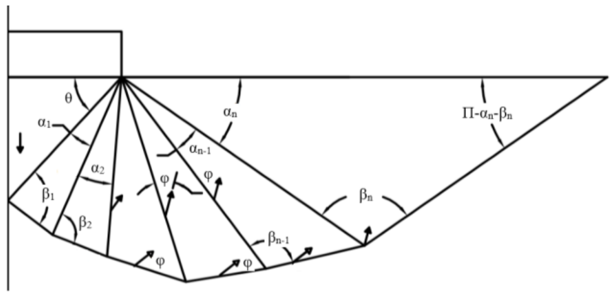

Fig. 4: Revised Prandtl failure mechanism.

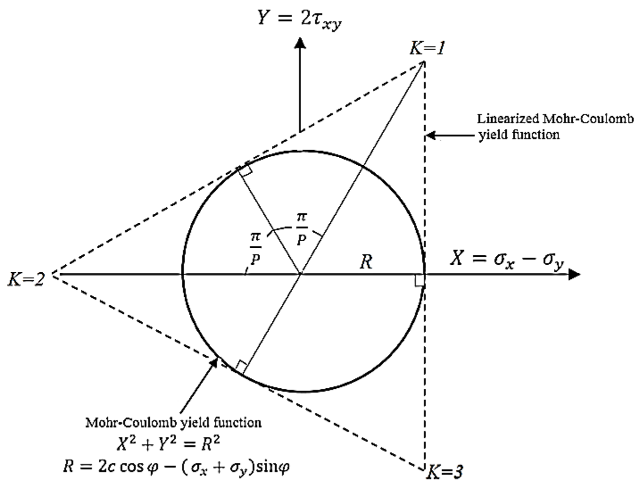

Fig. 5: Circumscribed linearization of the Mohr-Coulomb criterion [9].

3.2.2 The Lower-Bound Approach

The lower-bound theorem of the limit analysis states that the plastic collapse will not take place if any state of stress can be found that satisfies the equilibrium equations initially, and in the next, meets the traction boundary conditions and eventually, nowhere exceeds the yield function. Generally, in the lower-bound analysis, a stress field must be created which satisfies both equilibrium and yield condition, also referred to as a "statically admissible" stress field. A lower-bound load is also known as the safe load since it cannot lead the structure to plastically collapse.

The state of equilibrium can be changed either sharply or smoothly by assuming a limited number of stress discontinuities or taking a fan of discontinuity, correspondingly. The change of total stress across a stress discontinuity can be easily found by drawing the Mohr circles of two adjacent elements in the vicinity of discontinuity, which is simply related to undrained cohesion and rotation of the major principal stress's direction. There are many stress discontinuities and equally a great number of Mohr circles within the fan. Figure 6 shows the stress fan used in this study to calculate the lower-bound solution of bearing capacity of foundations on heterogeneous soil.

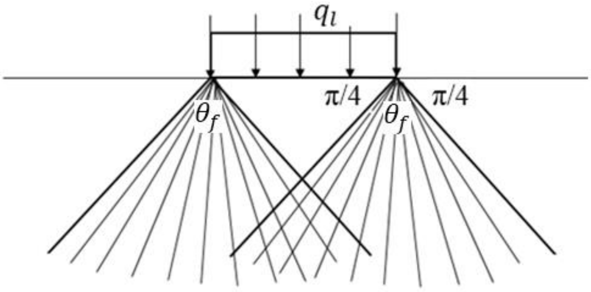

Fig. 6: Fan of stress field for lower bound limit analysis.

The lower-bound finite element method is the general form of a lower-bound approach in which no assumption is made for the stress field so as to enhance the accuracy of the results compared to the conventional lower bound solution. Just like the upper-bound finite element analysis, the yield function should be linearized by a polygon prior to using linear programming. In contrast to the upper-bound method, this polygon is interior to the Mohr-Coulomb criterion in the lower-bound approach, as depicted in Figure 7. The constraint equations and the objective function can be defined by applying the stress equilibrium, boundary and yield conditions. A detailed description of this method has been described by Sloan (1988) [8].

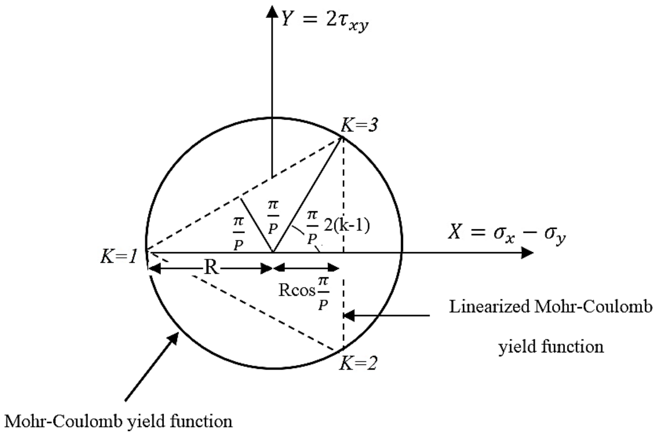

Fig. 7: Linearized Mohr-Coulomb function for the lower-bound solution [8].

3.3 The Characteristic Method

This theory is widely used for isotropic materials in which their behaviour is independent from the mean pressure. The numerical solution of these characteristic equations, developed by Sokolovskiĭ [20], is nowadays widely popular with researchers and geotechnical engineers dealing with plasticity problems, in which solving equilibrium and yield equations are demanded to estimate the ultimate load on foundations and retaining walls. The fundamental of stress characteristics method is given in details in the literature [20,21]. In this paper, the governing equations of the plastic stress field are completed and extended for the described heterogeneous soil in the plane strain condition and they are solved using the finite difference method (FDM). A combination of the equilibrium equation with the failure criterion leads to the two partial differential equations along with two further equations, defining the spatial variations of the orientation of the major principal stress, and the mean normal stress. By application of the finite difference method, these equations can be solved by establishing numerical integration along with the characteristic directions.

4. Results and Discussion

4.1 The Limit Equilibrium Method

The wedge angle (θ) was taken as ($\pi/4 + \varphi/2$) in this study to reach a more accurate solution. Figure 2 shows the general shear failure mechanism assumed for the limit equilibrium method by taking into account the linear variation of undrained shear strength with depth. Assuming $c \neq 0$ and $\varphi = q = \gamma = 0$, the forces acting on the wedge due to cohesion are the soil passive force (P_{pc}), the cohesion force acting on the length of ($\overline{bc}$), Rankine passive force due to the cohesion (P_{p1} for the effect of surface cohesion and P_{p2} for the linear variation term), and finally the cohesive force per unit area along the arc $\widehat{cf}$. Taking moment about point b, the resultant passive force (P_{pc}) can be simply determined in terms of surface cohesion (c_0), undrained shear strength density (λ), the wedge angle (θ) and foundation width (B). The ultimate load can then be found by considering stability of the rigid wedge directly under the foundation.

In order to find the ultimate load capacity, the value of θ for which the equation gives the minimum output should be determined. However, it can be inferred that the variation of undrained shear strength would result in different optimum angles in accordance with different amounts of λ.

Variation of the bearing capacity factors with different values of $\lambda B/c_0$ obtained by the general shear failure mechanism are presented and compared with those obtained by Terzaghi solutions and the other methods.

4.2 The Upper Bound Limit Analysis

The exact solution of the bearing capacity factors of N_c for homogenous soils can be found by the Prandtl method proposed in 1920 [42]. Hence, it would be rational to start with this mechanism for investigation of the effects of increasing shear strength with depth. Equation (1) gives the bearing capacity of the shallow foundation rested on non-homogeneous soil based on the Prandtl failure mechanism, as a function of mechanism angles (α, β), the width of foundation (B) and the rate at which the soil strength increases with depth (λ).

$$q=\begin{Bmatrix}\left[\left(c_0\theta_f+\frac{1}{2}\lambda\frac{B}{2}\frac{sin\alpha+cos\beta}{cos\beta}\right)\right]\\+\left[c_0\theta_f+\lambda\frac{B}{2}\frac{sin\alpha+cos\beta}{cos\beta}\right]\\+\left[c_0+\frac{1}{2}\lambda\frac{B}{2}\frac{sin\beta}{cos\beta}\right]tan\beta\\+\left[c_0+\frac{1}{2}\lambda\frac{B}{2}\frac{cos\alpha}{cos\beta}\right]\cot\alpha\end{Bmatrix} \tag{1}$$

Where $\theta_f=(90+\alpha-\beta)$. Figure 8 illustrates the variation of the optimum wedge angle beneath the foundation (β). It is worth mentioning that the angle α remains constant at 45° for different values of $\lambda B/c_0$. By increasing the rate of heterogeneity, the failure mechanism becomes shallower as the optimum wedge angle is decreased. In other words, assuming a constant values of the angle α, the farthest point of failure mechanism should become closer to the foundation to generate a narrower mechanism.

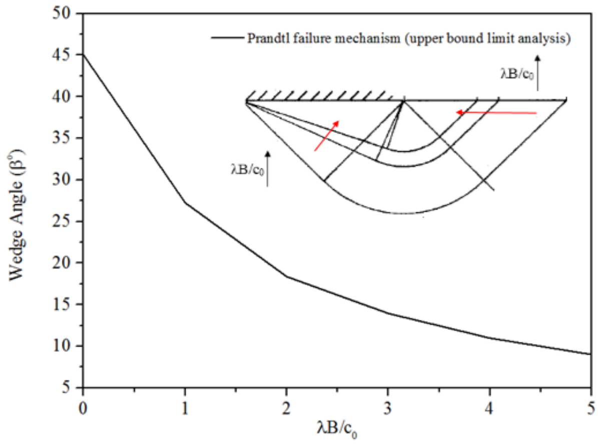

Fig. 8: Variation of the active wedge angle (β) with $\lambda B/c0$ for the Prandtl failure mechanism.

For the revised failure mechanism, the ultimate bearing capacity of the foundation can be found by equating the work done by external forces to that of established by internal stresses in terms of energy dissipated along the slip surface, as a function of (θ, α_{i}, β_{i}). Equation (2) gives the internal energy at the discontinuity with variable shear strength upon which the bearing capacity factor (N_c) can be found.

Genetic Algorithm (GA) was used in this study to find the least upper bound solution. Figure 9 represents some of the definitions of this optimization approach with regard to such particular problem. Obviously, by increasing the number of populations, iterations, and blocks, a more precsise solution can be found.

$$\begin{aligned}q&=c_0B\frac{\cos\varphi\cos(\beta_1-\theta-\varphi)\left(1+\frac{\lambda B\tan\theta}{4}\right)}{2\cos\theta\sin(\beta_1-2\varphi)}\\&+c_0B\frac{\cos(\theta-\varphi)\cos\varphi}{2\cos\theta\sin(\beta_1-2\varphi)}\\&\times\sum_{i=1}^{n}\left[\frac{\sin\alpha_i}{\sin(\alpha_i+\beta_i)}\prod_{j=1}^{i-1}\frac{\sin\beta_i\sin(\alpha_j+\beta_j-2\varphi)}{\sin(\alpha_j+\beta_j)\sin(\beta_{j+1}-2\varphi)}\left(1+\frac{\lambda B}{2}\right)\times\prod_{j=1}^{i}\frac{\sin\alpha_i}{\sin(\alpha_i+\beta_i)}\times\sin(\sum_{j=1}^{i}\alpha_j+\theta)\right]\\&+c_0B\frac{\cos(\theta-\varphi)\cos\varphi}{2\cos\theta\sin(\beta_1-2\varphi)}\times\\&\sum_{i=2}^{n}\left[\frac{\sin(\beta_{i-1}-\beta_i+\alpha_{i-1})}{\sin(\beta_i-2\varphi)}\prod_{j=1}^{i-1}\frac{\sin\alpha_i}{\sin(\alpha_i+\beta_i)}\prod_{j=1}^{i-2}\frac{\sin(\alpha_j+\beta_j-2\varphi)}{\sin(\beta_{j+1}-2\varphi)}\right]\\&\times\left(1+\frac{\lambda B\tan\theta}{4}\right)\times\prod_{j=1}^{i-1}\frac{\sin\beta_j}{\sin(\alpha_j+\beta_j)}\times\sin(\sum_{j=1}^{i-1}\alpha_j+\theta)\end{aligned} \tag{2}$$

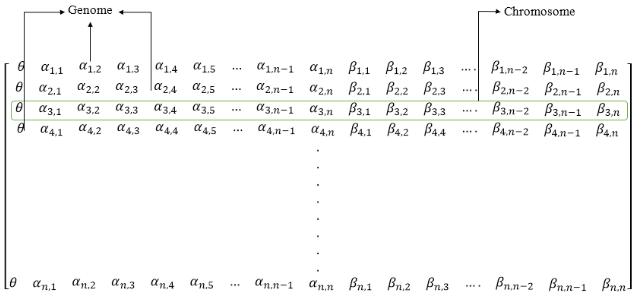

Fig. 9: Genetic algorithm definition for bearing capacity problem.

Figure 10 shows the finite element mesh discretization used in this study, for which the bearing capacity factors are calculated and presented in the following section for the sake of comparison. It is worth mentioning that, the displacement discontinuities were defined to occur at shared edges of adjacent elements, and one of these discontinues was magnified for illustration. By increasing the number of nodes, the analysis gives lower values to make the mesh finer. The number of linearization polygon sides (P) was assumed as $P=12$. Comparing the results of the finite element upper bound analysis for homogenous soils with those obtained by the exact solution of Prandtl failure mechanism, a discrepancy of *1.5%* is observed. This discrepancy arises due to the plastic behavior of triangular elements, which is in contrast with the rigid behavior of failure mechanisms used in the conventional methods. Figure 11 shows the displacement field for the mesh discretization, found to be similar to that of the Prandtl mechanism and Sloan finite element displacement field. Table 1 is presented to compare the bearing capacity factor (N_c) obtained by the upper bound limit analysis with those obtained by other methods as reported in the literature.

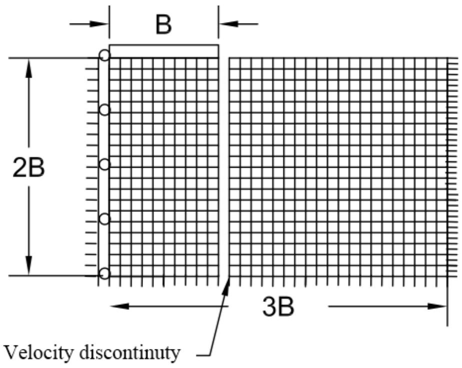

Fig. 10: Mesh discretization of the finite element upper-bound.

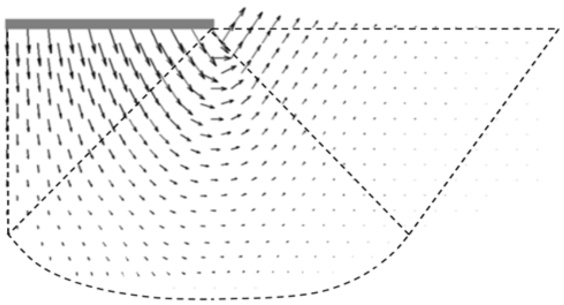

Fig. 11: Displacement fields of the mesh discretization.

Table 1. Comparison of Nc-values for non-homogenous soil.

$\lambda B/c_0$	Peck et al. [43]	Reddy and Srinivasan [44]	Livneh and Greenstein [45]	Reddy and Rao [46]	Revised failure mechanism	Upper bound Finite element
0	5.14	5.52	5.14	5.14	5.15	5.22
1	7.71	7.56	7.20	6.89	6.94	6.94
2	10.28	9.31	9.26	8.45	8.68	8.76
3	12.85	10.94	11.31	9.96	10.15	10.28
4	15.42	12.49	13.37	11.44	11.76	11.94
5	17.99	13.99	15.43	12.92	13.33	13.48

4.3 The Lower-Bound Limit Analysis

Assuming a fan of stress and drawing the Mohr circles of fan discontinuities, the solution of bearing capacity of non-homogenous clay can be found using Equations (3) and (4).

$$q=(2+2\Delta\theta_f)c_{ave}=2(1+\Delta\theta_f)\left(c_0+\frac{\lambda B\cot(\theta_f/2)}{4}\right) \tag{3}$$

where $\theta_f=\frac{\pi}{2}$

$$q_l=c_0(2+\pi)+1.28\lambda B \tag{4}$$

Figure 12 shows the stress field of the lower bound finite element method with 20 stress discontinuities, which is similar to the revised failure mechanism. Note that the angles between each pair of discontinuities are equal in contrast to the upper-bound solution. In fact, the stress fan of the lower-bound solution, velocity field of upper-bound and the revised failure mechanism all together simulate the Prandtl mechanism which represents the exact solution of the bearing capacity factors.

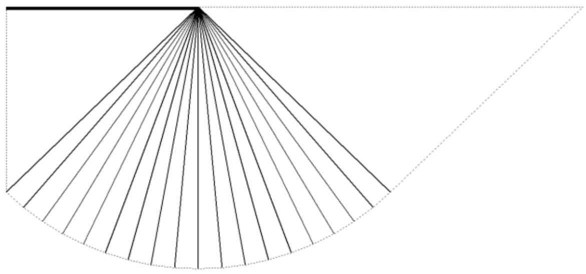

Fig. 12: Stress fan of the lower bound solution by the finite element method.

4.4 The Characteristic method

Curve-fitting to the characteristics analysis, Bransby (2001) proposed an approximate solution as Equation (5) for the bearing capacity factor (N_c)[47]. Figure 13 demonstrates the stress characteristics of the foundation's bearing capacity in non-homogenous soil condition.

$$N_c=(2+\pi)+1.646\left(\frac{\lambda B}{c_0}\right)^{0.662} \tag{5}$$

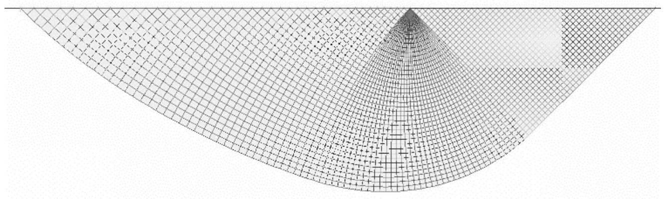

Fig. 13: Characteristics lines of bearing capacity problem for the non-homogenous soils.

4.5 Comparison of Different Approaches

Figure 14 compares the results of the current study calculated from different approaches including the limit equilibrium, limit analysis, and stress characteristics method with the results of a recent study by Ukritchon et al. [19]. They reported the value of bearing capacity factor (N_c) as a function of foundation roughness and heterogeneity of cohesion [19]. For this aim, the bearing capacity coefficient was compared for rough soil-foundation interface condition. Note that, the discrepancies between the results of the current study and those of Ukritchon et al. [19] are due to the discrepancies in optimization schemes. The lower bound finite element formulation of the current study lies in linearization of the yield surface while Ukritchon, et al. [19] employed the second-order cone programming scheme. The reported value of N_c is the ratio of ultimate bearing capacity calculated from only contribution of cohesion to the surface cohesion of soil, i.e. $Nc = q_u/c_0$, and $Nc = Nc_0$.

Tani and Craig (1995) conducted a series of Centrifuge model tests to validate the proposed bearing capacity calculation of circular foundation on heterogeneous marine clay. The circular model foundation had a radius of *304 mm* and thickness of *2 mm*, located at depth of *30 mm*. The model was rigidly connected to the strong-box of the Centrifuge apparatus to avoid any sliding or tilting. In the experimental test of Tani and Craig (1995), the vertical load was consequently increased with a constant rate up to the failure load, resulted in a considerable settlement [35]. Table 2 demonstrates a comparison of the results of numerical and experimental investigations of Tani and Craig (1995) with those obtained in this study. As can be seen, the bearing capacity factors of the current study are in good agreement with findings of Tani and Craig (1995).

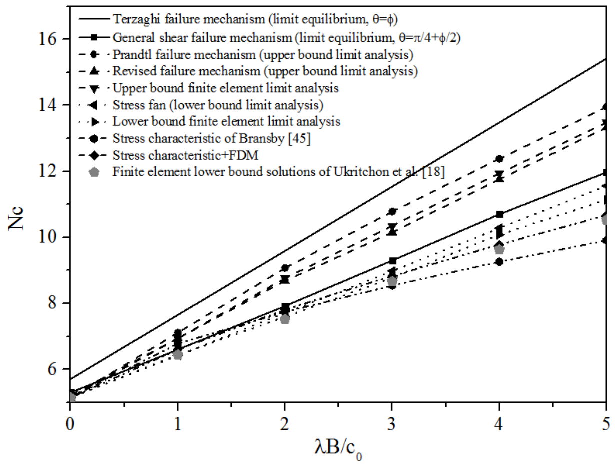

Fig. 14: Comparison of bearing capacity factor Nc of non-homogeneous soil with the results of Ukritchon et al. [19].

Table 2. Comparison between the results of the experimental and numerical bearing capacity factor of Nc_0.

Nc_0	$\lambda B/c_0$		
	2.13	4.07	5.38
Tani and Craig (1995)			
Theoretical	9.45	11.93	13.51
Experimental	8.12	10.16	12.77
Current study			
LE[1]- General shear failure mechanism	8.09	10.78	12.45
UP[2]- Prandtl failure mechanism	9.29	12.48	14.21
UP- Revised failure mechanism	8.87	11.86	13.38
UP- Finite element method	8.96	12.04	13.62
LB[3]- Stress fan	7.87	10.36	12.47
LB- Finite element method	7.74	10.15	12.31
Stress characteristics + FDM[4]	7.94	9.84	12.07

1- Limit equilibrium 2- Upper bound 3- Lower bound 4-Finite difference method

5. Conclusion

In this paper, different approximate methods including the limit equilibrium, limit analysis, and characteristic method, merged with the numerical techniques such as FEM and FDM were investigated to determine the ultimate bearing capacity of the purely cohesive nonhomogeneous soils in the plane strain condition. An effort was made to cover the problem by detailed investigations, evaluation, and comparison of the different methods. It can be useful to candidate the most appropriate method based on the accuracy required for the specific project. Accordingly, the following results were obtained in this study:

- Most of the approximate methods are iterative techniques; consequently, various possible failure mechanisms must be taken into account in the calculation of the collapse load so that the appropriate accuracy can be assured.
- Consideration of the shear strength variability can work reasonably well in simulating the real situation with naturally deposited clay. This leads to improvement of bearing capacity estimation because of the change in the failure zone pattern due to the shear strength increment with depth.
- Comparing the results obtained by different methods, the maximum bearing capacity was estimated by a considerable difference through the limit equilibrium approach, resulting in the most non-conservative design.
- In contrast, the characteristic method presented the most conservative estimations of bearing capacity in comparison to other approximate methods investigated in this study.
- The results of the Prandtl mechanism, revised failure mechanism, and finite element limit analysis are in good consistency with each other, suggesting the employment of the rather simpler mechanism of Prandtl in similar conditions as it demands a fewer computation effort.
- An explicit bracket of an exact solution can be achieved by the results of finite element upper and lower bound solutions. Therefore, it would be acceptable to estimate the bearing capacity using these two bounds and considering their mean value as the final solution.
- Putting into practice the finite element technique in the bound methods, the accuracy of the results can be enhanced compared to the conventional upper and lower bound solutions.

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