**Stress distribution and failures in partially overloaded support-removed flat slab floors**

**A. Mohammadi***, A. Pachenari**, B. Sadeghi**

**ARTICLE INFO**

Article history:
Received: February 2018.
Revised: May 2018.
Accepted: August 2018.

Keywords:
Reinforced Concrete Flat Slab; Column Removal Scenario; bar rupture; stress distribution; Near-collapse state.

**Abstract:**

Although concrete slabs have an extensive use in structures due to their architectural and executive benefits, the suitability of their behavior against the progressive collapse phenomenon has always been questioned. This study numerically investigates the step-by-step behavior of a support-removed flat slab floor with square panels under the effect of partial overloading. After validation of the modeling method, parts of the designed floor are exposed to increasing downward and uniformly distributed loading during three separate analyses that correspond to the removal of supporting corner, penultimate and interior columns. The pattern of stress in the slab reinforcement and propagation of cracks in the concrete are presented. The findings showed high concentration of slab damage around the corner columns located in the perimeter of overloaded panels and highlighted the role of slab add bars embedded in the vicinity of exterior columns against failure. It was also shown that, unlike the frame-type structural systems, stress redistribution occurs considerably along the diagonals of the slab panels directly connected to the failed support.

**1. Introduction**

Explosions, vehicle collisions, overloading, corrosion of materials and even fires can lead to the instability of structural elements and floors. Partial destruction of Ronan Point building due to a relatively small explosion raised the concerns for the possibility of replicating similar events and attracted significant attention from the research community to evaluate resistance of the structural systems against the progressive collapse phenomenon (Pearson and Delatte, 2005 [10]). The attack on WTC towers led to new series of research, particularly relying on notional column removal scenarios and more than ever revealed the necessity to provide alternative load paths in structures (Sadek et al., 2011 [16]).

Two-way slab floor systems along with architectural and executive benefits, have been widely favoured by structural designers because of their proper load distribution (Prasad and Hutchinson, 2014 [12]). However, occurrence of disasters such as the collapse of Pipers Row car park and the federal Murrah building, have motivated researchers to focus, more than ever, on the performance of the concrete flat slabs under abnormal loading (Osteraas, 2006 [8]; Russell, 2015 [14]). Muttoni, 2008 [5] addressed the issue of flat slabs failure in the laboratory, which has been considered heretofore by Hawkins and Mitchel, 1979 [3] and Mitchel and Cook, 1984 [4]. He emphasized the important effect of the slab reinforcements in preventing progressive collapse. Also Qian and Lee, 2015 [13] evaluated the crack formation pattern after an interior column loss by testing a scaled flat slab consisting of 4 panels and showed that, due to the low percentage of reinforcement and large ratio of bay length to slab thickness, the flexural collapse mode dominates primarily on the punching shear mechanism. The findings of the mentioned study strongly confirmed that the
pattern proposed by the yield-line theory for predicting the flexural collapse mechanism corresponds to reality. Subsequently, further studies were carried out to calculate the critical load of other flat slab collapse mechanisms including punching shear and local modes (Pachenari and Bagherzadeh, 2019 [9]).

In recent years, the development of computational tools and the dramatic growth in the finite element (FE) models have facilitated the course for a more accurate evaluation of the structures’ behaviour under support failure scenarios (Trivedi and Singh, 2013 [19]). For example, Pham et al., 2017 [11] studied the behaviour of beam and beam-slab sub-assemblages under different loadings after removing a column and showed that the lack of penultimate column could be even more critical than the corner column loss. Bredean and Botez, 2018 [2] investigated the resisting mechanisms of slabs in addition to studying the performance of beams, which are commonly considered in progressive collapse analysis. Russell et al., 2018 [15] evaluated the effect of two-way slab design parameters such as length, width, and thickness of panels as well as concrete compressive strength on the nonlinear behaviour considering different column removal locations. The study reported slab deflections and cracking pattern assuming that the panels were directly connected to the removed column subjected to a static overloading equal to the initial load on the entire floor.

Summary of previous studies indicates that stress distribution and steel bar failures in support-removed flat slabs even at relatively small deflections have not been addressed enough by researchers. The present work innovatively investigates the behaviour of such slabs at near-collapse state to pave the way for getting closer to a better performance against progressive collapse. Additionally, taking advantage of modelling features like bar rupture and concrete damage highly increases the credibility and quality of the results. For this purpose, in this study, first, a FE model is built in Abacus (Simulia, 2016 [18]) software to simulate a flat slab’s behaviour under a support failure scenario. The performance accuracy of the model is verified by using data of an existing experimental program. A case study on a conventionally designed slab floor is then conducted. Applying an increasing uniformly distributed load (UDL) to the panels adjacent to the removal location, the step-by-step behaviour is presented in 3 scenarios including interior, corner and penultimate column loss. Finally the pattern of stress and failures in reinforcement and gradual propagation of cracks in concrete are compared.

2. Description of case study structure and support removal scenarios

In order to evaluate the influence of the removed support location on the flat slab behaviour, a typical slab floor with 6 square panels with dimensions of 4000 mm by 4000 mm and the uniform thickness of 150 mm is designed according to Building Code Requirements for Structural Concrete ACI 318-14 (ACI, 2014 [1]) . As shown in Fig. 1, this floor contains 3 panels in the X-direction and 2 panels in the Y-direction and is located on 12 reinforced concrete (RC) square columns with dimensions of 500 mm. The slab overhang length in all edges is 100 mm. The design dead load includes the self-structural weight plus 1 kN/m² corresponding to the flooring weight and the live load is equal to 2.5 kN/m². Also, the floor is assumed to be restrained against lateral displacements meaning that lateral loads do not contribute to the design of the floor. The criteria of minimum reinforcement is observed in determination of slab steel reinforcement, by placing the minimum number of add bars. However, with respect to Table 1, the use of add bars in the vicinity of some columns is inevitable. Detail of upper layer of slab reinforcement has been shown in Fig. 2. In both directions, the lower layer has continuous rebar mesh of 12mm bars at 300mm centre to centre, the same as upper layer, but excludes add bars.

![Fig. 1: An overview of the geometry of case study structure (left) and mesh detail around column B1 (right)](image)

Table 1: Number of 2 meters length add bars at the top of the slab section

<table>
<thead>
<tr>
<th>Column location</th>
<th>B1,C1,B3,C3</th>
<th>B2,C2</th>
<th>A2,D2</th>
</tr>
</thead>
<tbody>
<tr>
<td>X-direction</td>
<td>2 bars</td>
<td>5 bars</td>
<td>-</td>
</tr>
<tr>
<td>Y-direction</td>
<td>-</td>
<td>5 bars</td>
<td>2 bars</td>
</tr>
</tbody>
</table>

The effect of support removal scenarios on the slab behaviour are implemented by defining 3 separate analyses named SC, SI and SP. The first letter S denotes square panels while the latter indicates the column removal location. In fact, the letters C, P and I correspond to the corner, penultimate and internal column loss scenarios,
respectively. The locations of the lost columns have been shown in Fig. 1.

![Fig. 2: Detail of upper layer of slab reinforcement. 10 mm diameter add bars were used.](image)

3. FE model

3.1 Material modelling

To evaluate the behaviour of RC flat slab after the column removal, a FE model was modelled in Abaqus software. The steel reinforcement was modelled with B31 elements. Each element contained two nodes and used a linear shape formulation. Hexagonal cubic elements with 8 nodes (C3D8R) were also used for modelling concrete elements. The element uses a linear shape function and reduced integration points to prevent shear locking problems under certain conditions. Moreover, it takes advantage of accuracy and economy in comparison with tetrahedral elements. The assumed concrete stress-strain curves are presented in Fig. 3 (Russell et al., 2018 [15]; Okamoto and Maekawa, 1991 [6]).

![Fig. 3: Concrete stress-strain curves: uniaxial compression (top), uniaxial tension (bottom)](image)

As can be seen in the compressive part, at first, the curve rises parabolically from the initial elastic area up to the point which corresponds to the concrete compressive strength (equal to 30 MPa). After this point, with respect to the concrete cracking, a descending branch is observed. Also, a linear elastic zone followed by a nonlinear softening model is used to model the concrete tensile behaviour.

Concrete damaged plasticity (CDP) model is used to consider the stiffness reduction after cracking. The tensile and compressive damage indices ($d_t$ and $d_c$) change from 0 for undamaged concrete to 1 for totally damaged state (Simulia, 2016 [18]). The value of these indices have been calculated according to Russell, 2015 [14] and presented in Fig. 4. Other required parameters for using this damage model were extracted from Russell's 2015 [14] and Oliver et al.’s 1989 [7] researches. For modelling steel bars, nominal stress-strain curve of Fig. 5 was used. Moreover, the bar rupture potential is taken into account by utilizing ductile damage model. The limit strain of 0.13 is regarded as rupture point. It is notable that the stress triaxiality limit values are defined by values between +0.3333 and −0.3333. The fracture energy is also set to zero in order to make bar rupture immediately occur at the moment in which plastic strain equals to 0.119 (Bredean and Botez, 2018 [2]).

![Fig. 4: Nominal stress-strain curve of steel bars](image)

![Fig. 5: Nominal stress-strain curve of steel bars](image)
The mesh dimension is one of the important parameters in matching the results of FE models with reality. In fact, on the one hand, the very large dimension of the elements diverts the results from the actual behaviour of the structure, and on the other hand, their very small dimension leads to prolonging the computation time. Although it is common to use elements with relatively equal dimensions in meshing process, the slab was meshed with elements lacking such a specification. This lowered the possibility of hourglassing phenomenon which can emerge in case of reduced-integration solid elements (Russell, 2015 [14]). In this way, Element length and width of 25 mm and height of 6.67 mm (along slab thickness) was used for modelling, which is the same as those used for validation of FE models. Cubic elements with a dimension of 25mm were used for the columns, however. Mesh detail around column B1 is visible in Fig. 1.

3.2 Boundary conditions

As seen in Fig. 1, the lower columns represent the columns connected to the ground. Thus, their bottom nodes are restrained against both rotation and translation. The top surfaces of 1.5 meters high upper columns (half of the floor height) are also modelled in a manner that only their rotational degree of freedom is kept free. It should be noted that tie bond is used at the interface of columns and the slab. Reinforcing bars of columns (16 bars with diameter of 25 mm) are continuous and pass through the slabs. The bond between steel and concrete was modelled by defining concrete as host region and steel as embedded region.

3.3 Loading

At the beginning of each analysis, a UDL is applied to the surface of all the slab panels. The loading value increases linearly from 0 to \( W_{ac} \) according to Eq. (1):

\[
W_{ac} = 1.2DL + 0.5LL
\]  

(1)

Where DL and LL are dead and live loads, respectively, and \( W_{ac} \) is the random load combination according to Progressive collapse analysis and design guidelines (GSA, 2013 [20]). After initial loading, an increasing UDL is applied only to the panels adjoining to the removed column and the behaviour of concrete slab and bars are evaluated.

3.4 Validation

A part of Russell’s 2015 [14] experimental program is used to validate the numerical models of this study. As seen in Fig. 6, the test included exertion of a uniform increasing load on the entire surface of a 2 panel slab located on 5 supports (lacking a column support at its left-hand panel corner). The deformations at the missing column location and the middle of the right-hand panel have been reported during the experiment. Regarding Fig. 7, good conformity is observed between the results of the experimental research and the numerical model, which approves the accuracy and convergence characteristics of the FE model. The results of sensitivity analysis indicate that an element size of 25x25x6.67 mm for the slab elements could be appropriate. It should be noted that after the cracking expansion stage in the laboratory specimen, stiffness reduction occurred suddenly. Conversely, the reported displacements in the numerical model gradually increased, which can be attributed to the gradual reduction of the plastic capacity in the model.

![Fig. 6: Details of Russell’s experimental model (Russell, 2015)](image)

![Fig. 7: Normalized displacements of the slab at different locations versus the applied distributed load in experimental research and the numerical model.](image)

4. Results and discussion

The relationship between UDL per unit area of the panels adjacent to the lost column and vertical displacement of the slab at the removed column location normalized to the slab thickness (called the load-displacement curve for short) can be drawn for each of the three cases SP, SI, and SC. Although this curve is presented only for case SP in Fig. 8, its general scheme is relatively similar in all other cases, i.e. there is an ascending branch followed by a descending area; however, the normalized peak load is 3.95 and 5.54 for models SC and SI, respectively. As the case SC has the smallest peak load, the loss of corner column seems to cause...
the most critical situation compared to other support failure scenarios.

In order to better understand the slab behaviour and present appropriate outputs, 7 points corresponding to 4 main and 3 subordinate stages are highlighted on the diagram. The four main stages are defined as follows:

- **Stage 1**: completion of initial loading on the surface of all panels according to equation (1). With respect to the curve slope, this stage almost coincides with the beginning of the concrete elements’ cracking.
- **Stage 3**: yielding of the first series of slab bars, which are specially located around the columns adjacent to the removal location.
- **Stage 4**: failure of the first series of slab bars.
- **Stage 7**: chain rupture of a significant number of slab longitudinal bars (in case SC) or all the reinforcement around a column(s) adjacent to the removed column (in cases SI and SP). This stage is accompanied by very large deformations in the slab, that is, the slab is in near-failure state.

Furthermore, the result of the analyses are presented in 3 subordinate stages:

- **Stage 2**: a state between the first and third stages in the load-displacement curve, which is defined to report the expansion of cracks.
- **Stages 5 and 6**: These stages lie between the main stages 4 and 7 and are used to illustrate the order of occurrence and expansion of failure in the slab bars.

It is worth saying that the first 3 stages are used to indicate cracks in the concrete elements, whereas the last 5 stages are applied to show the status of the bars.

**Fig. 8**: Cracks in Slab bottom surface at the end of loading: predictions of the numerical model (top), crack pattern in the laboratory test (bottom) (Russell, 2015)

### 4.1 Pattern of concrete cracking

**a) Model SC:**

Fig. 9 shows variation of the concrete plastic strains for the model corresponding to removal of corner column A1 at the predefined stages of 1, 2 and 3. At the end of stage 1, there are slight cracks on the slab top surface just around the two columns A2 and B1, and also no cracking is seen on the bottom surface of the slab. By increasing applied loads and reaching stage 2 in the load-displacement curve, not only do the cracks develop around the same columns on the slab top surface, but also some cracks are initiated on the bottom surface. As the first series of bars reach yielding stress (stage 3 in the load-displacement curve), the top surface cracks join together at both continuous edges of the A1-B2 panel (the panel whose diagonal passes through two columns A1 and B2) while the bottom surface cracks have been diagonally propagated from discontinuous edges toward column B2.

**b) Model SP:**

In stage 1 of this model, corresponding to the loss of penultimate column B1, slight cracks are created on the slab top surface at the adjacency to the removed column’s adjacent supports but no cracks are initiated on the slab bottom surface. In stage 2, the cracks not only spread on the slab top surface but also form gradually on the slab bottom surface. However, regarding the slab continuity on the right side of the panel B1-C2, more limited positive cracks (due to the positive bending moment) are found in this panel compared to the end panel A1-B2. Significant crack growth occurs at the bottom and top surfaces of the slab by increase in the vertical displacements at the removed column location.
and reaching Stage 3. Fig. 10 shows the plastic strains in the concrete elements in model SP during stages 2 and 3.

c) Model SI:
The cracking pattern in stages 1 to 3 of the SI model (corresponding to removal of the internal column B2) is somewhat similar to the models SP and SC, but as shown in Fig. 11, the created cracks look more extensive in stage 3 of the SI model. It should be noted that in this model, most of the top surface cracks either grew in the vicinity of the columns (at the discontinuous edges) or stretched almost in parallel to the longitudinal reinforcement (at the continuous edges). Also, the formed cracks at the bottom surface of the slab (due to positive bending moments) run diagonally between the remaining columns. However, the cracks have not exceeded the overloaded panels. It should be mentioned that the damage severity and the cracks expansion at the slab bottom surface in the continuous section seems lower than the discontinuous section.

4.2 Stress pattern and failures in steel bars

Determination of the stress distribution pattern in slab bars located in the vicinity of removal location is of crucial importance because it not only shows the areas where the reinforcements start to fail and how to develop, but also, finally depicts a schema of some more vulnerable areas in the slab after loss scenarios. Hence, the reinforcement Stress states are evaluated in successive stages 3 to 7 of each numerical model.

a) Model SC:
As shown in Fig. 12, in the third stage, stress in some slab bars around columns A2 and B1 has reached yielding. Thereafter, the tensile strain in the bars increases until in stage 4, three bars (including two add bars and one longitudinal bar) near column B1 and one add bar near column A2 rupture simultaneously. By continuing the loading process, not only does the number of the ruptured bars provided at the top layer increase, but also failures begin around column B1 in the bottom layer reinforcement. In stage 6, rupture of a greater number of reinforcements occurs (in both top and bottom layers) at the location of column A2 as well as column B1. It should be considered that failures in reinforcement follow a relatively specific pattern in these moments: “a bar at the top and a bar at the bottom at the same location, fail consecutively. The trend continues and subsequent bars are torn like an opening zipper”. Finally, in stage 7, the failures in a lot of bars, in both top and bottom layers, and also at both locations of columns A2 and B1 are reported, and the slab is in near-failure state.
Fig. 12: Patterns of stress and failure in bars during different stages of model SC

b) Model SP:
Fig. 13 demonstrates the patterns of stress and failure in the slab bars in different stages of the model SC, starting from the moment of the reinforcement yielding around the columns A1, C1 and B2 (corresponding to stage 3 in the load-displacement curve). By increasing the bars’ tensile strain in stage 4, the first series of bars get ruptured, including two top longitudinal bars around column A1 and one add bar around column C1. In stages 5 and 6, although none of the bars around column B2 have been torn, more bars are ruptured in both top and bottom layers of slab reinforcement in the vicinity of columns A1 and C1. In stage 7, the slab remaining bars around the corner column A1 (an area located on the discontinuous edge of the overloaded panels of the slab which lacks add bars) are torn and the column is completely separated from the slab. This provides the basis for the collapse of the flat slab. In summary, at the end of this stage, the areas around column C1 are in worse condition compared to the areas around column B2 because a greater number of its bars have ruptured. In addition, since slab top add reinforcement, like the previous model, have been among the first group of yielded or ruptured bars, it seems that the increase of add bars in the slab design plays a useful role in better performance against the support failure scenarios.
Table 2 summarizes the stress status for the flat slab bars around several columns at different stages of the model SI. As seen, the bar rupture starts in an add bar around the column A2 (adjacent and coaxial to the removal location) and then propagates to other areas of the slab in the next stages. Although at first glance, the load-bearing mechanisms of the flat slabs due to an internal column loss, may look quite similar to the frame systems, there are also differences. In the frames, it is expected that only columns adjacent and coaxial to the removed column have a significant share in redistribution of forces and subsequent damage will mainly occur in beams bridging over the missing columns (Sagiroglu, 2012 [17]). However, examining the stress patterns in flat slab bars at different stages of analysis in model SI shows that, due to the absence of beams, the missing column load share is considerably transferred along the diagonal of the slab panels (for brevity, only stages 4 and 7 have been shown in Fig. 14). It is worth saying that a similar result is also true for the cases of the corner and interior columns loss (see Fig. 12).

Failures in all the slab bars around a specified column can be somehow considered as a basis for the completion of the punching shear occurrence, and thus providing the field for collapsing a part of the slab. At the end of stage 7, this occurred practically around external columns A1, A2 and A3 that lacked or had two add bars only in the Y-direction and were located on the discontinuous edges of the overloaded panels. Therefore, it can be inferred that providing the exterior slab panels with add bars in the vicinity of external columns can be effective in total collapse behaviour of support-removed flat slabs.

Fig. 13: Stress and failure in bars during different stages of model SP
### Table 2: Summary of stress status of flat slab bars at different locations in model SI

<table>
<thead>
<tr>
<th>Stage</th>
<th>Around Column A1</th>
<th>Around Column A2</th>
<th>Around Column A3</th>
<th>Around Column B1</th>
<th>Around Column B3</th>
<th>Around Column C2</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>- YL of T Bars</td>
<td>- R of a Bar from TM in X.</td>
<td>- YL of T Bars</td>
<td>- R of 2 Bars from TM in Y.</td>
<td>- R of 2 Bars from BM in Y.</td>
<td>- R of 2 Bars from TM in X.</td>
</tr>
<tr>
<td>7</td>
<td>- R of a Bar from BM in X.</td>
<td>- R of 2 Bars from BM in Y.</td>
<td>- R of an add. Bar in Y.</td>
<td>- R of 3 Bars from TM in X.</td>
<td>- R of a Bar from TM in Y.</td>
<td>- R of 2 Bars from BM in Y.</td>
</tr>
</tbody>
</table>


**Fig. 14:** Stress and failure of bars in stages of 4 and 7 of model SI
5. Conclusion

This paper investigated the step-by-step cracking and stress distribution in a support-removed flat slab. The floor was designed and partially overloaded under the effect of three separate support failure scenarios (removal of corner, penultimate and interior columns) until the near-collapse state was reached. To this end, at first, the load-displacement curve and the cracks pattern formation of an existing laboratory test were simulated by a finite element model in the Abacus software; a similar numerical model was then developed for the designed slab. Based on the results presented in this article, the following conclusions were drawn:

- According to the load-displacement curves obtained, the slab stiffness starts to decrease after the initial elastic stage due to crack formation. The negative cracks on the slab top surface at the continuous edge of the overloaded area form approximately parallel to the longitudinal reinforcement and also among the columns, whilst at the discontinuous edges, some cracks tend to grow around the supporting columns. Moreover, fewer positive cracks are seen on the slab bottom surface at the continuous edge of the slab rather than the discontinuous edges.

- The case SC has the smallest peak load, and the loss of corner column seems to cause the most critical situation compared to other support failure scenarios. However, in the model corresponding to the internal column loss, the formed cracks and subsequent concrete damage at yielding of the first series of slab bars look more extensive in comparison with the other 2 models.

- Stress state monitoring of flat slab bars at successive analysis stages shows that the missing column load share is significantly transferred along the diagonals of the slab panels, particularly in the model corresponding to the removal of the interior column. In other words, the rupture of slab bars near columns adjacent but not coaxial to the removed column provides evidence that unlike the frame-type structural systems, these columns also have a key role in redistribution of forces.

- Rupture of all the slab bars occurs around some columns located in the discontinuous edge of the overloaded panels. The slab floor lacks or has few add bars just in these locations. Therefore, it can be inferred that providing the exterior slab panels with add bars in the vicinity of external columns affect the destructive punching shear mode and also consequently, the total collapse of the flat slabs due to support failure scenarios.

References
